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Effects of different initial conditions on the emergence of chimera states

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ABSTRACT

Chimeras are fascinating spatiotemporal states that emerge in coupled oscillators. These states are characterized by the coexistence of coherent and incoherent dynamics, and since their discovery, they have been observed in a rich variety of different systems. Here, we consider a system of non-locally coupled three-dimensional dynamical systems, which are characterized by the coexistence of fixed-points, limit cycles, and strange attractors. This coexistence creates an opportunity to study the effects of different initial conditions – from different basins of attraction – on the emergence of chimera states. By choosing initial conditions from different basins of attraction, and by varying also the coupling strength, we observe different spatiotemporal solutions, ranging from chimera states to synchronous, imperfect synchronous, and asynchronous states. We also determine conditions, in dependence on the basins of attraction, that must be met for the emergence of chimera states.

1. Introduction

In 2002, Kuramoto and Battogtokh [1] discovered a remarkable spatiotemporal pattern in a ring network of symmetrically coupled phase oscillators, in which the array of oscillators splitted into two parts of: one coherent and phase-locked and the other incoherent and desynchronized. This phenomenon was named “Chimera” two years later by Abrams and Strugatz [2]. Chimera state has been investigated in many studies since then [3–7], including in phase oscillators [8] chaotic and periodic maps [9], pendulum-like oscillators [10] and delay-coupled oscillator systems [11], as well as experimentally in different chemical [12], mechanical [13], optical [14] and electronic [15] systems. Through these studies, some special types of chimera state were discovered. Kemeth et.al [16] presented a classification scheme that can classify three categories of stationary, turbulent and breathing chimeras. Zakharova et.al. [17,18] found a novel pattern called chimera death, combination of amplitude chimera and oscillation death, in dynamical networks. Kapitaniak et.al. [19] showed another pattern called imperfect chimera in a network of coupled Huygens clocks, in which a few oscillators escape from the synchronized chimera's cluster.

Some of the studies have investigated the properties or the effect of parameters on chimera states. Wolfrum et.al. [20] presented the analysis of spectral properties for the trajectories in chimera state. Omelchenko et.al. [21] examined the effect of coupling strength on non-locally coupled FitzHughNagumo oscillators, and reported multi-chimeras occurrence dependence on coupling strength. Since the chimera state is strongly related to many real world phenomena such as unihemispheric sleep, epileptic seizure, atrial fibrillation and etc. [22–25], determining the effect of different initial conditions on chimera state can provide beneficial information.

Multistablity is a very important phenomenon in nonlinear dynamical systems [26–29]. While in routine engineering problems multistablity is mostly unwanted, in biological and complex systems it is natural phenomenon causing many advantages. Obviously in every multistable system there should be at least two coexisting attractors. Recently many chaotic systems with coexisting attractors have been reported in literature [30–33].

In this paper, we study a network of simple three-dimensional and autonomous system with only quadratic nonlinearities that has all three different common types of attractors (fixed-point, limit cycle and strange attractors (coexisting side by side. This system has been reported by Sprott et.al. [34] in 2013. Using non-local coupling between the oscillators, we aim to consider the

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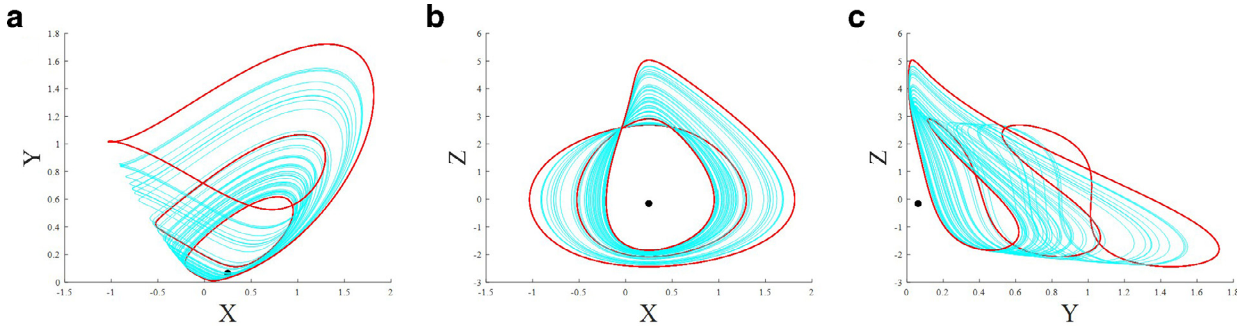

