



ELSEVIER

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Temporal distribution of recorded magnitudes in Serbia earthquake catalog



Srđan Kostić^{a,b}, Nebojša Vasović^c, Matjaž Perc^{d,e,f,*}

^a Department of Geology, University of Belgrade, Faculty of Mining and Geology, Đušina 7, 11000 Belgrade, Serbia

^b University of Banja Luka, Faculty of Mining, Save Kovačevića bb, 79000 Prijedor, Republic of Srpska, Bosnia and Herzegovina

^c Department of Applied Mathematics, University of Belgrade, Faculty of Mining and Geology, Đušina 7, 11000 Belgrade, Serbia

^d Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, SI-2000 Maribor, Slovenia

^e Department of Physics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia

^f CAMTP – Center for Applied Mathematics and Theoretical Physics, University of Maribor, Krekova 2, SI-2000 Maribor, Slovenia

ARTICLE INFO

Keywords:

Statistical seismology
Earthquakes
Nonlinear time series analysis
Surrogate data testing
Autocorrelation
Temporal distribution

ABSTRACT

We focus on earthquakes that were recorded in Serbia between 1970 and 2011 within shallow parts of the Earth's crust, having local magnitudes from the 1.2–5.8 interval. The main goal of the performed analysis is to examine whether the temporal sequence of these recorded magnitudes exhibits some deterministic pattern or whether it simply represents a series of random events. For this purpose, the temporal distribution of earthquake magnitudes above the magnitude of completeness is analyzed by means of nonlinear time series analysis and surrogate data testing, as well as by means of the autocorrelation function. Piece-wise low cross-prediction errors, with 75% of segment pairs having the error smaller than its average value, indicate stationary properties of the examined sequence. Results of surrogate data testing indicate high zeroth-order prediction error that is independent of prediction time for the original dataset and 20 different surrogates, implying that the observed magnitude sequence is a series of independent random events drawn from some fixed but unknown distribution. These findings are supported further by a low value of the determinism factor for an earthquake treated as a system with four degrees of freedom (epicentral latitude and longitude, hypocentral depth and magnitude). The randomness in observed data is indicated further by the properties of the autocorrelation function, whose values for different time lags fall within the 95% confidence limit without an apparent pattern.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

Basic statistical properties of seismicity are implied by a special temporal pattern of earthquake occurrence along a single fault or a fault segment (i.e. recurrent events) and by spatial and temporal distribution of earthquakes recorded in one tectonic (seismic) area (i.e. interoccurrent events), which are typically examined by analyzing the corresponding earthquake catalogs [1]. Extensive seismological studies of these seismic databases have shown that temporal distribution of earthquakes in one region usually follows a discrete Poisson distribution, indicating temporal independence of the recorded seismic events [2,3]. This time-independent occurrence is a prominent feature of large earthquakes, which are assumed to occur

* Corresponding author at: Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, SI-2000 Maribor, Slovenia.

E-mail addresses: srdjan.kostic@rgf.bg.ac.rs (S. Kostić), nvasovic@rgf.bg.ac.rs (N. Vasović).

as a stationary Poisson process inside a specific region [4–7]. Besides the assumption of Poisson distribution, some authors also propose non-Poisson models, which are more consistent with underlying physics and take into account the occurrence history, like Markov processes [8]. Another frequent hypothesis on temporal seismic distribution relies on the assumption that magnitudes of all the seismic events (including large events, foreshocks and aftershocks) are independent random variables, which is the main starting point of a widely used epidemic-type aftershock sequence (ETAS) model. This ETAS model describes the space–time magnitude distribution of earthquake occurrences, by presuming that the squared distance between an aftershock and its triggering event follows a Pareto distribution [9]. Following the same assumption of earthquakes as random events, Ben-Naim et al. [10] showed that the series of recorded earthquakes is consistent with a random process for magnitudes in the range $M \in [7.0, 8.3]$.

In contrast to aforementioned models of earthquakes as predominantly independent events, there are certain claims of periodic, quasi-periodic and chaotic temporal distribution of recorded earthquakes, as a result of extensive analyses in the area of nonlinear dynamics and chaos theory [11,12]. Supporting this point of view, Beltrami and Mareshal [13] tried to reconstruct the strange attractor for the earthquake time series recorded in the Parkfield seismic region between 1969 and 1987. They came to ambiguous results – either this series cannot be distinguished from a random one, or it has a strange attractor with dimension higher than 12. Matcharashvili et al. [14] found evidence of low-dimensional attractor for earthquakes in Caucasian region by using the inter-event times between successive events. Tiwari et al. [15] applied a nonlinear forecasting approach in a reconstructed phase space of earthquake frequency in the Central Himalayan Region. Results of their studies indicated a low positive correlation between predicted and observed data suggesting that the earthquake dynamics in this area is characterized by a mix of stochastic and chaotic behavior.

