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Chaos and complexity in a fractional-order financial system with time delays

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ABSTRACT

Finance and economics are complex nonlinear systems that are affected by various external factors, including of course human action, bilateral relations, conflicts, and policy. Time delays in a financial system take into account the amount of time that passes from a particular policy or decision being made to it actually taking effect. It is thus important to consider time delays as an integral part of modeling in this field. Moreover, many features of financial systems cannot be expressed sufficiently precisely by means of integer-order calculus. Fractional-order calculus alleviates these shortcomings. The aim of this paper is therefore to study the dynamics and complexity in a fractional-order financial system with time delays. We observe fascinating transitions to deterministic chaos, including cascading period doubling, as well as high levels of complexity. This is particularly true in response to variations of derivative orders, which are thus identified as key system parameters.

1. Introduction

Over the past decades, chaos and its applications have attracted remarkable consideration in a variety of fields, including chemistry, ecology, and economy [1]. The high sensitivity to the variations of the environmental situation, system parameters, and initial conditions are some features of chaotic systems. Due to the importance of chaos and its influence in economic and financial systems, the behavior of these systems has been explored frequently and in great detail [2–4].

Nowadays, fractional calculus is likewise attracting a lot of research efforts from various fields in the social and natural sciences. In comparison with integer calculus, the significant advantage of fractional calculus is its memory, and the ability for the description of hereditary properties [5,6]. As it is evident, financial variables, including foreign exchange rates, interest rates, gross domestic product, and stock market prices have long memory. Therefore, it is advantageous, and indeed necessary, to use fractional modeling to describe economic systems [7,8]. Chen has introduced a fractional-order financial system. Dynamic behaviors of the system such as periodic motions, fixed points, chaotic motions, and period-doubling have been studied by Chen [9].

Time delay in a financial system indicates a period of time from one policy or decision being made to taking effect. Although it is challenging to precisely calculate the delay in a financial system, considering time delay in the real system is necessary [10]. Hence, several research studies on economic systems have incorporated time delay into dynamic models. Firstly, Kalecki has proposed economic processes with time delay [11]. Then, due to the application of this system, the study on economic systems with time delay has become a subject of interest in recent years [7,12,13].

Complexity measure is an important technique to analyze the dynamics of a chaotic and hyperchaotic system. There are various methods to measure the complexity of chaotic systems including statistical complexity measure (SCM) [14], fuzzy entropy [15,16], sample entropy [17], spectral entropy (SE) [18], and C_0 algorithm [19]. Among these methods, SE and C_0 algorithms are proper methods to estimate the complexity of a time series accurately without any over-coarse graining preprocessing [18,19].

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A complexity measure C_0 has been proposed by En-hua et al. [19]. This method has two significant properties: (1) obtaining robust estimation even with a short data set; (2) It can be used even for continuous signals without any over-coarse graining preprocessing. The C_0 complexity algorithm removes the regular part of the signal in the frequency domain and leaves the irregular part. The higher the proportion of the energy of the irregular part, that is, the greater the complexity measure result the time series holds. Several research studies have used C_0 algorithms to measure the complexity of chaotic systems [20,21]. However, few papers have used complexity analysis for fractional-order chaotic systems. Also, based on the best of our knowledge, there is no study on the C_0 algorithms for fractional-order financial systems.

Though some dynamical properties of economic and financial systems with time delay have been studied in the literature, there are still other meaningful behaviors of these systems that required to be further understood. In this study, a fractional-order financial system with time delay is investigated. Dynamics and complexity of this system with the variation of derivative orders and system parameters have been studied by means of bifurcation diagram and a complexity measure algorithm.

The rest of this article is planned as follows: Section 3 describes some preliminaries and details the preliminaries and mathematical modeling of the fractional-order time-delayed chaotic financial system. In Section 3, the dynamical behaviors of the system through phase portraits and bifurcation diagrams have been studied. In Section 4, the complexity of the system is analyzed via the multiscale C_0 complexity measure algorithm, followed by conclusions, presented in Section 5.

2. Mathematical model

Among various definitions of fractional integral and derivative, Caputo method has been used in this study. According to Ref. [22], the definition of the Caputo integral and derivative is expressed in the following.

Definition 1. The fractional integral of function $f(t)$ is

$$I^q f(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-\tau)^{q-1} f(s) ds \tag{1}$$

where $t \geq t_0$ and $q > 0$ denotes an integral order. Also, $\Gamma(q)$ indicates the Gamma function and could be obtained as

$$\Gamma(s) = \int_0^{\infty} t^{s-1} e^{-t} dt \tag{2}$$

Definition 2. According to Caputo's definition, fractional-order derivative of function $f(t) \in C^n([t_0, +\infty), R)$ is as follow

$$D^q f(t) = \frac{1}{\Gamma(n-q)} \int_{t_0}^{t} \frac{f^{(n)}(s)}{(t-s)^{q-n+1}} ds, \tag{3}$$

where $t \geq t_0$ and $q > 1$ Also, n is a positive integer parameter which $n-1 \leq q < n$. Furthermore, when $0 < q < 1$ the fractional-order derivative of function $f(t)$ is defined as

$$D^q f(t) = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} \frac{f'(s)}{(t-s)^q} ds \tag{4}$$