Fig. 1. State space of system (1) with coexistence of limit cycle (red), strange (cyan) and fixed-point (black) attractors. The initial conditions are respectively, (**1,1,1**), (**1,1,0**) and (**0.2,0,0**). (a) X_Y plane, (b) X_Z plane and (c) Y_Z plane.

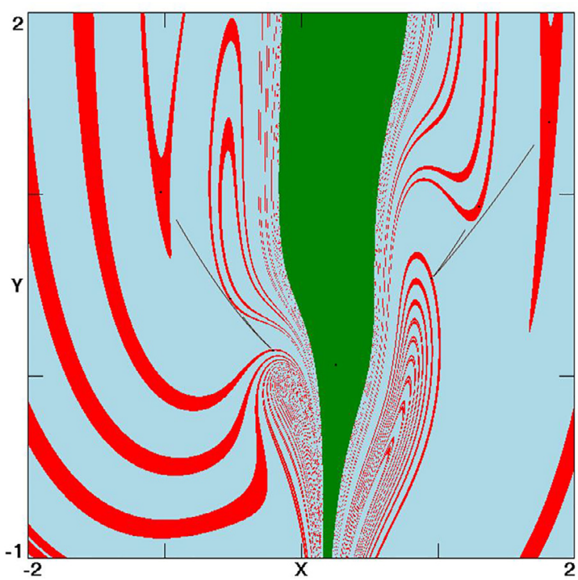

Fig. 2. Basins of attraction of system, limit cycle in red color, strange in blue color and the fixed-point in green color, in X-Y cross section [34].

effect of initial conditions on chimera emergence by choosing the initial conditions in different regions of the basins of attraction. We are aware of no similar investigation on such a strongly multistable dynamical system.

The rest of this paper is organized as follows. The mathematical equations of the system are described in the following section. Section three presents the results gained by numerical simulation of the network and finally the conclusions are given in section four.

2. Multistable oscillator

We consider a network of non-locally coupled 3D system which was proposed by Sprott and coauthors in 2013 [34]:

$$\begin{aligned} \dot{x} &= yz + a \\ \dot{y} &= x^2 - y \\ \dot{z} &= 1 - 4x \end{aligned} \tag{1}$$

This system has only one equilibrium in $\left(\frac{1}{4}, \frac{1}{16}, -16a\right)$. For $a = 0.01$, System (1) exhibits coexistence of a limit cycle attractor, a strange attractor and one attracting fixed-point. Fig. 1 Shows the state space of the system in X-Y, X-Z and Y-Z planes with initial conditions given in the caption. The basins of attraction of the system is shown in Fig. 2.

To construct the network, N oscillators (System (1)) are coupled on a one dimensional ring. The coupling is non-local and each oscillator is coupled with its $2P$ nearest neighbors. The equation of the network is given as:

$$\begin{aligned} \dot{x} &= yz + 0.01 + \frac{d}{2P}\sum_{j=i-P}^{i+P}(X_j - X_i) \\ \dot{y} &= x^2 - y \\ \dot{z} &= 1 - 4x. \end{aligned} \tag{2}$$

where $i = 1, 2, \ldots, N$ is the index of ith oscillator, parameter d is the strength of coupling, and P describes the number of coupling neighbors for each oscillator. In all simulation the value of $P = 20$ and $N = 100$ are fixed.