Having in mind these previous divergent evidences and assertions on temporal distribution of seismic events, we apply a series of tests in order to examine whether there is some underlying pattern of temporal distribution of earthquake magnitudes recorded in Serbia, between 1970 and 2011. The research is done by applying the methods of nonlinear time series analysis [16], which were previously rarely used in the field of seismology [17], even though they were successfully applied in many other fields of geophysics [18,19].

The scheme of the paper is as follows. Seismic activity in Serbia is described in Section 2, while the applied methods are detailed in Section 3. The obtained results are presented in Section 4, while in the last section we give a brief discussion on the applied methods and obtained results, with suggestions for further research.

2. Seismic activity in Serbia

According to Advanced National Seismic System composite earthquake catalog (ANSS), hosted by Northern California Earthquake Data Center [20], 757 earthquakes of local magnitudes $M_L \in [1.2, 5.8]$ were recorded in Serbia between 1970 and 2011 (Fig. 1). In this period only four moderate earthquakes of local magnitudes $M_L \in [5.2, 5.8]$ were recorded, with epicenters located at a wider area of Kopaonik, Mionica, Trstenik and Kraljevo. One could note from Fig. 1 that the major seismic activity in this period was caused by the fault motion in west/northwest-east/southeast direction, due to compression along the contact of Adriatic table and Dinarides, on one hand, and extension generated by the regressive roll-back of the subducted lithosphere in Carpathian zone, on the other hand [21,22]. Majority of earthquakes in this period was recorded during 2002 (Fig. 2) with most frequent magnitude of 2.7 (Fig. 3a). Hypocentral depth was less than 40 km, with the most frequent value of 10 km, implying that only shallow seismic events were registered in the observed period (Fig. 3b).

3. Applied methods

In present paper, we analyze temporal distribution of earthquake magnitudes recorded in Serbia between 1970 and 2011, because there are no instrumental recordings of earthquakes before 1970. Since the observed seismic data set contains many earthquakes with magnitude under the completeness of the catalog, it means that the corresponding analysis would be missing many low magnitude earthquakes, which could likely affect the results. In other words, a first and compulsory step in our analysis would be to calculate a magnitude of completeness M_c , as the lowest magnitude at which 100% of the earthquakes in a space–time volume are detected [23]. In present paper, magnitude of completeness was calculated in ZMAP software [24], by applying Maximal Curvature technique, as a catalog-based method to assess M_c . This technique represents fast and straightforward way to estimate M_c and consists in defining the point of the maximum curvature by computing the maximum value of the first derivative of the frequency–magnitude curve. The advantages of applying this technique are its easy applicability and the fact that it requires fewer events than other techniques to reach a stable result [25].

After determining the magnitude of completeness, a series of main shocks, without foreshocks and aftershocks, with local magnitude equal or larger than M_c is examined by the means of nonlinear time series analysis. In order to conduct this analysis, we had to embed the observed scalar series into the appropriate phase space via the Takens embedding procedure [26] by using the open-source software [27]. The optimal embedding delay is calculated using average mutual information method [28], while the minimum embedding dimension is examined by the method of false nearest neighbors [29], considering that two points are false neighbors if the normalized distance between their embedding coordinates is larger than a given threshold (R_{tr}). According to [29], the value of $R_{tr} = 10$ proves to be a good choice for most data sets.

