Applied Mathematics and Computation 372 (2020) 124996

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Multivariable coupling and synchronization in complex networks

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ARTICLE INFO

Article history:
Received 27 September 2019
Revised 12 December 2019
Accepted 15 December 2019

Keywords:
Multivariable coupling
Synchronization
Complex network
Chaos
Nonlinear dynamics

ABSTRACT

Synchronization in complex networks is an evergreen subject with numerous applications in biological, social, and technological systems. We here study whether a transition from a single variable to multivariable coupling facilitates the emergence of synchronization in a network of circulant oscillators. We show that the network indeed has much better synchronizability when individual dynamical units are coupled through multiple variables rather than through just one. In particular, we consider in detail four different coupling scenarios for a simple three-dimensional chaotic circulant system, and we determine the smallest coupling strength needed for complete synchronization. We find that the smallest coupling strength is needed when the coupling is through all three variables, and that for the same level of synchronization through a single variable a much stronger coupling strength is needed. Our results thus show that multivariable coupling provides a significantly more efficient synchronization profile in complex networks.

1. Introduction

Chaos can be observed in both real-world phenomena and computational models [1,2]. Scholars have analyzed chaos and found how chaotic attractors are created [3,4]. For many years there was a claim that chaotic attractors are related to saddle point equilibrium [5–7]. However, some systems with different kinds of equilibria have been recently introduced as counterexamples [8,9]. Therefore, the only certain way of finding chaos is to do exhaustive computer search after selecting the general structure of the system (e.g. jerk or circulant) [10,11]. Recently, studying the dynamical properties of networks of chaotic systems has attracted much attention [12–14]. Moreover, the reconstruction of networks to reach the desired topology is a significant subject [15,16]. One of the most important collective behaviors of networked systems is synchronization, which is observed in many fields, ranging from physics to biology and social systems [17,18]. For instance, synchronization of neurons has been a hot topic in neuroscience [19,20]. Synchronization of networks of chaotic flows has received much attention due to unique property of chaotic flows which is sensitivity to initial condition (butterfly effect) [21–23]. Various types

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https://doi.org/10.1016/j.amc.2019.124996
0096-3003/

of synchronization have been studied such as phase synchronization [24]. Wu et al. have studied synchronization of chaotic flow without equilibria [25]. In [26], synchronization and fractional-order form of a no-equilibrium chaotic flow have been investigated. Synchronization of Lur's system using a time-delay feedback control has been discussed in [27]. Chen et al. have used adaptive control as a useful tool in the synchronization of chaotic systems [28].

Determining synchronizability of a dynamical network is one of the fundamental topics in complex networks receiving significant attention within the community [29,30]. The first general stability of the synchronization manifold by the help of the extended Lyapunov matrix approach was proposed by Fujisaka and Yamada [31]. Then, Wu and Chua proposed their eigenvalues conjectures in linearly coupled arrays of oscillators [32]. Master stability function, which provides necessary conditions for linear stability of the synchronization manifold in identical networks, was introduced by Pecora and Carrol [33]. They showed that one could decuple nodal dynamics and network structure, and obtain a synchronisability index that is solely dependent on the network structure.

Generally, synchronization emerges when two or more dynamical systems have strong enough coupling strength. A class of systems show synchrony patterns when the coupling strength is higher than a threshold, and remain synchronized for all values of the coupling higher than this threshold. Whereas, some other systems can be synchronized only for the coupling strength in a certain range. In the synchronization of two coupled systems, the coupling can be applied to different state variables. For example, in three-dimensional differential equations with three x, y, and z variables, the systems can be coupled through either of the variables or a combination of them. If we consider the effect of coupling as a linear feedback function, the coupling on x variables means that the term $d \times (x_2 - x_1)$ is added to the $\dot{x}_1$ and $d \times (x_1 - x_2)$ is added to the $\dot{x}_2$, where d is the coupling strength and x_i is the x variable of system i. The previous works have mainly considered coupling through one of the variables or all of them, and there is yet no systematic study comparing synchronization in single-variable versus multivariable couplings. This manuscript provides a systematic study to compare these two coupling forms. Our results show that multivariable coupling results in better synchronisability than single-variable coupling of the same cost.

2. Test design

In this section, first, the circulant chaotic systems are presented, and their chaotic dynamics are discussed. Then, various types of coupling configurations are studied. Finally, two criteria are presented to investigate the synchronization of mutually coupled systems.

2.1. Circulant system

A circulant system is a special case in which the variables are cyclically symmetric. The general form of a circulant system is as follows,

$$\begin{aligned} \dot{x} &= f(x, y, z) \\ \dot{y} &= f(y, z, x). \\ \dot{z} &= f(z, x, y) \end{aligned} \tag{1}$$

All the velocities have the same function, expect that their variables are rotated [4]. In this paper, we use three circulant systems.

The first circulant system is Halvorsen system:

$$\begin{aligned} \dot{x} &= -1.3x - 4y - 4z - y^2 \\ \dot{y} &= -1.3y - 4z - 4x - z^2. \\ \dot{z} &= -1.3z - 4x - 4y - x^2 \end{aligned} \tag{2}$$

The system has a chaotic solution with initial conditions $(x_0, y_0, z_0) = (-6.4, 0, 0)$, and its attractor is shown in Fig. 1(a). Lyapunov exponents of the chaotic attractor of the system are $(\lambda_1, \lambda_2, \lambda_3) = (0.6928, 0, -4.5928)$.