3. The effect of initial conditions on coupled oscillators

In most cases, the chimera state is very sensitive to initial conditions. Regarding this, we investigate the effect of initial conditions on networks dynamics, by setting a box in various regions of basins of attractions and choosing initial conditions randomly from them.

3.1. Fixed-point's basin of attraction

In Fig. 2, the basin of attraction for the stable equilibrium is shown in green color. Firstly, we consider a box of initial conditions in this region and examine the network by changing coupling strength. Fig. 3 shows the results. For small coupling strengths, the network demonstrate imperfect synchronization state as shown in Fig. 3(a) for $d = 0.06$. In this case, all the oscillators oscillate coherently except six of them. Although all of the oscillators are limit cycle, the attractor of the coherent cluster differs from the escaped ones. Fig. 3(b) shows the attractor of the coherent cluster and Fig. 3(c) shows a different attractor which belongs to a solitary oscillator ($i = 20$). When the coupling coefficient grows, even with the same initial conditions, all the oscillators are attracted by a stable fixed-point, as is shows in Fig. 4 for $d = 0.5$.

3.2. Strange attractor's basin of attraction

In the basin of attraction diagram, the area which is colored in blue, relates to the strange attractor's basin of attraction. In this part, the initial conditions of all oscillators are selected from this region and the dynamics are investigated with different coupling coefficients. It is observed that for low couplings, chimera state occurs and the oscillators have strange attractors. Fig. 5(a) shows the space time plot for $d = 0.11$ and Fig. 5(b) and (c) show the state spaces of respectively coherent and incoherent oscillators. By increasing d, again all the oscillators are attracted by one stable fixed-point.

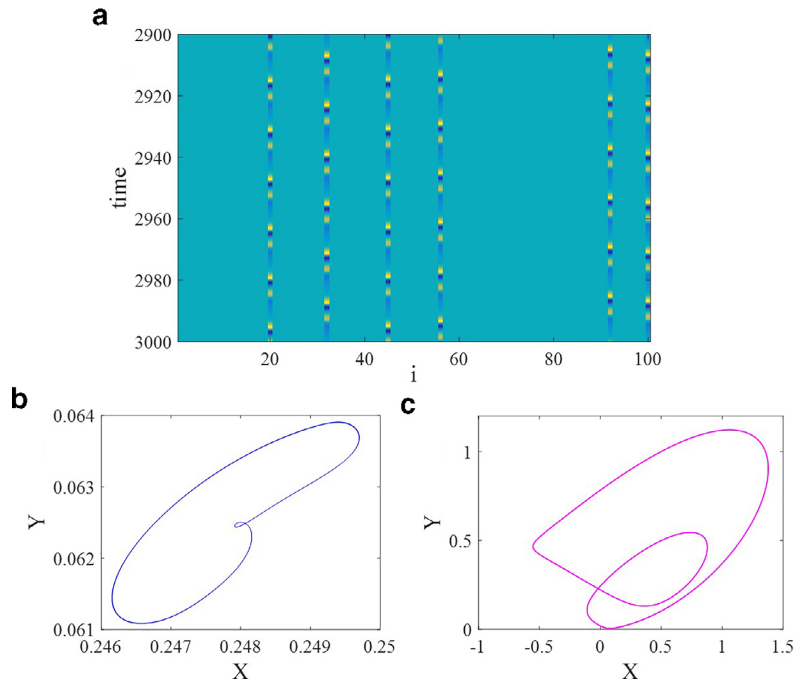

Fig. 3. (a) spatiotemporal pattern when initial conditions are from fixed-point basin of attraction for $d = 0.06$, (b) Attractor of coherent oscillators, (c) attractor of oscillator $i = 20$.