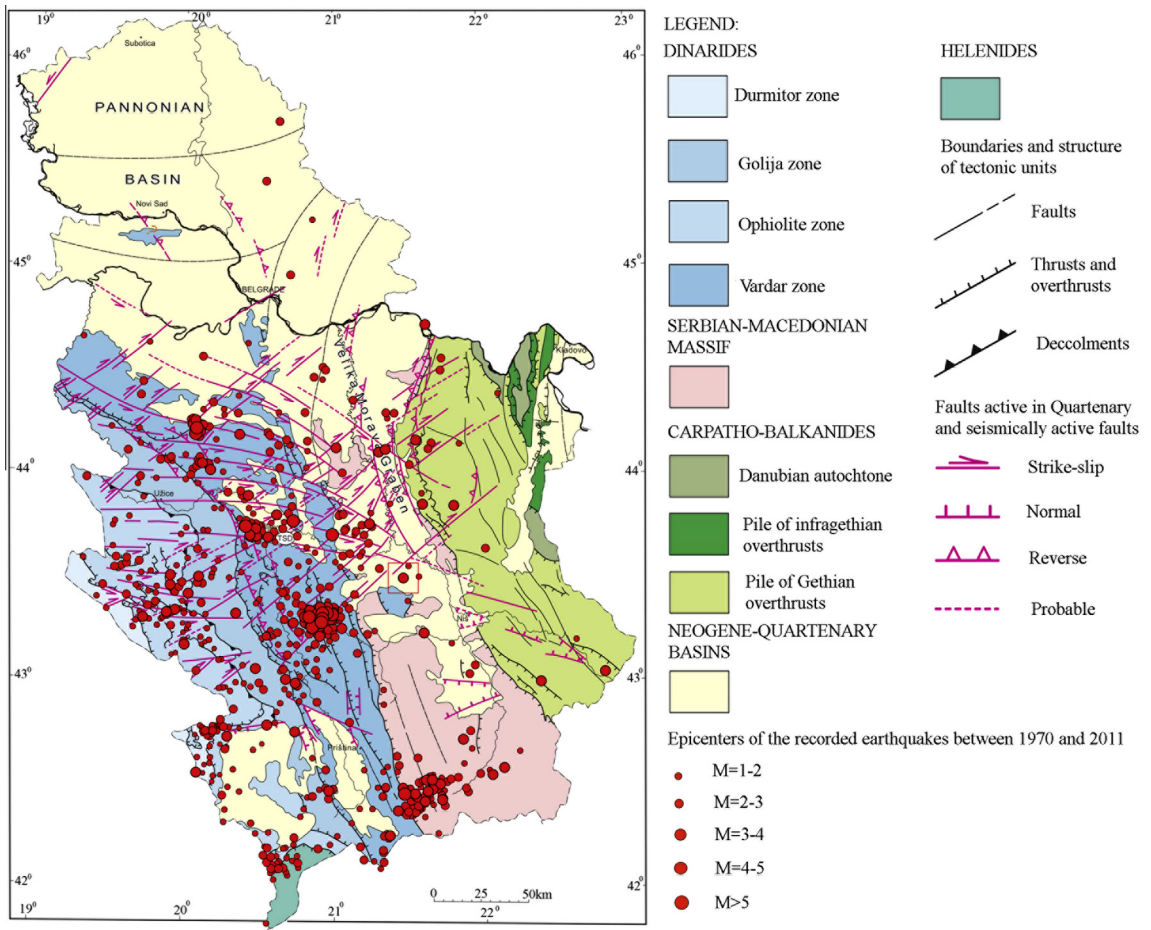


Fig. 1. Seismicity of the territory of Serbia, for the period 1970–2011, with tectonic map including major tectonic units and distribution of seismically active faults. It is clear that central and western part of Serbia represent seismically active areas, whereas eastern and, particularly, northern part of Serbia are not prone to frequent occurrence of earthquakes.

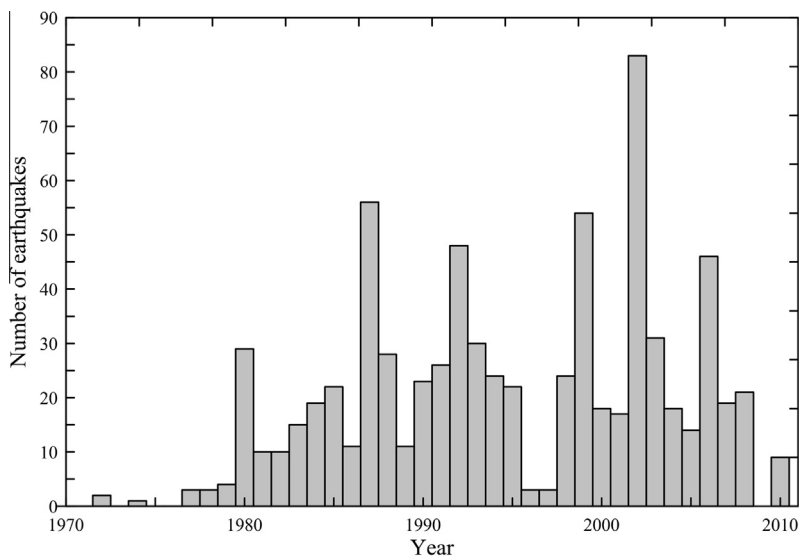


Fig. 2. Annual number of earthquakes in Serbia, recorded between 1970 and 2011. Maximum number of earthquakes is recorded in 2002 (83), 1987 (56), 1999 (54), 1992 (48) and 2006 (46).