Fractional-order financial system with time delay has been proposed in [23] as:

$$\begin{cases} D^{q_1} x_1(t) = x_3(t) + (x_2(t-\tau) - a)x_1(t) \\ D^{q_2} x_2(t) = 1 - bx_2(t) - x_1^2(t-\tau) \\ D^{q_3} x_3(t) = -x_1(t-\tau) - cx_3(t), \end{cases} \tag{5}$$

where state variables $x_1, x_2,$ and x_3 respectively indicate the interest rate, investment demand and price index. $q_i \in (0,\ 1]$ denotes the order of derivatives, τ is a time delay. The parameters $a, b,$ and c represent the saving amount, cost per investment and elasticity of demand of the markets respectively.

3. Dynamics

3.1. Phase portraits

Here, in this section, p–splots of the 0–1 test [24] are used to verify the existence of chaos where a segment of the time series of targeting system is needed. The bounded p–s trajectories imply the underlying dynamics is regular (i.e. periodic or convergent), while Brownian like (unbounded) trajectories imply the underlying dynamics is chaotic. For the given time series $\{x(n), n = 0, 1, 2, \cdots, N-1\}$, the following two real-valued sequences are defined as [24]:

$$\begin{cases} p(n) = \sum_{j=1}^{n} x(j) \cos(\theta(j)) \\ s(n) = \sum_{j=1}^{n} x(j) \sin(\theta(j)) \end{cases}, \tag{6}$$

$$\theta(j) = j\eta + \sum_{i=1}^{j} x(i), \text{ and } \eta \in \left[\frac{\pi}{5}, \frac{4\pi}{5}\right]. \tag{7}$$

Let $q_i = 0.95$ $(i = 1, 2, 3)$, $a = 3$, $b = 0.1$, $c = 1$ and $\tau = 0.03$ the chaotic phase diagram is given in Fig. 1 (a) and its corresponding p–s plot is presented in Fig. 1 (d). When $q_i = 0.95$ $(i = 1, 2, 3)$, $a = 3$, $b = 0.25$, $c = 1$, and $\tau = 0.03$, the periodic circuit is shown in Fig. 1 (b), and the p–s plot is given in Fig. 1 (e). Choosing $q_i = 0.95$ $(i = 1, 2, 3)$, $a = 3$, $b = 0.1$, $c = 2$, and $\tau = 0.03$, the convergent state in the system is observed. Thus, the system has different states with different parameters and chaos is verified.

3.2. Bifurcation diagrams

The bifurcation diagrams of the system with varying derivative orders are plotted, and the results are shown in Fig. 2, where $a = 3$, $b = 0.1$, $c = 1$, and $\tau = 0.03$. In Fig. 2 (a), $q_1 = q_2 = q_3 = q$ and derivative order q varies from 0.8 to 1 with step size of 0.0004. For the rest of the bifurcation diagrams, we let two of the derivative orders equal to one and the third one varies. As a result, q_1 varies from 0.55 to 1 with step size of 0.0009, q_2 varies from 0.8 to 1 with step size of 0.0004, and q_3 varies from 0.825 to 1 with step of 0.00035. It shows in Fig. 1 that the system enters to chaotic state via different route. Fig. 2 (a) (b) and (c), illustrate that the system becomes to chaotic state after a periodic or convergent state although Fig. 2 (d) shows that the system enters to chaotic state after a period-doubling bifurcation. Thus, rich dynamics have been found in the system with variation of derivative orders

Bifurcation diagrams of the financial system with the variation of parameter a, b and c are shown in Figs. 3–5, respectively. The parameter a varies from −5 to 10 while step size is 0.027, and $b = 0.1$, $c = 1$, $\tau = 0.03$. When $q = 0.9$, it is shown in Fig. 3 that the system has a wider range of chaos with the variation of the parameter a comparing with that of $q = 0.95$. The parameter b varies from −0.1 to 0.25 with a step size of 7.0140×10^{-4}, and $a = 3$, $c = 1$, $\tau = 0.03$. It is illustrated in Fig. 4 that the system has the period-doubling bifurcation and goes to chaos, but the inner crisis bifurcation is observed when $q = 0.9$. The parameter c varies from 0.5 to 5 with a step size of 0.003, and $a = 3$, $b = 0.1$, $\tau = 0.03$. Comparing bifurcation diagrams of $q = 0.95$ and $q = 0.9$ depicts that when $q = 0.9$, the responses of the system have different ranges for chaos. The derivative orders can change the bifurcation types and dynamics of the system. It again indicates that the derivative order is a bifurcation of the fractional-order financial chaotic system.