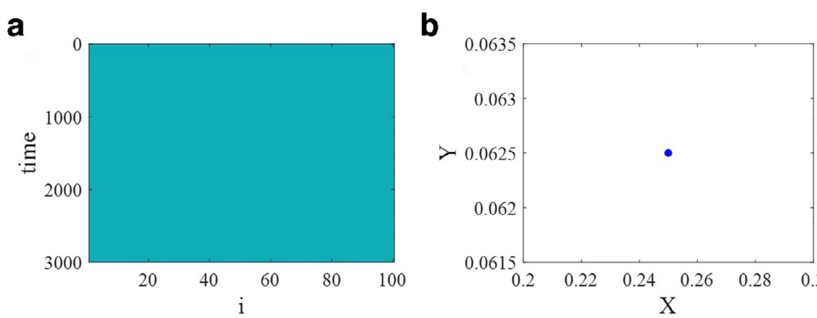

Fig. 4. (a) spatiotemporal pattern when initial conditions are from fixed-point basin of attraction for $d = 0.5$, (b) the oscillators attractor.

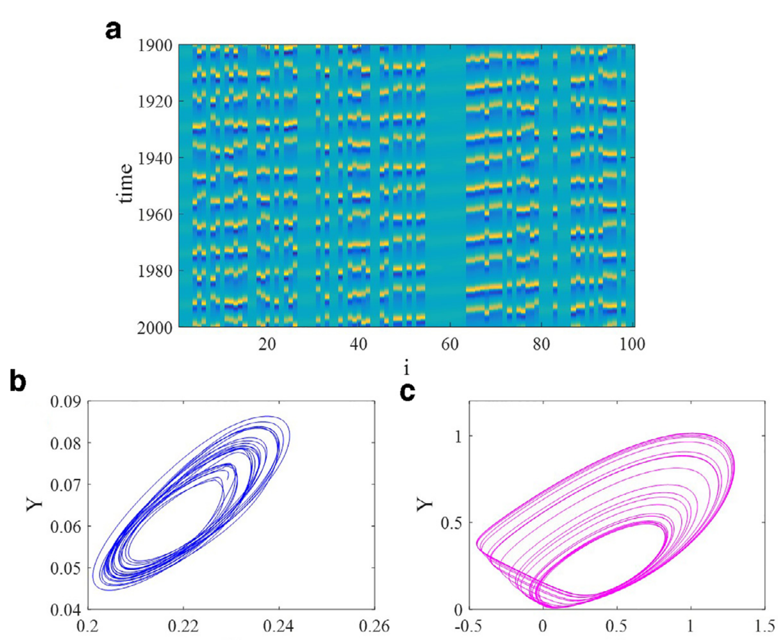

Fig. 5. (a) spatiotemporal pattern of network when initial conditions are selected from strange attractor's basin of attraction, for $d = 0.11$, (b) attractor of synchronous oscillators, (c) attractor of asynchronous oscillators.