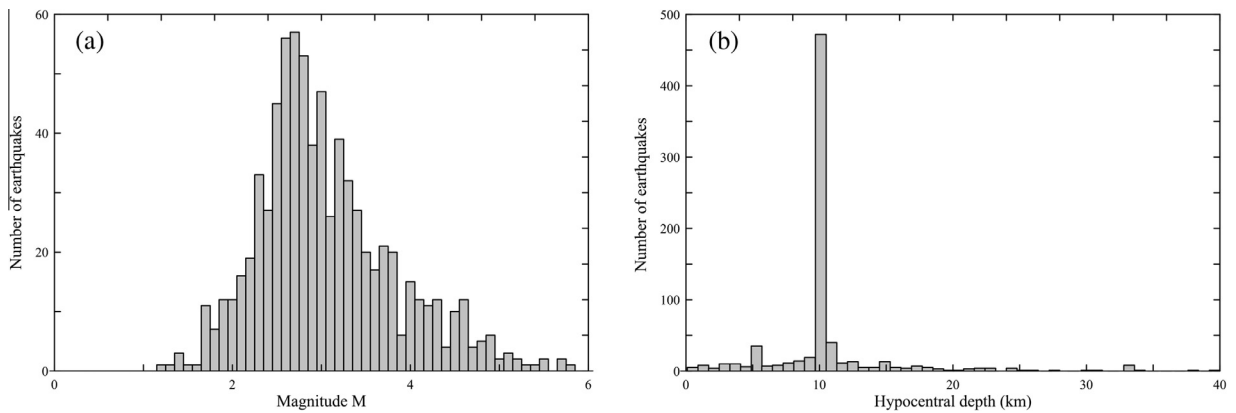


Fig. 3. Number of recorded earthquakes between 1970 and 2011 as a function of: (a) magnitude; (b) hypocentral depth. Hypocenter of the largest number of earthquakes is at 10 km, with the most frequent magnitude of 2.7.

With the observed series properly reconstructed in phase space, we were able to conduct a stationarity test [16], which is a necessary prerequisite for a random dataset. This test is based on the quality of prediction of unknown data value, using similar events happened in the past (neighboring points). For each point of the equally sized non-overlapping segment i at time t , predictions of the value of an unknown data are performed in the segment j at time $t + \Delta t$. Afterwards, the accuracy of obtained predictions is evaluated by calculating the average prediction error δ_{ij} , which is repeated for all combinations of i and j . The resulting high prediction error δ_{ij} is a clear indicator that the stationarity requirements in the examined series are not fulfilled.

As a next step in our analysis, we conducted the surrogate data testing, by assuming that the observed data are independent random numbers drawn from some fixed but unknown distribution [30]. For this purpose, we generated 20 surrogates, as already proposed in [16], using Matlab toolkit MATS developed in [31]. Then, in order to compare the original data and generated surrogates, we calculated the zeroth-order prediction error ε [30], according to the algorithm in C suggested in [16]. If this error for the original dataset (ε_0) is smaller in comparison to the calculated error for surrogate data (ε), then a null hypothesis can be rejected. On the other hand, if $\varepsilon_0 > \varepsilon$ at any instance of the test, the null hypothesis cannot be rejected. Usually, more than one wrong result out of 20 is not considered acceptable [16].

As a final step in the performed time series analysis, we applied a determinism test [32]. This test is based on the assumption that if a time series originates from a deterministic process, it can be described by a set of the first-order ordinary differential equations, whose vector field consist solely of vectors that have unit length. In other words, if the system is deterministic, the average length of all directional vectors κ will be 1, while for a completely random system, $\kappa \approx 0$ [32]. The calculation was also done by using the open-source software presented in [27]. Finally, results of nonlinear time series analysis were additionally verified by calculating the autocorrelation function [33].

4. Results

Application of Maximal Curvature technique has shown that the function of cumulative number of recorded earthquakes against their magnitude abruptly changes its direction at the value of magnitude of completeness, $M_c = 2.7$ (Fig. 4). In present study, only the earthquakes with local magnitudes over the magnitude of completeness are taken into account for inquiry of the possible randomness in the observed dataset, in order to exclude possible artifacts in any kind of analysis. In this way, the observed dataset is made even shorter (512 events), but it better preserves the physics of the phenomenon under study.

Regarding the optimal values of embedding parameters, the performed analysis showed that mutual information takes the first local minimum for $\tau = 3$ (Fig. 5), while fraction of false nearest neighbors rises with the increase of embedding dimension, which could indicate random signature in the input data. However, in order to be able to perform stationarity and determinism test, we have chosen $m = 4$ as an optimal embedding dimension, since earthquakes as macroscopic events have four degrees of freedom (epicentral latitude and longitude, hypocentral depth and magnitude).

In order to question the stationarity of the magnitude distribution under study, the original dataset is divided into short series, each occupying approximately 10 points. In that way, a total of 52 segments is obtained, with 52^2 possible combinations to evaluate the statistics. The color of each map segment indicates the cross-prediction error of using segment i as the neighbor source for making predictions in segment j . The average cross-prediction errors for all possible combinations of i and j are presented in Fig. 6, from which it is clear that the piece-wise low cross-prediction error prevails (green to blue color), confirming that the random process underlying the generation of the earthquakes did not change over the observed period. Approximately 75% of segment pairs exhibits cross-prediction error $\delta_{ij} < 0.93$, which is the average value of the observed error range [0, 1.78].