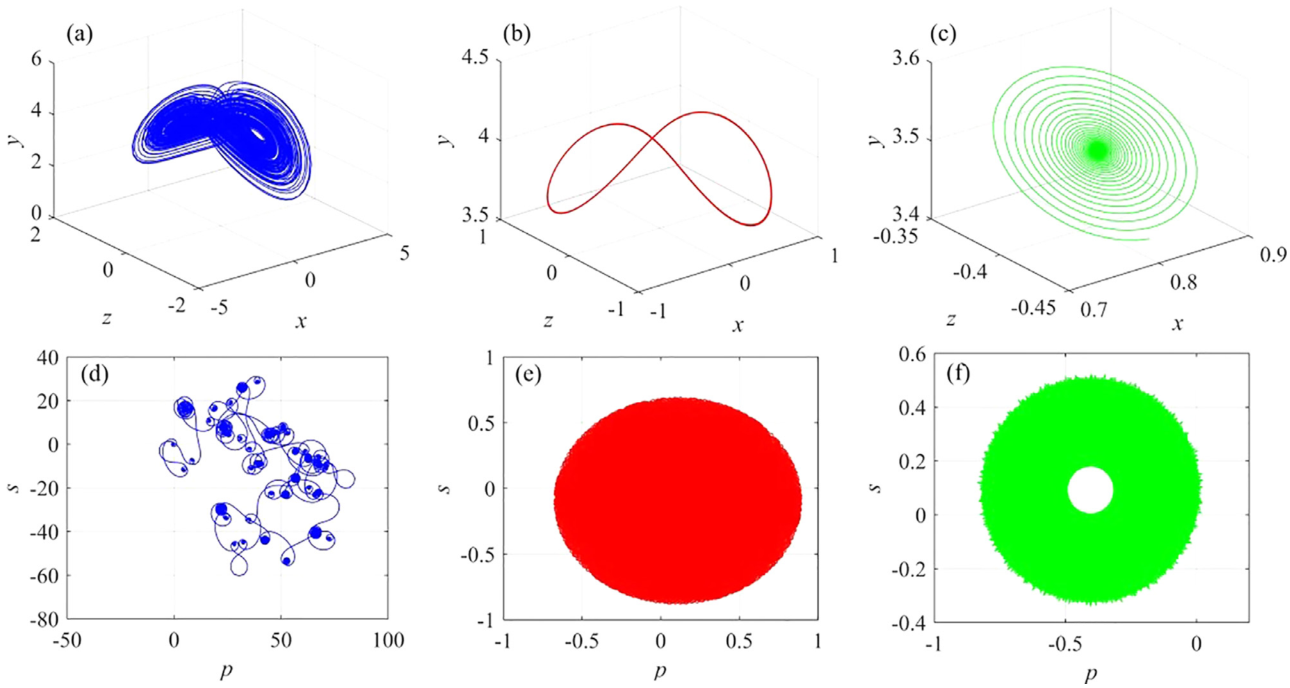

Fig. 1. Phase diagrams and p–s plots of the fractional-order delayed financial system.