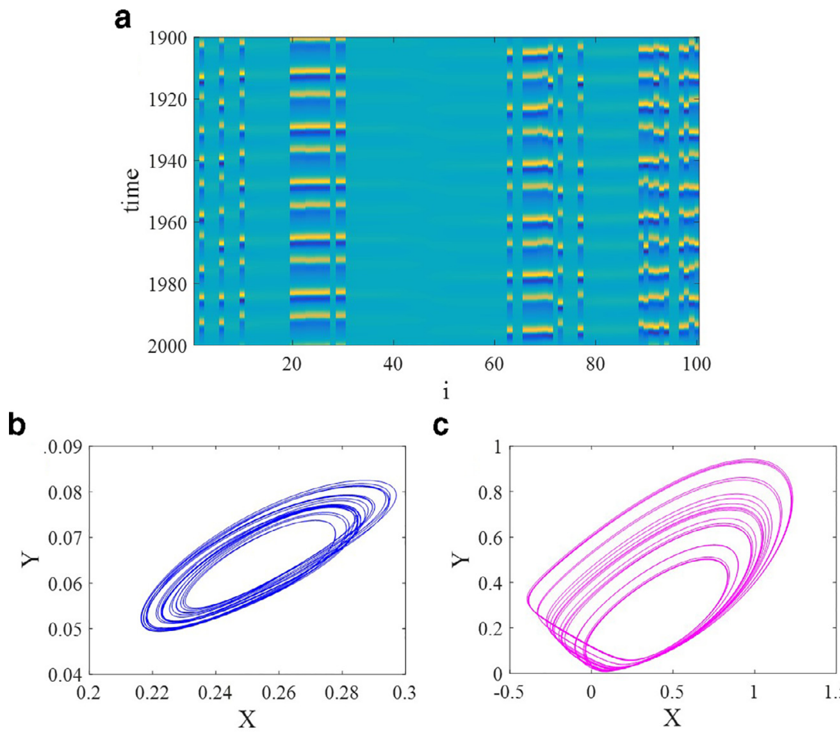

Fig. 6. (a) spatiotemporal pattern with strange and fixed-point basin initial conditions for $d = 0.11$, (b) attractor of oscillator $i = 57$ and (c) attractor of oscillator $i = 93$.

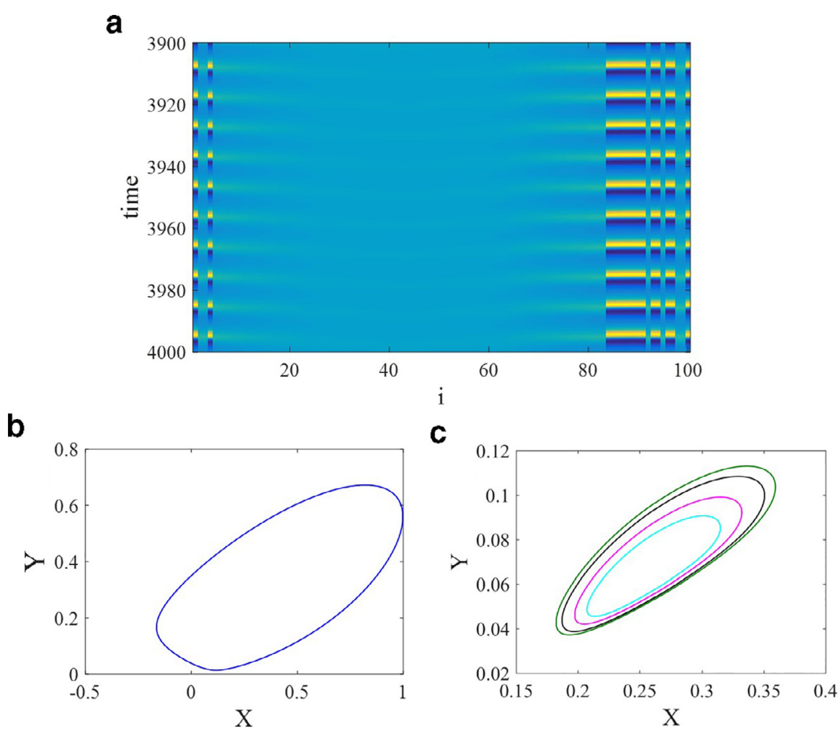

Fig. 7. (a) spatiotemporal pattern for $d = 0.2$ with strange and fixed-point basin initial conditions. (b) attractor of oscillator $i = 91$ in coherent cluster, (c) attractors of oscillators $i = 70$ (cyan), $i = 73$ (magenta), $i = 76$ (black) and $i = 77$ (green), in incoherent cluster. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3.3. Strange attractor's and fixed-point's basin of attraction

Now considering two boxes in the blue and green regions in Fig. 2, the initial conditions are selected from both strange attractor's and fixed-point's basin of attractions randomly. Although some oscillators have initial conditions in the fixed-point's basin, for a specified range of d, chimera state is achieved and all coherent and incoherent oscillators oscillate chaotically, as shown in Fig. 6 for $d = 0.11$.