In order to test the null hypothesis that the data are independent random numbers drawn from some fixed but unknown distribution, 20 surrogates are generated by randomly shuffling the data (without repetition), thus yielding surrogates with

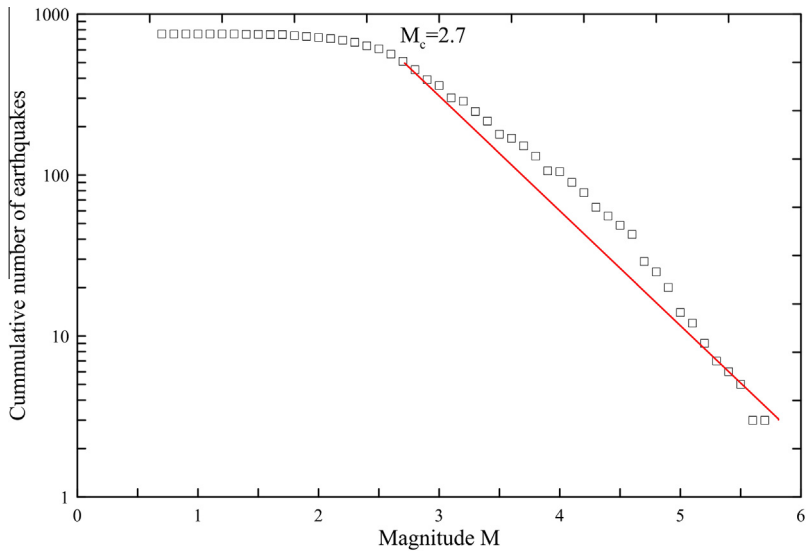


Fig. 4. Cumulative number of earthquakes versus recorded magnitudes for the overall catalog. It is obvious that magnitude of completeness (M_c) is equal to 2.7, where the function of cumulative number of recorded earthquakes against their magnitude abruptly changes its direction. Only the earthquakes with local magnitude equal or higher than magnitude of completeness are taken into account for the performed analysis.

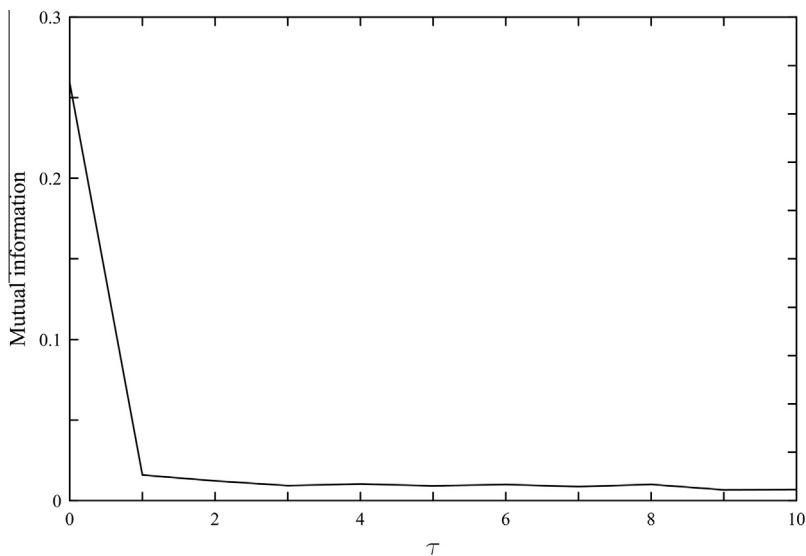


Fig. 5. Determination of the proper embedding delay – mutual information has the first minimum at minimum embedding delay $\tau = 3$, meaning that the values of the recorded magnitudes at dimensionless discrete time units, $t + 3, t + 6, \dots$ (depending on the value of optimal embedding dimension) could be used for the embedding of the observed dataset in phase space.

exactly the same distribution yet independent construction. Then the zeroth-order prediction error is calculated for the original dataset (ε_0) and for each of the 20 generated surrogates (ε). It is clear from Fig. 7 that ε_0 is well within ε in all the cases, so the null hypothesis cannot be rejected.

In order to further exclude the possible deterministic signatures in the observed dataset, we applied determinism test by coarse-graining the embedding space into 26 boxes in one dimension, which is a half of the maximum number of boxes, due to the relatively small data set (further coarse-graining would give spurious results). Only those boxes visited more than one time by the trajectory are included in the analysis. The obtained value of determinism factor, $\kappa = 0.76$, indicates possible randomness in the sequence of recorded earthquake magnitudes.

As an additional test for hypothesis of random magnitude series, apart from nonlinear time series analysis, we examined the autocorrelations of the observed dataset, in order to find any possible repeating pattern, which would indicate a deterministic signature. However, as it is shown in Fig. 8, the majority of autocorrelations fall within the 95% confidence limits (dashed lines), without the apparent pattern, which we expect to see if the data are random. A few lags slightly outside