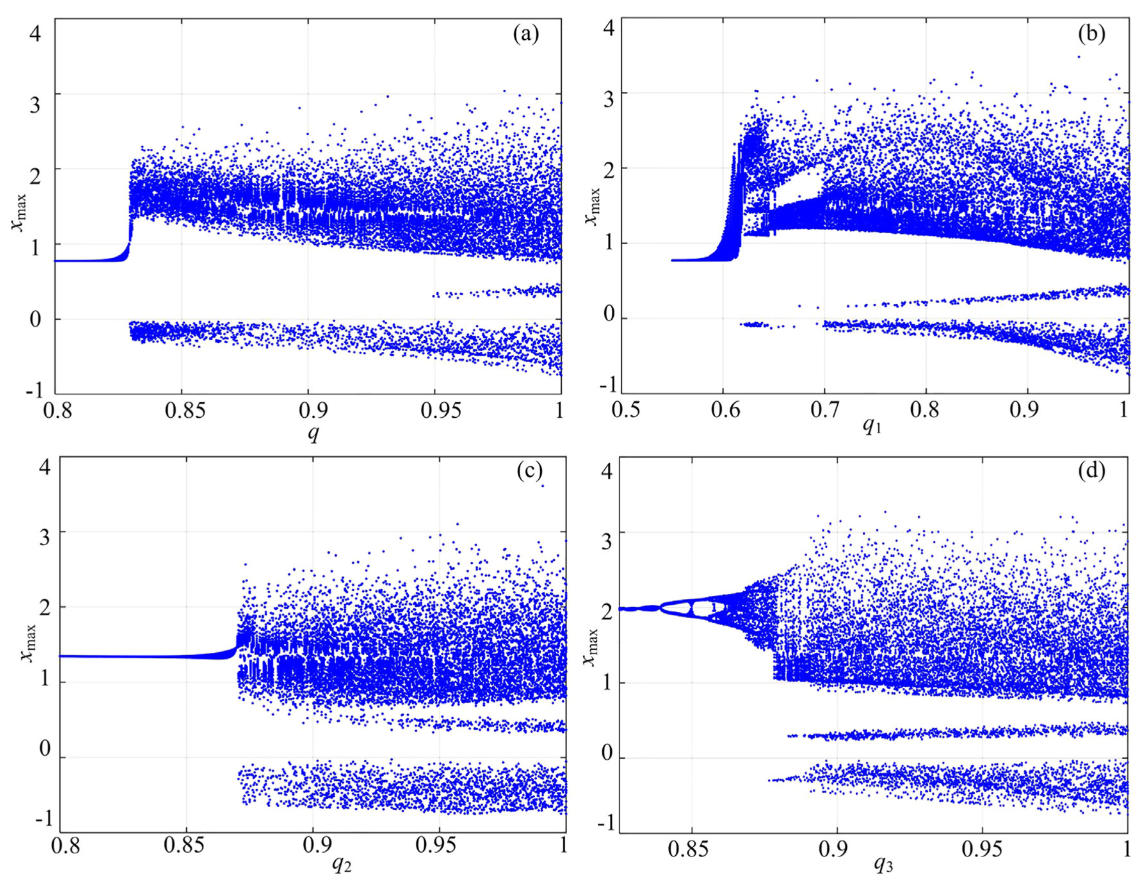

Fig. 2. Bifurcation diagrams of the fractional-order delayed financial system with derivative order varying (a) $q = q_1 = q_2 = q_3$; (b) q_1 varying, $q_2 = q_3 = 1$; (c) $q_2$varying, $q_1 = q_3 = 1$; (d) q_3 varying, $q_1 = q_2 = 1$.

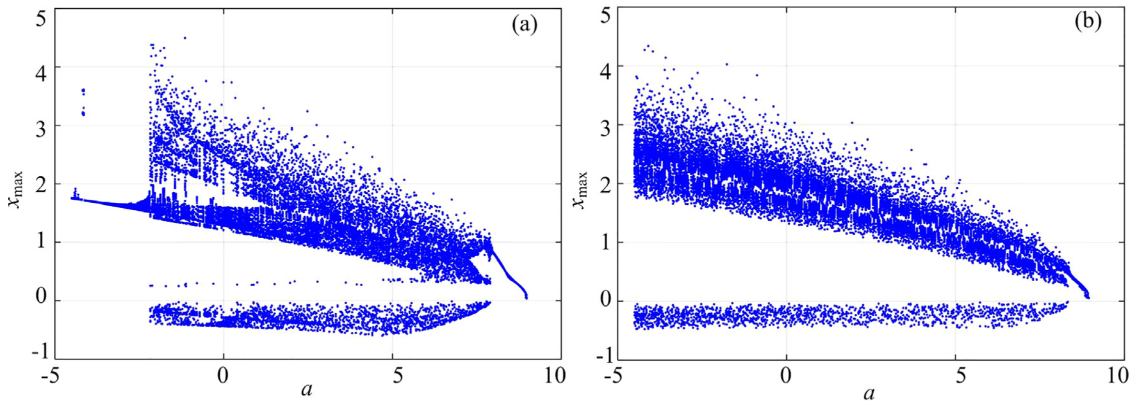

Fig. 3. Bifurcation diagrams of the fractional-order delayed financial system with parameter a varying, $b=0.1$, $c=1$ and $\tau=0.03$ (a) $q=0.95$; (b) $q=0.9$.

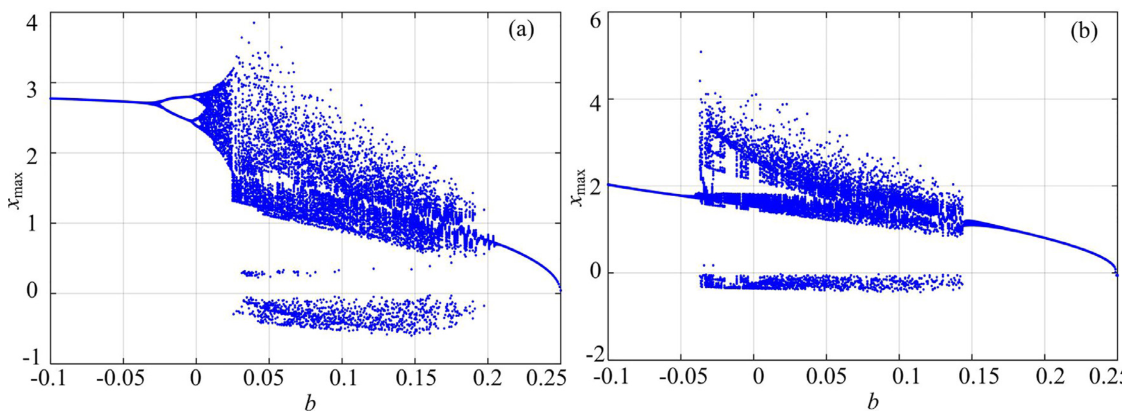

Fig. 4. Bifurcation diagrams of the fractional-order delayed financial system with parameter b varying, $a=3$, $c=1$ and $\tau=0.03$ (a) $q=0.95$; (b) $q=0.9$.