Fig. 7(a) represents a chimera state that has been emerged for $d = 0.2$. However, observing the spatiotemporal pattern give rise to derive cluster synchronization, but actually the large cluster be-

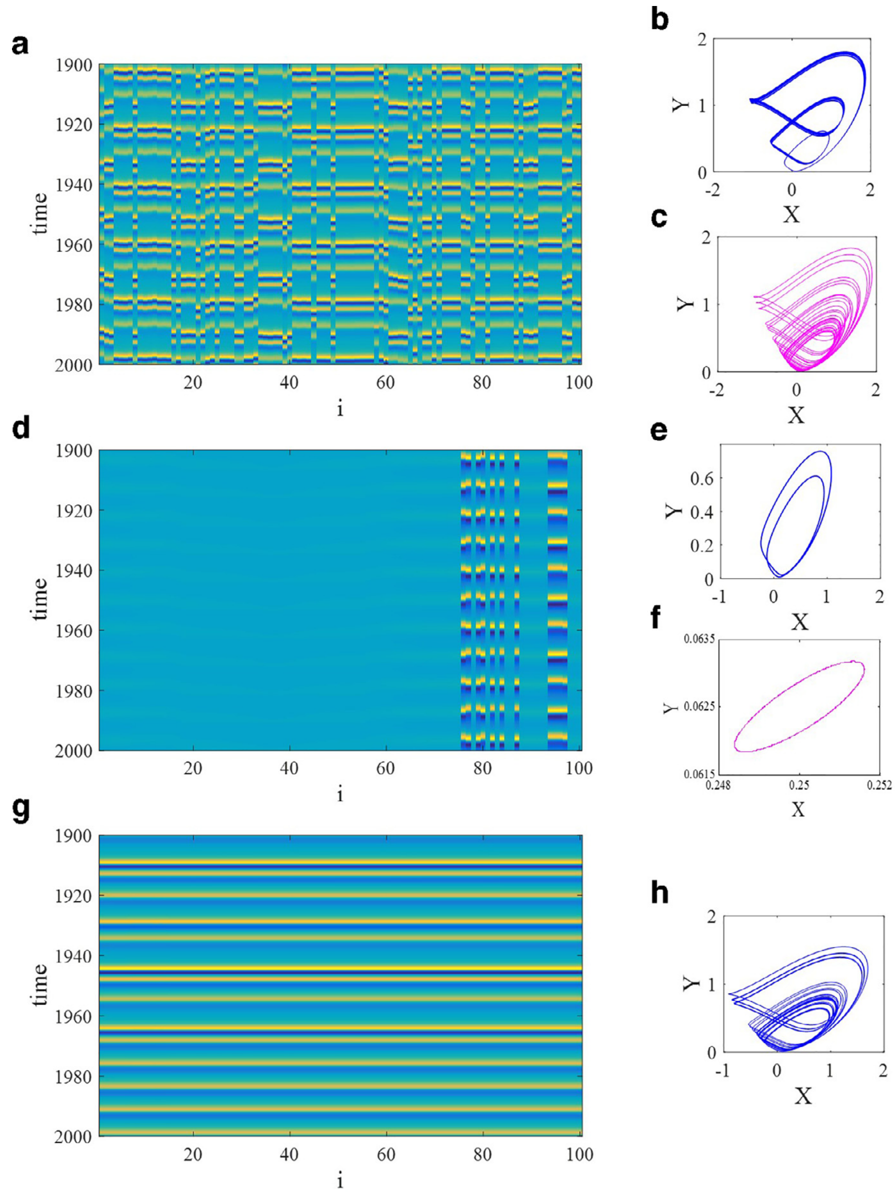

Fig. 8. The results of network with strange and limit cycle basins initial conditions. (a) spatiotemporal pattern for $d=0.01$, (b) attractor of oscillator $i=53$ in coherent cluster, (c) attractor of oscillator $i=1$ in incoherent cluster, (d) spatiotemporal pattern for $d=0.14$, (e) attractor of oscillator $i=79$ in coherent cluster, (f) attractor of oscillator $i=41$ in incoherent cluster, (g) spatiotemporal pattern for $d=2$, and (h) attractor of coherent oscillators.

longs to incoherent limit cycles that their attractors get smaller by getting away from the coherent cluster. (b) shows some of the oscillators attractors.