The second circulant system is piecewise linear one [4]:

$$\begin{aligned} \dot{x} &= 1 - x - y - 4|y| \\ \dot{y} &= 1 - y - z - 4|z|. \\ \dot{z} &= 1 - z - x - 4|x| \end{aligned} \tag{3}$$

It has chaotic solution when initial conditions are $(x_0, y_0, z_0) = (0.4, 0, 0)$ with Lyapunov exponents as $(\lambda_1, \lambda_2, \lambda_3) = (0.0975, 0, -3.0975)$. The chaotic attractor is shown in Fig. 1(b).

The third circulant system is a cubic system as follows [4],

$$\begin{aligned} \dot{x} &= y^3 - z \\ \dot{y} &= z^3 - x. \\ \dot{z} &= x^3 - y \end{aligned} \tag{4}$$

Initial values for the chaotic solution of this system are $(x_0, y_0, z_0) = (-0.77, 0.35, 1.13)$ and its Lyapunov exponents are $(\lambda_1, \lambda_2, \lambda_3) = (0.0059, 0, -0.0059)$. Fig. 1(c) shows the chaotic sea of System (4).

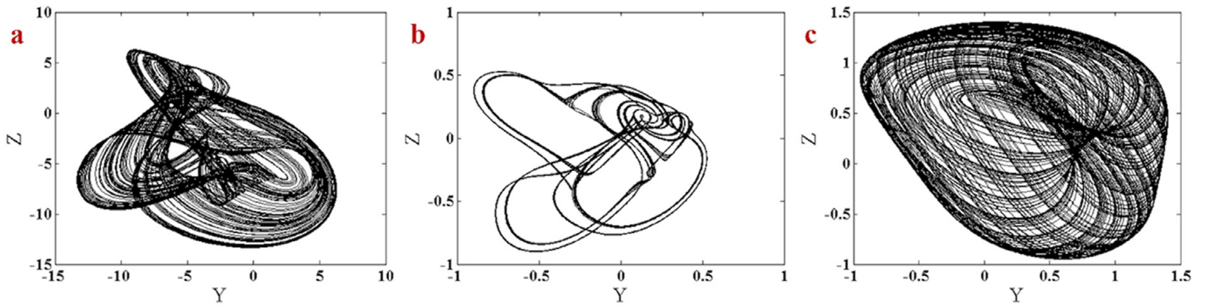

Fig. 1. Two-dimensional projections of the studied strange attractors. (a) Chaotic attractor of the Halvorsen system. (b) The chaotic attractor of the piece-wise linear system. (c) Chaotic sea of the cubic system.