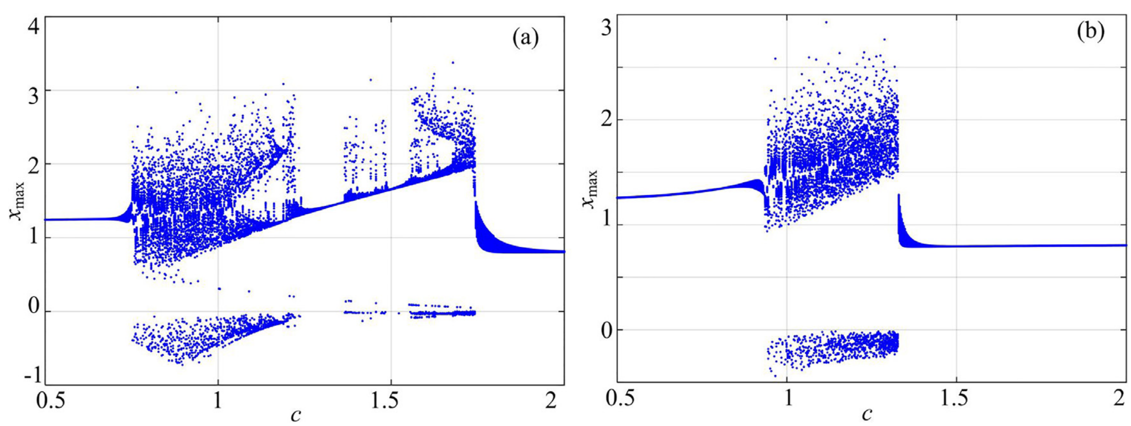

Fig. 5. Bifurcation diagrams of the fractional-order delayed financial system with parameter c varying, $a=3$, $b=0.1$ and τ=0.03 (a) $q=0.95$; (b) $q=0.9$.

4. Complexity

In this section, the complexity of the chaotic system is analyzed using the multiscale C_0 complexity measure algorithm [19,25]. Firstly, the calculation processes of the C_0 algorithm are presented.

Step 1: Remove the average value. For a given time series, $\{x(n), n=0,1,2,\ldots,N-1\}$, the average value is removed by

$$x(n)=x(n)-\bar{x} \tag{8}$$

where $\bar{x}=\frac{1}{N}\sum_{n=0}^{N-1}x(n)$.

Step 2: Fourier transform. The Fourier transform of the time series is given by

$$X(k)=\sum_{n=0}^{N-1}x(n)e^{-j\frac{2\pi}{N}nk}, \tag{9}$$

where $k=0,1,2,\ldots,N-1$.