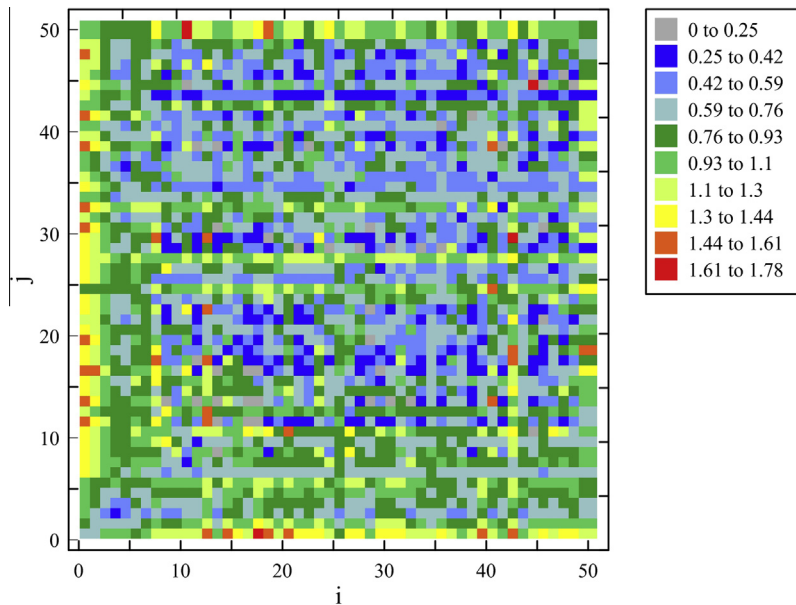


Fig. 6. Stationarity test. The whole time series is partitioned into 52 non-overlapping segments each occupying approximately 10 data points. The color map displays average cross-prediction errors δ_{ij} in dependence on different segment combinations (i, j) . The obtained results indicate that the piece-wise low cross-prediction error prevails (green to blue color), confirming stationarity in the observed dataset. This further corroborates the randomness in magnitude distribution, since random series have relatively uniform statistical characteristics over time. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

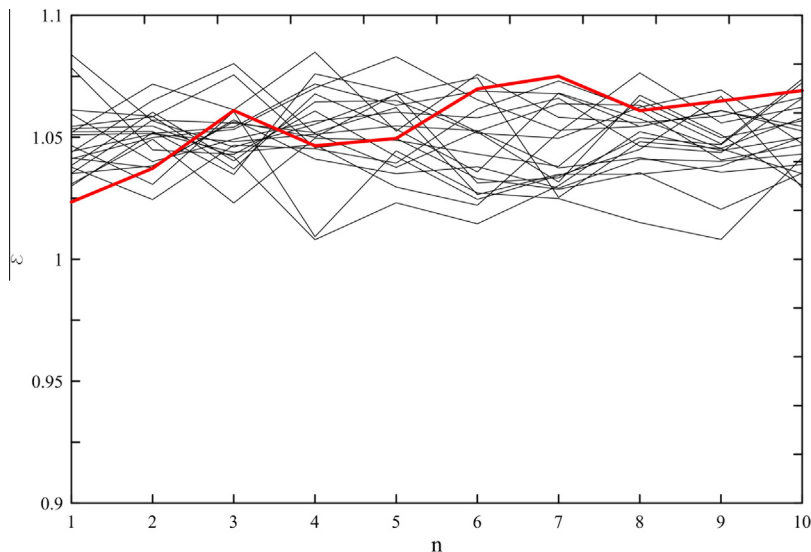


Fig. 7. Surrogate data test for the hypothesis that the data are independent random numbers drawn from some fixed but unknown distribution. Red line denotes the zeroth-order prediction error for the original series and black lines – zeroth-order prediction error for the surrogates, for n prediction units. The obtained results indicate that the error for the original dataset (ϵ_0) is within the error for surrogate data (ϵ), following that the null hypothesis could not be rejected. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

the 95% confidence limits do not necessarily indicate non-randomness, since we might expect approximately one out of twenty lags to be statistically significant due to random fluctuations [33].

5. Discussion

Results of the performed analyses indicate random distribution of earthquake magnitudes between 1970 and 2011 in Serbia, which might put under suspicion the possibility of deterministic feature in dataset of this type, including the plausible chaotic behavior, previously claimed by some authors [14,15]. Moreover, the absence of determinism could question

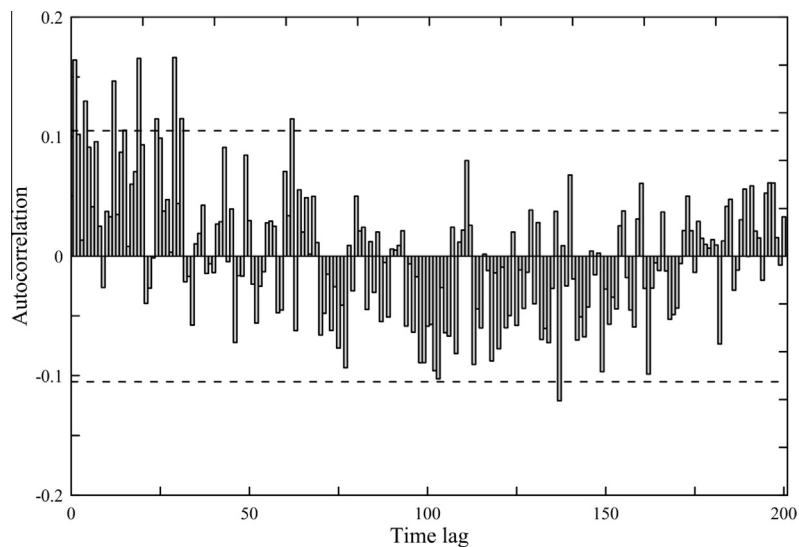


Fig. 8. Autocorrelation plot for temporal magnitude distribution. It is clear that the majority of the autocorrelations fall within the 95% confidence limits (dashed lines), which indicates possible randomness in the observed dataset.