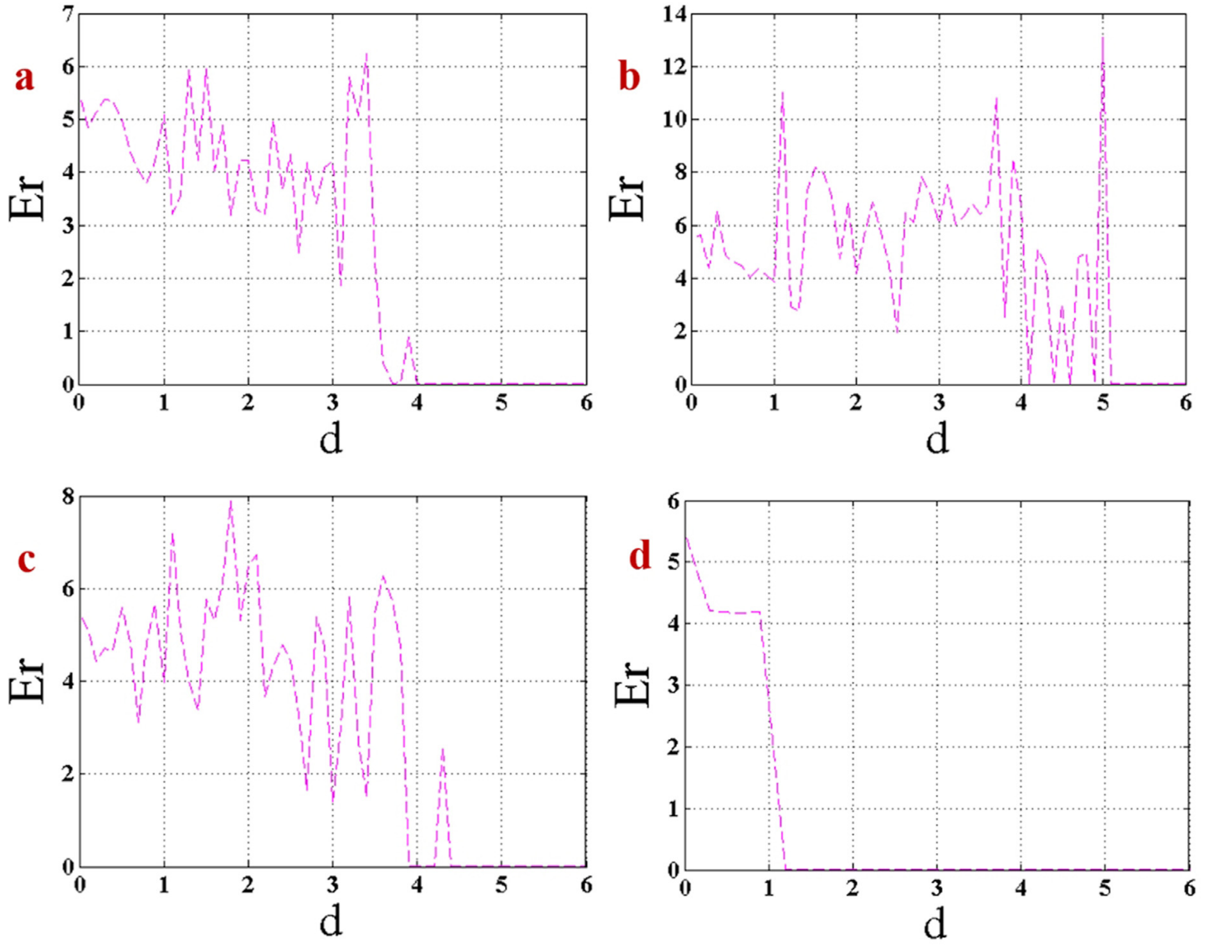

Fig. 2. The error function of two Halvorsen systems as a function of the coupling parameter, and for the cases where (a) coupling on x variable. (b) Coupling on y variable. (c) Coupling on z variable. (d) Coupling on x, y, z simultaneously. The results show that multivariable coupling is more efficient than single-variable coupling.

2.2. Type of coupling

To investigate the coupling effect on the synchronization of two mutually coupled systems, four types of coupling are studied. In the first case, the coupling term is only applied to the x variables. The structure of this coupling is shown in Eq. (5).

$$\begin{array}{ll} \dot{x}_1 = f(x_1, y_1, z_1) + d(x_2 - x_1) & \dot{x}_2 = f(x_2, y_2, z_2) + d(x_1 - x_2) \\ \dot{y}_1 = f(y_1, z_1, x_1) & \dot{y}_2 = f(y_2, z_2, x_2) \\ \dot{z}_1 = f(z_1, x_1, y_1) & \dot{z}_2 = f(z_2, x_2, y_2) \end{array} \quad (5)$$