Step 3: Remove the irregular part. Set the

$$G_N=\frac{1}{N}\sum_{k=0}^{N-1}|X(k)|^2$$

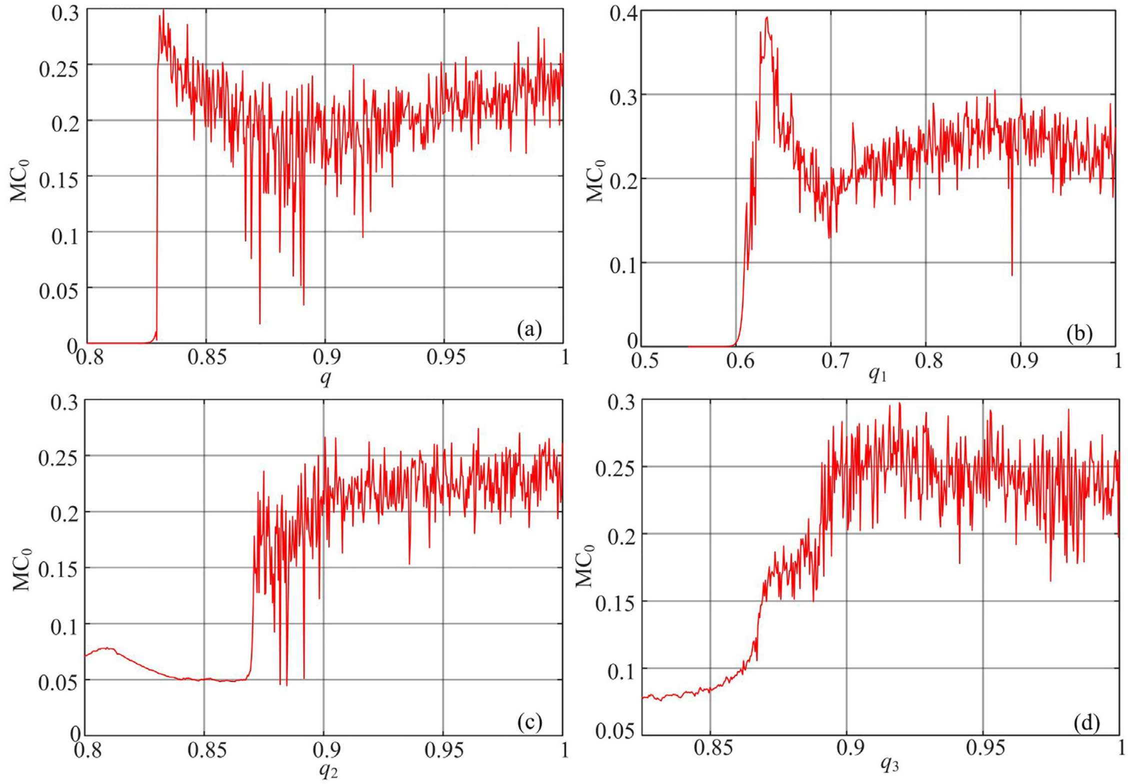

Fig. 6. Complexity plots of the fractional-order delayed financial system with derivative order varying (a) $q = q_1 = q_2 = q_3$; (b) q_1 varying, $q_2 = q_3 = 1$; (c) q_2 varying, $q_1 = q_3 = 1$; (d) q_3 varying, $q_1 = q_2 = 1$.

Introduce a control parameter r. Keep the frequency which is larger than r times of GN, and set the rest as zero, which is

$$\tilde{X}(k) = \begin{cases} X(k) & if\,|X(k)|^2 > rG_N \\ 0 & if\,|X(k)|^2 \le rG_N \end{cases} \tag{10}$$

The inverse DFT of $\tilde{X}(k)$ is

$$\tilde{x}(n) = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}(k) e^{\frac{j2\pi nk}{N}} \tag{11}$$

where $n = 0, 1, \ldots, N-1$ and $\tilde{x}(n)$ reflects the regular part of the time series with detail information removed.

Step 4: Calculate the C_0 complexity. By comparing the summation of the irregular part and the summation of the original time series, the C_0 complexity is defined by

$$C_0(x, r, N) = \frac{\sum_{n=0}^{N-1} |x(n) - \tilde{x}(n)|^2}{\sum_{n=0}^{N-1} |x(n)|^2} \tag{12}$$

The multiscale coarse graining of the $\{x(n), n = 1, 2, \ldots, N-1\}$ is given by [26]:

$$y^s(j) = \frac{1}{s}\sum_{i=(j-1)s+1}^{js} x(i), \tag{13}$$

where $1 \le j \le \lfloor N/s \rfloor$, s is the scale factor, $\lfloor \cdot \rfloor$ is the floor function and y^s is the multiscale time series. Finally, the multiscale C_0 algorithm is denoted as

$$MC_0(x, r, s, N) = \frac{1}{s}\sum_{i=1}^{s} C_0(y^s, r, N) \tag{14}$$

The MC_0 analysis results of the fractional-order financial time-delayed system with derivative orders and parameters varying are shown in Figs. 6–9. Here, the same parameter setting and step size are used as with the corresponding bifurcation diagrams. As it is demonstrated in these MC_0 plots, they match well the corresponding bifurcation diagrams. When the system is chaotic, higher complexity measure results are obtained, while if the system is non-chaotic, the measured results are relatively smaller. According to Fig. 6, complexity of the system changes with derivative orders, and it is shown in Fig. 6(a) and (b) that complexity of the system could be higher when the derivative order q and q_1 take relative smaller values. Meanwhile, as shown in Figs. 7 and 8 that complexity of the decrease with the increase of the parameters a and b. However, according to Fig. 9, MC0 complexity of the system increases with the increase of parameter c. Obviously, MC0 provides different information on the dynamics of the system comparing with the bifurcation diagrams. We can know complexity variation trend of the system clearly.

To better analyze the complexity of the fractional-order chaotic system, MC_0 complexity in the parameter plane is investigated. Fig. 10 shows the MC_0 complexity analysis results of the system in the parameter q-a plane. The derivative order q varies from 0.8 to 1 with a step size of 0.002, and the parameter a varies from −4.5 to 9 with step size of 0.135. As shown in Fig. 10, the system has wide region of high complexity in the q-a parameter plane. Generally, complexity of the system decreases with increase of derivative order q and the parameter a. Let the parameter b vary from −0.1 to 0.25 with step size of 0.0035. The contour plot in the parameter q-b is presented in Fig. 11. It illustrates that the range of high complexity region over parameter b changes with the derivative order q. Meanwhile, it also shows that the system has relative lower complexity when the parameter b increases. Moreover, MC_0 complexity in the parameter q–c plane is investigated by Fig. 12, where c varies from 0.5 to 2 with step size of 0.015. It shows the high complexity region in the parameter plane. Since the complexity measure result of the system can be obtained