the reliability of prediction that is solely based on temporal distribution of recorded main shocks within a specific region. However, before reaching the general conclusion, one should bear in mind that the results of this analysis are valid only for the seismicity detected on the territory of Serbia, which represents an area of weak to moderate seismicity with rare and medium-size events, as it was already stated in Section 2. The question of possible determinism in earthquake magnitude distribution still remains open for the seismic active areas, with more frequent events of greater magnitudes.

One should be aware of the fact that the analysis of a small data set could lead to ambiguous results. However, regarding the research on dynamics of the recorded earthquakes, the analysis of short series is not an exception. For example, De Santis et al. [18] also considered limited number of data (782 earthquakes) and showed that the seismic sequence of foreshocks culminating with the $M_w = 6.3$ main shock on April 6, 2009 in L'Aquila (Central Italy) evolved as a chaotic process, by using the method based on the Accelerated Strain Release analysis in time and on the nonlinear approach in a reconstructed phase space.

Another important issue that has to be emphasized is the analysis of the relatively short period of seismicity, which is not a standard approach in seismological research within a specific area. Usually, the recurrence time of great earthquakes is taken as an optimal period (100 years in Serbia). Nevertheless, previous studies on seismicity in different areas also considered relatively short time period of the observed seismicity, including the analysis of seismicity in Pakistan for the similar period, 1973–2008 [34] or principal component and cluster analysis of seismicity in Iran for the period 1957–2006 [35].

Regarding the reliability of the applied techniques and methods, it has to be emphasized that the method of false nearest neighbors did not give the specific value of optimal embedding dimension. Instead, it showed the increase of the percentage of false neighbors with the increase of embedding dimension, which could intuitively lead to a conclusion of a random magnitude distribution. However, since the specific value of embedding dimension is needed for stationarity and deterministic test, its value had to be assumed, which, in this case, was chosen to be equal to the number of degrees of freedom for earthquakes ($m = 4$). We believe this approximation did not affect the results obtained by nonlinear time series analysis in any significant way, which was additionally verified by surrogate data testing and analysis of autocorrelations.

In order to further examine the temporal distribution of recorded earthquakes, it would be interesting to separately investigate two different features (if possible for the area of higher frequency of earthquakes): interoccurrent events, for the whole area, and recurrent events, which appear only along a single fault. In that way, by comparing these events, and, in the same time, by confronting the results of the research in different seismic areas, we could determine the general nature of the earthquake magnitude distribution.

Acknowledgments

This work is partly supported by the Ministry of Education, Science and Technological development of the Republic of Serbia (Contract No. 176016 and 171017) and by the Slovenian Research Agency (Program P5-0027). Special thanks go to M. Toljić from University of Belgrade regarding the quality of Fig. 1.