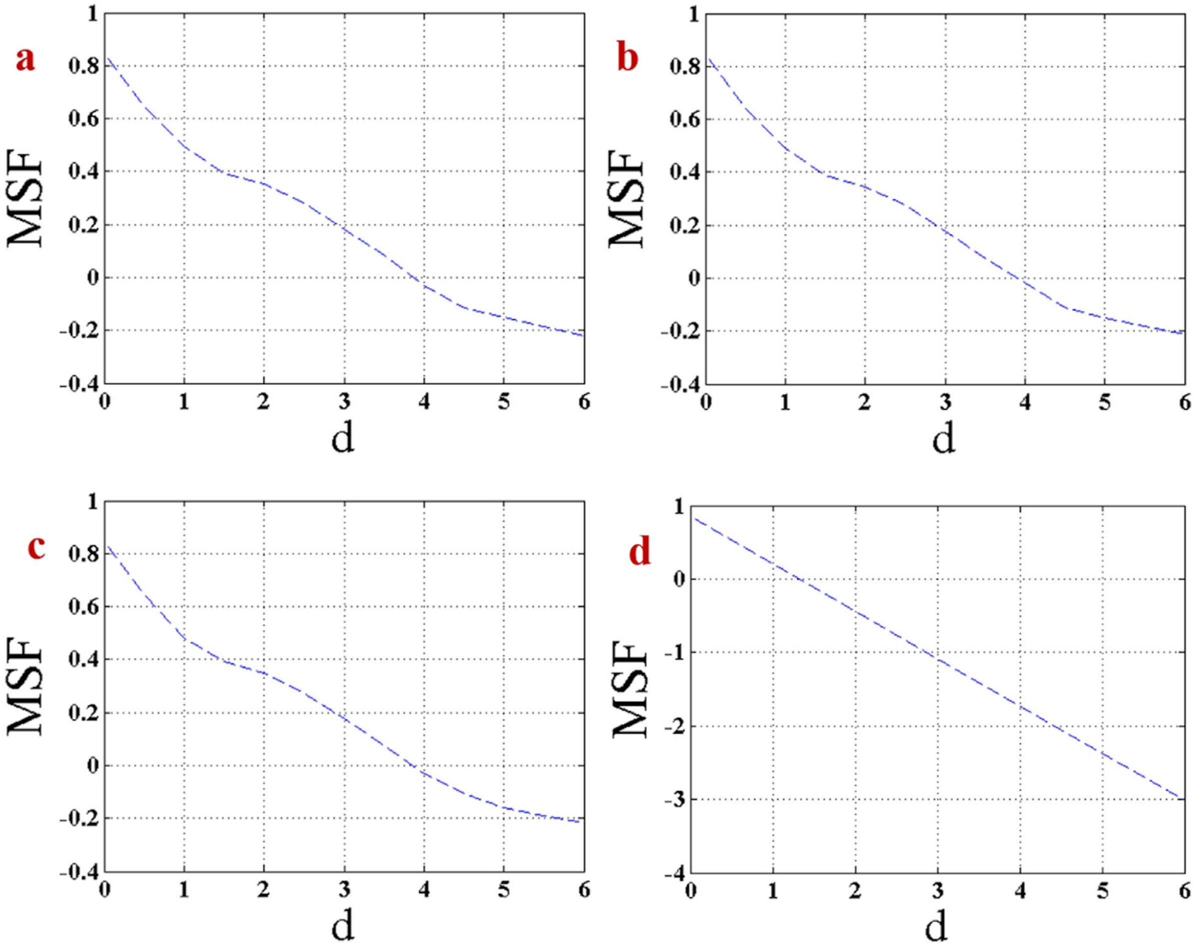

Fig. 3. Master stability function of two Halvorsen systems for changing coupling parameter and (a) coupling on x variable. (b) Coupling on y variable. (c) Coupling on z variable. (d) Coupling on x, y, z simultaneously. The smallest synchronization couplings on x, y, and z variables are $d_x = 3.869, d_y = 3.8956, d_z = 3.8555$, respectively. The smallest synchronization coupling of the multivariable coupling case is $d_{xyz} = 1.3155$, which shows it is more efficient than single-variable coupling.

In the second case, the coupling term is applied to the y variables. This case can be written as Eq. (6).

$$\begin{aligned} \dot{x}_1 &= f(x_1, y_1, z_1) & \dot{x}_2 &= f(x_2, y_2, z_2) \\ \dot{y}_1 &= f(y_1, z_1, x_1) + d(y_2 - y_1) & \dot{y}_2 &= f(y_2, z_2, x_2) + d(y_1 - y_2) \\ \dot{z}_1 &= f(z_1, x_1, y_1) & \dot{z}_2 &= f(z_2, x_2, y_2) \end{aligned} \tag{6}$$

In the third case, the coupling is applied to the z variables, which is formulated in Eq. (7).

$$\begin{aligned} \dot{x}_1 &= f(x_1, y_1, z_1) & \dot{x}_2 &= f(x_2, y_2, z_2) \\ \dot{y}_1 &= f(y_1, z_1, x_1) & \dot{y}_2 &= f(y_2, z_2, x_2) \\ \dot{z}_1 &= f(z_1, x_1, y_1) + d(z_2 - z_1) & \dot{z}_2 &= f(z_2, x_2, y_2) + d(z_1 - z_2) \end{aligned} \tag{7}$$

In the fourth case, the coupling is applied to all three variables of the system; however, the sum of coupling on all variables is remained equal to d. Its structure is as follows,

$$\begin{aligned} \dot{x}_1 &= f(x_1, y_1, z_1) + \frac{d}{3}(x_2 - x_1) & \dot{x}_2 &= f(x_2, y_2, z_2) + \frac{d}{3}(x_1 - x_2) \\ \dot{y}_1 &= f(y_1, z_1, x_1) + \frac{d}{3}(y_2 - y_1) & \dot{y}_2 &= f(y_2, z_2, x_2) + \frac{d}{3}(y_1 - y_2) \\ \dot{z}_1 &= f(z_1, x_1, y_1) + \frac{d}{3}(z_2 - z_1) & \dot{z}_2 &= f(z_2, x_2, y_2) + \frac{d}{3}(z_1 - z_2) \end{aligned} \tag{8}$$