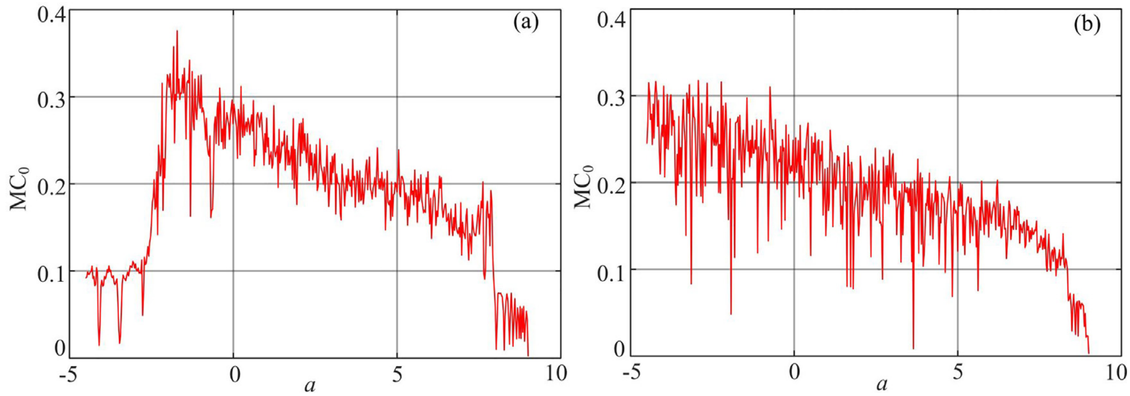

Fig. 7. Complexity plots of the fractional-order delayed financial system with parameter a varying, $b = 0.1$, $c = 1$ and $\tau = 0.03$ (a) $q = 0.95$; (b) $q = 0.9$.

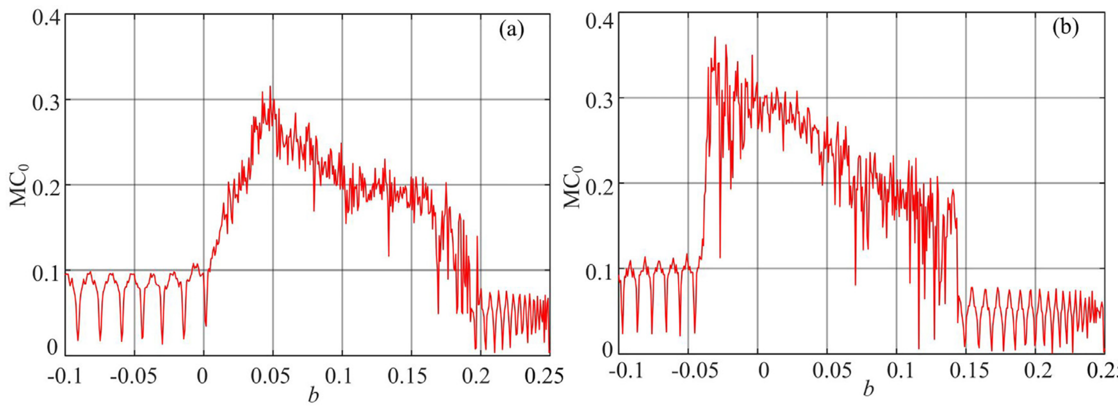

Fig. 8. Complexity plots of the fractional-order delayed financial system with parameter b varying, $a = 3$, $c = 1$ and $\tau = 0.03$ (a) $q = 0.95$; (b) $q = 0.9$.

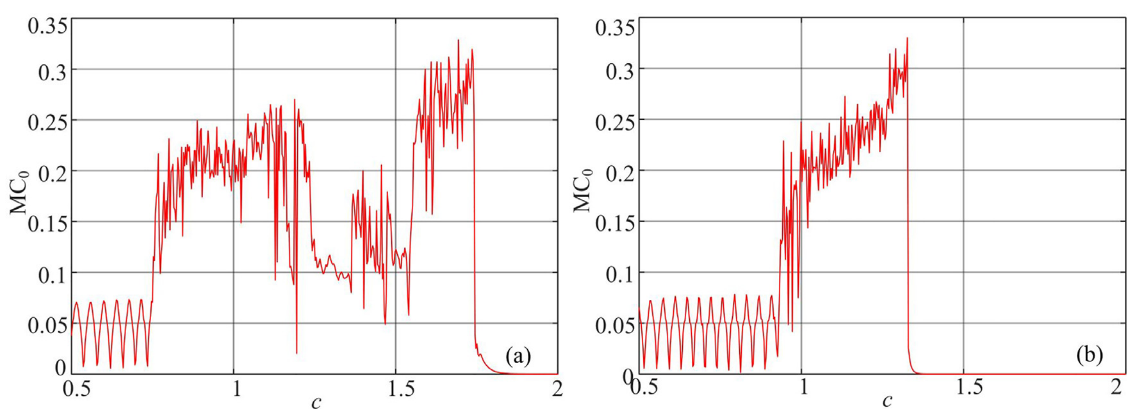

Fig. 9. Complexity plots of the fractional-order delayed financial system with parameter c varying, $a = 3$, $b = 0.1$ and $\tau = 0.03$ (a) $q = 0.95$; (b) $q = 0.9$.

with a segment of time series. Meanwhile, because a time series with higher complexity has better randomness and security for real applications, MC_0 algorithm analysis results provide a useful reference for the real applications of the fractional-order delayed financial system with incommensurate orders.

5. Discussion

As it is well known and quite obvious, we do not need chaos in the financial system, at least in most cases. It means that the system is unstable and the economic situation has become unpredictable. In this paper, we diagnosed the statue of the financial system by means of complexity method using the time series. And MC_0 complexity measure algorithm provides an effective method. If high complexity measure result is observed, it means that the system has complex dynamical behavior and some necessary control or policies should be considered.