References

- [1] Y.Y. Kagan, Statistical distributions of earthquake numbers: consequence of branching process, *Geophys. J. Int.* 180 (2010) 1313–1328.
- [2] J.K. Gardner, L. Knopoff, Is the sequence of earthquakes in Southern California, with aftershocks removed, Poissonian?, *B Seismol. Soc. Am.* 64 (1974) 1363–1367.
- [3] A. Corral, Long-term clustering, scaling, and universality in the temporal occurrence of earthquakes, *Phys. Rev. Lett.* 92 (2004) 108501.
- [4] Y. Ogata, Statistical models for earthquake occurrence and residual analysis for point processes, *J. Am. Stat. Assoc.* 83 (1988) 9–27.
- [5] A.J. Michael, Random variability explains apparent global clustering of large earthquakes, *Geophys. Res. Lett.* 38 (2011) L21301.
- [6] T. Parsons, E.L. Geist, Were global $M \geq 8.3$ earthquake time intervals random between 1900 and 2011?, *Br Seismol. Soc. Am.* 102 (2012) 1583–1592.
- [7] P.M. Shearer, P.B. Stark, Global risk of big earthquakes has not recently increased, *Proc. Natl. Acad. Sci. USA* 109 (2012) 717–721.
- [8] E. Garavaglia, R. Pavani, About earthquake forecasting by markov renewal processes, *Method. Comput. Appl.* 13 (2011) 155–169.
- [9] Y. Ogata, Space-time point-process models for earthquake occurrences, *Ann. Inst. Stat. Math.* 50 (1998) 379–402.
- [10] E. Ben-Naim, E.G. Daub, P.A. Johnson, Recurrence statistics of great earthquakes, *Geophys. Res. Lett.* 40 (2013) 3021–3025.
- [11] K.M. Scharer, G.P. Biasi, R.J. Weldon, T.E. Fumal, Quasi-periodic recurrence of large earthquakes on the southern San Andreas fault, *Geology* 38 (2010) 555–558.
- [12] D.R. Shelly, Periodic, chaotic, and doubled earthquake recurrence intervals on the deep san andreas fault, *Science* 328 (2010) 1385–1388.
- [13] H. Beltrami, J. Mareshal, Strange seismic attractor?, *Pure Appl Geophys.* 141 (1993) 71–81.
- [14] T. Matcharashvili, T. Chelidze, Z. Javakhishvili, Nonlinear analysis of magnitude and interevent time interval sequences for earthquakes of the Caucasian region, *Nonlinear Proc. Geophys.* 7 (2000) 9–19.
- [15] R.K. Tiwari, S. Sri Lakshmi, K.N.N. Rao, Characterization of earthquake dynamics in northeastern India regions: a modern nonlinear forecasting approach, *Pure Appl. Geophys.* 161 (2004) 865–880.
- [16] H. Kantz, T. Schreiber, *Nonlinear Time Series Analysis*, Cambridge University Press, Cambridge, 2004, p. 388.
- [17] S. Kostić, N. Vasović, M. Perc, M. Toljić, D. Nikolić, Stochastic nature of earthquake ground motion, *Physica A* 392 (2013) 4134–4145.
- [18] A. De Santis, G. Cianchini, E. Qamili, A. Frepoli, The, L'Aquila (Central Italy) seismic sequence as a chaotic process, *Tectonophysics* 496 (2010) 44–52.
- [19] R. Donner, S. Barbosa, J. Kurths, N. Marwan, Understanding the Earth as a complex system – recent advances in data analysis and modeling in Earth sciences, *Eur. Phys. J. Spec. Top.* 174 (2009) 1–9.
- [20] ANSS composite earthquake catalog, <<http://quake.geo.berkeley.edu/cnss>>, 2013.
- [21] R.A. Bennett, S. Hreinsdóttir, G. Buble, T. Bašić, Ž. Bačić, M. Marjanović, G. Casale, A. Gendaszek, D. Cowan, Eocene to present subduction of southern Adria mantle lithosphere beneath the Dinarides, *Geology* 36 (2008) 3–6.
- [22] F. Horváth, G. Bada, P. Szafián, G. Tari, A. Ádám, S. Cloetingh, Formation and deformation of the Pannonian Basin: constraints from observational data, *Geol. Soc. London Mem.* 32 (2006) 191–206.
- [23] A. Mignan, J. Woessner, Estimating the magnitude of completeness for earthquake catalogs, *Community Online Resource for Statistical Seismicity Analysis* (2012) 1–45, <http://dx.doi.org/10.5078/corssa-00180805>. available at <http://www.corssa.org>.
- [24] S. Wiemer, A software package to analyze seismicity: ZMAP, *Seismol. Res. Lett.* 72 (2001) 373–382.
- [25] S. Wiemer, M. Wyss, Minimum magnitude of complete reporting in earthquake catalogs: examples from Alaska, the Western United States, and Japan, *Br. Seismol. Soc. Am.* 90 (2000) 859–869.
- [26] F. Takens, Detecting strange attractors in turbulence, in: D.A. Rand, L.S. Young (Eds.), *Lecture Notes in Mathematics* 898, Springer, Berlin, 1981, pp. 366–381.
- [27] S. Kodba, M. Perc, M. Marhl, Detecting chaos from a time series, *Eur. J. Phys.* 26 (2005) 205–215.
- [28] A. Fraser, H. Swinney, Independent coordinates for strange attractors from mutual information, *Phys. Rev. A* 33 (1986) 1134–1140.
- [29] M. Kennel, R. Brown, H. Abarbanel, Determining embedding dimension for phase-space reconstruction using a geometrical construction, *Phys. Rev. A* 45 (1992) 3403–3411.
- [30] M. Perc, A.K. Green, C. Jane Dixon, M. Marhl, Establishing the stochastic nature of intracellular calcium oscillations from experimental data, *Biophys. Chem.* 132 (2008) 33–38.
- [31] D. Kugiumtzis, A. Tsimpiris, Measures of analysis of time series (MATS): A MATLAB toolkit for computation of multiple measures on time series data bases, *J. Stat. Softw.* 33 (2010) 1–30.
- [32] D. Kaplan, L. Glass, Direct test for determinism in a time series, *Phys. Rev. Lett.* 68 (1992) 427–430.
- [33] G.E.P. Box, G. Jenkins, G. Reinsel, *Time Series Analysis: Forecasting and Control*, John Wiley & Sons, Hoboken, New Jersey, 2007, p. 784.
- [34] M.N.M. Van Lieshout, A. Stein, Earthquake modelling at the country level using aggregated spatio-temporal point processes, *Math. Geosci.* 44 (2012) 309–326.
- [35] S.N. Hashemi, Seismicity characterization of Iran: a multivariate statistical approach, *Math. Geosci.* 45 (2013) 705–725.