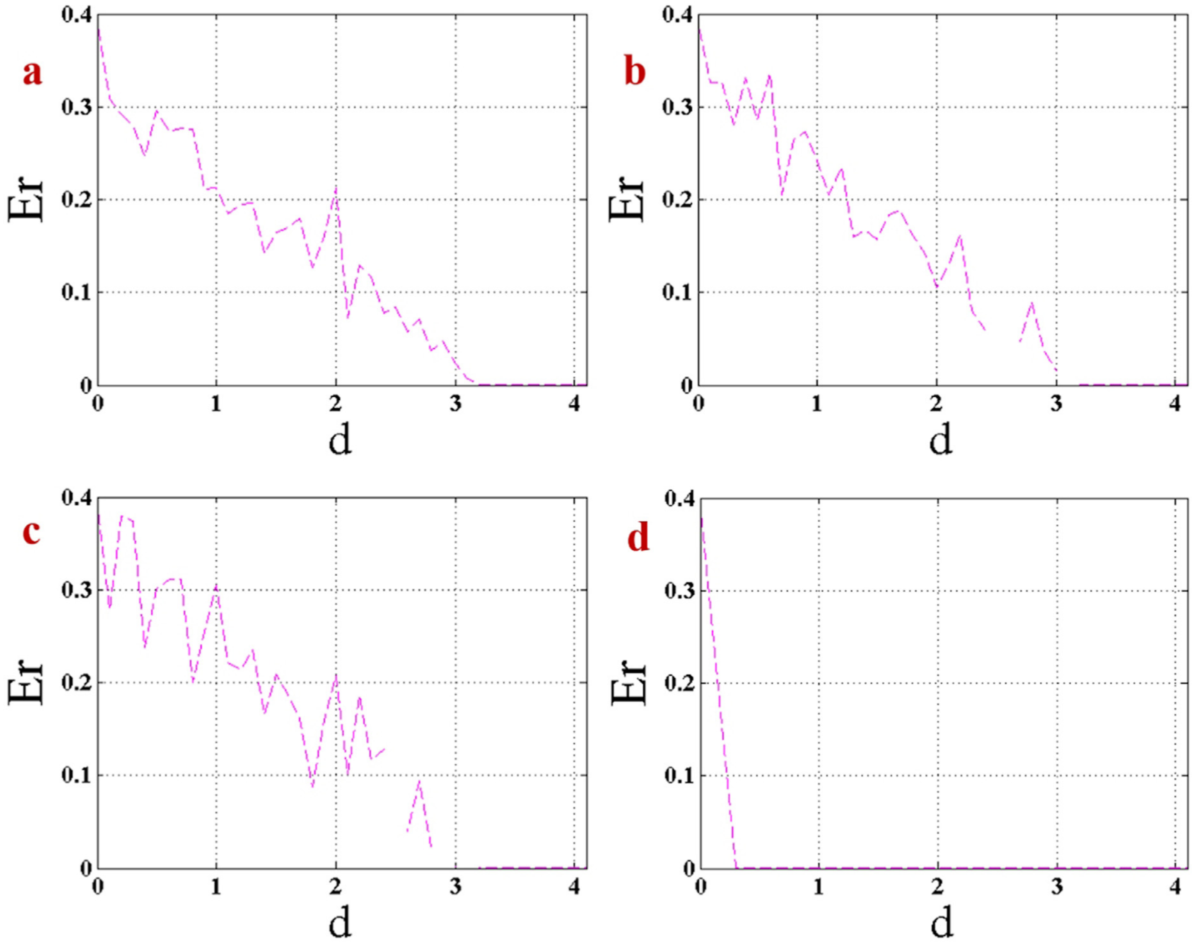

Fig. 4. The error function of two piecewise linear systems as a function of the coupling parameter, and for the cases where (a) coupling on *x* variable. (b) Coupling on *y* variable. (c) Coupling on *z* variable. (d) Coupling on *x, y, z* simultaneously. Accordingly, the multivariable coupling is more efficient than single-variable coupling.

It should be mentioned that in all cases above, the two coupled systems are identical but with different initial conditions. As the variables have exactly the same role in the differential equations, one can argue that the coupling cost of Eq. (8) may be the same as Eqs. (5)–(7).

2.3. Evaluation criteria

In this study, the full synchronization of two mutually coupled circulant systems is studied. Two criteria are used to detect the network's synchronization as follows.

2.3.1. Error function

To compare the results in each coupling scheme, an error function is used. This error is defined on the difference of the corresponding variables as follows,

$$Er = \frac{1}{M}\sum_{i=1}^{M}\sqrt{\frac{(x_{1,i}-x_{2,i})^2+(y_{1,i}-y_{2,i})^2+(z_{1,i}-z_{2,i})^2}{3}} \tag{9}$$

where M is the number of samples of the time series. In each system, we have obtained the time series for approximately 450 cycles. The time series are discretized in the intervals of 0.01 s, and the error is calculated after removing 50% of time series as the transient part. Such error is approximately zero in the case of full synchronization.

2.3.2. Master stability function

Master Stability Function (MSF) is a tool to investigate the stability of the synchronization manifold in coupled identical systems [33]. The synchronization manifold $\mathbf{s}(t)$ of an identical network with N individual units $(\mathbf{s}(t) = \mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots =$