3.4. Strange attractor's and limit cycle's basin of attraction

Fig. 8 displays the results that are obtained by selecting the initial conditions from two limit cycle's and strange attractor's basins. When the coupling is small, a chimera state occurs that the oscillators are chaotic as shown in Fig. 8(a) for $d=0.01$. By increasing d, still chimera state is observed, but the attractors change to limit cycles as shown in Fig. 8(d) for $d=0.14$. Finally, further increase in d causes fully chaotically synchronization. An example of that with $d=2$ is shown in Fig. 8(g).

3.5. Initial conditions from all basins

Finally, we choose the initial conditions randomly from the whole basins shown in Fig. 2. In this case, the chimera is similar to Fig. 9(a) and both the synchronous and asynchronous oscillators have strange attractors. Fig. 9(b) and (c) show respectively the attractors of the oscillator $i=50$ in the coherent cluster and oscillator $i=40$ in the incoherent cluster.

4. Conclusion

We have studied a network of non-locally coupled 3D system that have coexistence of all types of attractors (fixed-point, limit cycle and strange attractor) and discussed the effect of initial conditions on the emerging spatiotemporal pattern. The initial conditions were chosen randomly in specific regions of basins of attraction. Numerical results showed that when the initial conditions were selected from fixed-point basin of attraction, the network just exhibited synchronous and asynchronous states and in other cases, chimera state was observed. Interestingly for all kinds of initial conditions, when chimera occurred, the attractors of the network were all from one type, all strange or all limit cycle.

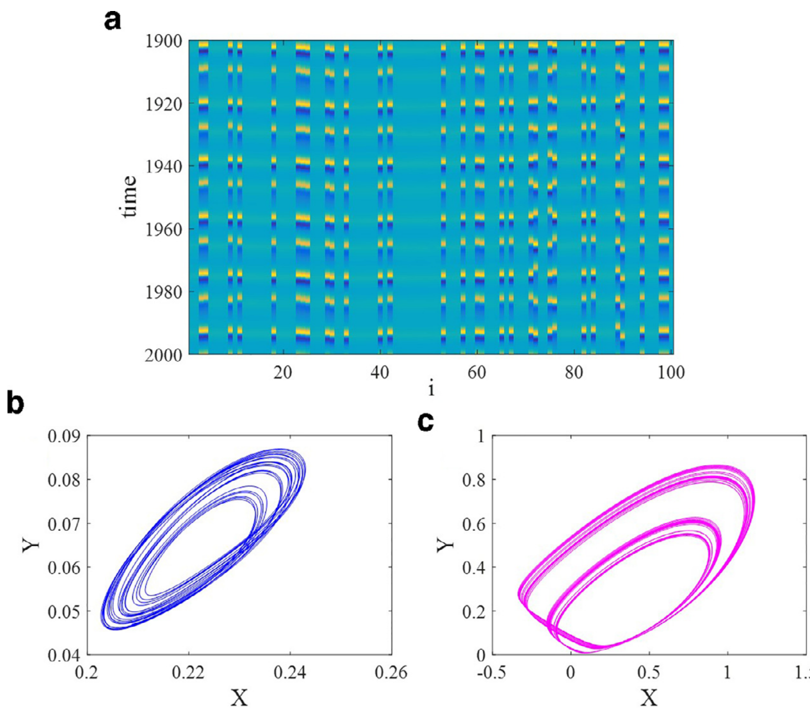

Fig. 9. (a) Spatiotemporal pattern for $d = 0.12$, (b) attractor of oscillator $i = 50$, and (c) attractor of oscillator $i = 40$.

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