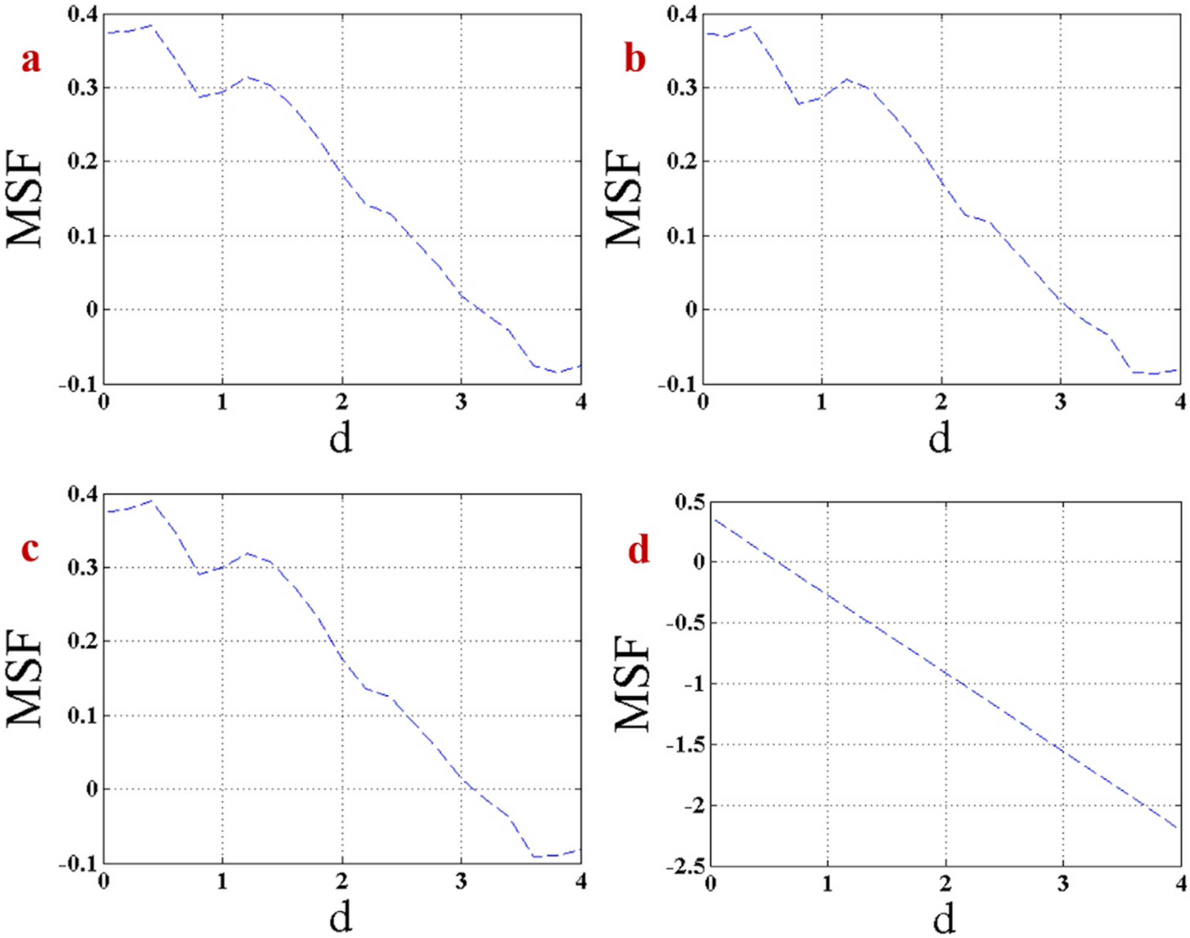

Fig. 5. Master stability function of two piecewise linear systems concerning changing the coupling parameter and (a) coupling on x variable. (b) Coupling on y variable. (c) Coupling on z variable. (d) Coupling on x, y, z simultaneously. The results of the master stability function illustrate that the smallest synchronization coupling on three variables $d_{xyz} = 0.5811$ is much less than $d_x = 3.1505$, $d_y = 3.2903$, and $d_z = 3.1016$.

$\mathbf{x}_N(t))$ is where all the states of the networks are in the same dynamics. The undirected and unweighted identical network is ruled by:

$$\dot{\mathbf{x}}_i = F(\mathbf{x}_i) + d\sum_{j=1}^{N} g_{ij}H(\mathbf{x}_j) \tag{10}$$

where $F(.)$ is the dynamics of each individual oscillators, and d is the uniform coupling strength. $G = (g_{ij})$ is the Laplacian matrix of the connection graph, and H is the projection matrix determining from which variables the individual systems are coupled.

The variational equations of Eq. (10) are as follows:

$$\dot{\xi}_i = DF\xi_i + d\sum_{j=1}^{N} g_{ij}DH\xi_i \tag{11}$$

where D is Jacobian. According to the block diagonalized coupling function and the eigenvectors of the matrix G, Eq. (11) can be rewritten as:

$$\dot{\eta}_i = (DF + d\lambda_i DH)\eta_i \qquad i = 1, \cdots, N \tag{12}$$

where λ_i are the eigenvalues of the Laplacian matrix. As it is stated before, all the cases which have been introduced in Section 2.2 have the same topology. The stability of the synchronization manifold can be studied through the MSF approach. The largest Lyapunov exponent of Eq. (12) is called the MSF, and accounts for the local stability of the synchronization manifold. The synchronization manifold is stable if the MSF is negative.

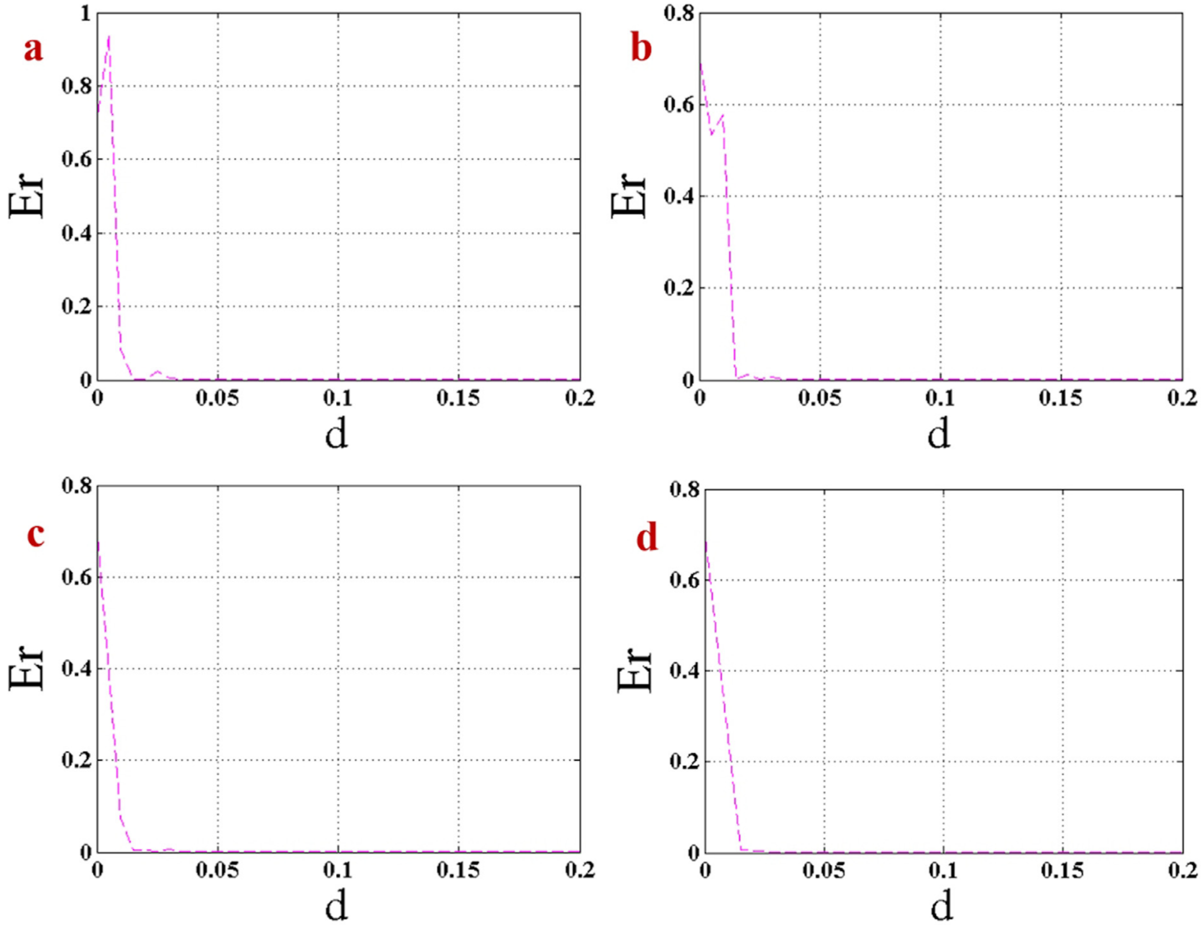

Fig. 6. The error function of two cubic systems as a function of the coupling parameter, and for the cases where (a) coupling on *x* variable. (b) Coupling on *y* variable. (c) Coupling on *z* variable. (d) Coupling on *x, y, z* simultaneously. The multivariable coupling exhibits better performance than single-variable coupling.

3. Results

Coupled oscillators have been a hot topic in various fields such as physics, for example, in arrays of Josephson junctions [34,35] and coupled lasers [36]. Multivariable coupled networks are one of the main branches of coupled systems. In [37], the multivariable coupling of two Van der Pol Oscillators has been studied in which microwave oscillators were the inspiration. In the operation of microwave oscillators close to each other, the output of each oscillator may be affected by other ones [38]. Multivariable coupling of chaotic Chua's circuit has been discussed in [39]. Another example is the collective behavior of the nerve impulse in brain, such as dynamical network of electromagnetic Hindmarsh–Rose neuronal model with multivariable coupling [40,41]. In this paper, the effects of single variable coupling and multivariable coupling in the synchronization of coupled circulant oscillators are investigated. According to the previous section, a test was designed to investigate these effects. To analyze the results of the test, in each case, the synchronization of two coupled systems is studied by varying the coupling parameter, and numerically finding the minimum coupling parameter required for complete synchronization. As it is mentioned in the previous section, the synchronization of the networks is quantified using two criteria: error function and MSF. Let us denote the smallest synchronizing coupling strength for x-coupled case by d_x, that of y-coupled case by d_y, z-coupled case by d_z, and the one for the case where the systems are coupled through all variables by d_{xyz}. In the following, we compare these values in different dynamical systems.

3.1. Halvorsen system

All simulations on the Halvorsen system are performed over 400 s. The error function of Eq. (9) is calculated to express the synchronization of two coupled systems for changing the coupling parameter d. Initial values of the first system are $(x_{1,0}, y_{1,0}, z_{1,0}) = (-6.4, 0, 0)$ while initial conditions of the second system are $(x_{2,0}, y_{2,0}, z_{2,0}) = (-4.93, 2.273, -1.26)$. The second initial conditions are selected on the strange attractor of the system. Fig. 2 shows the error functions computed on

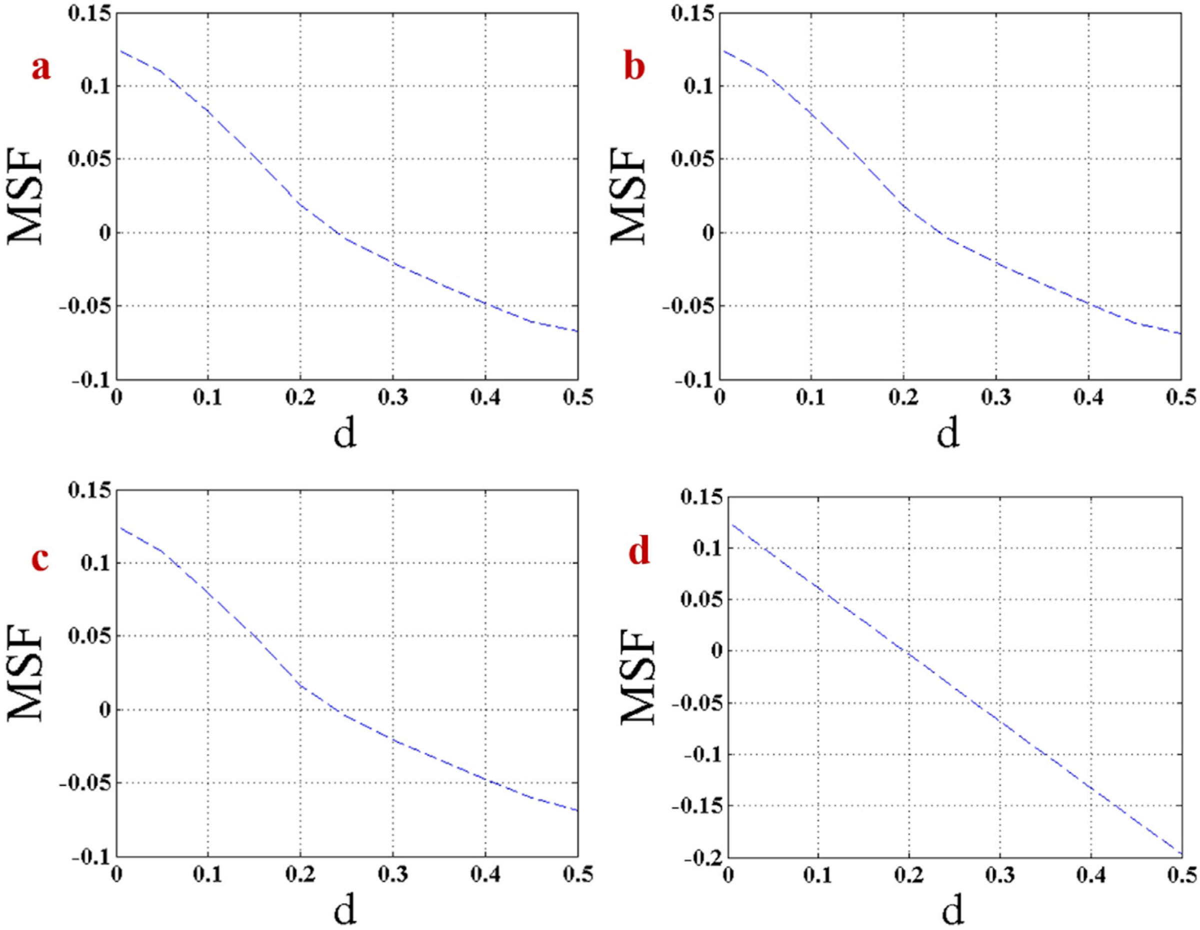

Fig. 7. Master stability function of two cubic systems for changing coupling parameter and (a) coupling on x variable. (b) Coupling on y variable. (c) Coupling on z variable. (d) Coupling on x, y, z simultaneously. Master stability function shows the same results as the error function.

the variables of the system for different cases. Fig. 2(a) depicts the error function where the two systems are coupled on the x variables. Parts (b) and (c) of the figure shows error functions where the two systems are coupled on y and z variables, respectively. The results of multivariable coupling are shown in Fig. 2(d). As it is illustrated in the figure, $d_x = 3.7, d_y = 4.1$ and $d_z = 3.9$, while $d_{xyz} = 1.2$, indicating that multivariable coupling is more efficient than single-variable coupling. Fig. 3 shows the MSF of the system for the four cases. The minimum synchronizing coupling strength is the one on which the sign of MSF changes from positive to negative. The results show that the smallest synchronization couplings on x variable, y variable, and z variable are $d_x = 3.869, d_y = 3.8956, d_z = 3.8555$, while for the multivariable coupling case it is $d_{xyz} = 1.3155$. This shows that the multivariable coupling is also more efficient than single-variable coupling in terms of the MSF criterion.

3.2. Piecewise linear system

All the simulations for the system are computed over 1000 s, and the error function is calculated on the second-half of the time series as the transient time is removed. The calculated error function of the coupled piecewise linear system on x variables, y variables, z variables, and x, y, z variables are shown in Fig. 4(a), (b), (c) and (d), respectively. In some couplings, the system's dynamic becomes unbounded, and thus the error function cannot be defined. The results show that $d_{xyz} = 0.6$, much less than $d_x = d_y = d_z = 3.2$, indicating better performance for the multivariable coupling. Similar results are also obtained for the MSF criterion, where $d_{xyz} = 0.5811$ is much less than $d_x = 3.1505$, $d_y = 3.2903$, and $d_z = 3.1016$ (Fig. 5).

3.3. Cubic system

The coupled cubic systems are simulated over 1500 s. The calculated error functions of the two coupled cubic systems which are coupled with different strategies are shown in Fig. 6, where the multivariable coupling exhibit better performance than single-variable coupling. The same is true in terms of the MSF criterion (Fig. 7).

4. Conclusions

The aim of this paper was to compare the effects of single variable coupling and multivariable coupling in the synchronization of coupled circulant oscillators. To this end, three individual dynamical systems were considered, and their synchronization was studied when two identical systems were mutually coupled. The minimum synchronizing parameter was identified using two criteria, namely the synchronization error and the master stability function. Our numerical simulation results showed that multivariable coupling is always more efficient (requiring smaller coupling strength for synchronization) than coupling through only a single variable.

Acknowledgment

This work was supported by Iran's National Elites Foundation (Grant No. M-97171) and by the Slovenian Research Agency (Grant Nos. J4-9302, J1-9112, P1-0306, and P1-0403).

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