As presented in the above analysis, the system has different states with different system parameters, time delay and derivative orders. However, for the decision-makers, the derivative order relates to the model itself. It provides some better explanations when the integer model cannot. On the other hand, the decision-makers can control the system by adjusting the system parameters, because the system with some parameters has low complexity even with different derivative orders. Meanwhile, the time delay is an important factor for the system. It is one of the important reason for generating chaos in the system. Thus, we hold the opinion that, the delay in the interest rate and investment demand can make the system become chaotic. In conclusion, the financial

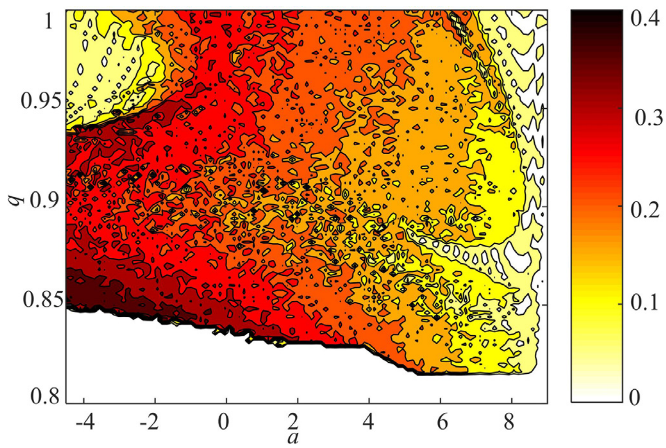

Fig. 10. Contour plot of the fractional-order delayed financial system in the *q*-*a* parameter plane.

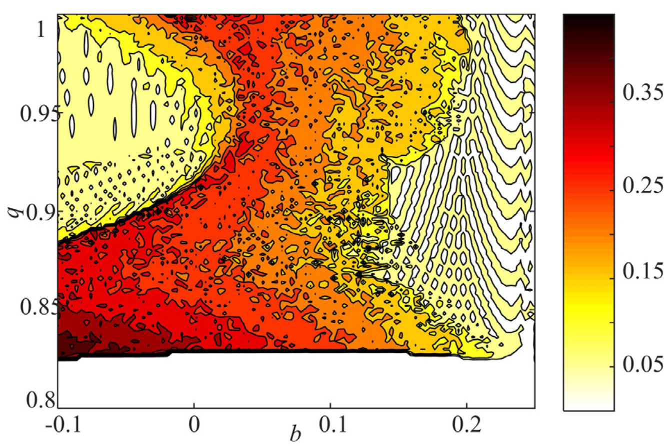

Fig. 11. Contour plot of the fractional-order delayed financial system in the *q*-*b* parameter plane.

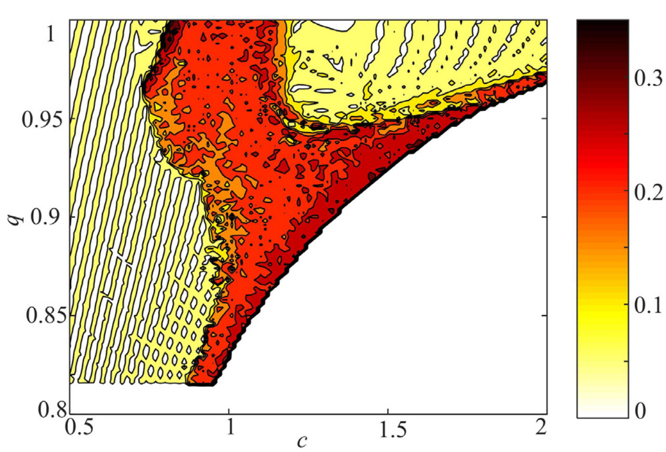

Fig. 12. Contour plot of the fractional-order delayed financial system in the *q*-*c* parameter plane.

system has complex behaviors due to many reasons, and MC_0 algorithm provides an effective index for the complexity analysis of the system. The decision-makers should pay close attention to such complex systems, and the regulatory lapses which relate to the delay should be avoided.

6. Conclusions

In the present study, the dynamic behavior of a fractional-order time-delayed chaotic financial system was investigated. The different aspects of dynamic behaviors of the system including phase diagrams and their corresponding *p*–*s* plots, bifurcation diagrams with derivative orders and parameters, C_0 complexity with derivative orders and parameters were studied. It was found that chaos and different states can be observed with different parameters. Especially, we analyzed the complexity of the system in different parameter planes by showing the contour plots. The derivative orders are the bifurcation parameters essentially since the system has rich dynamics with those derivative orders. Meanwhile, C_0 complexity shows the changes of the complexity in the system. It also shows that the system has wide region of high complexity in the corresponding parameter planes.

Declaration of Competing Interest

Matjaz Perc is Editor of Chaos, Solitons & Fractals. In keeping with Elsevier's guidelines on potential editorial conflicts of interest, manuscripts coauthored by one of the Editors will be handled fully by other Editors or the Editor-in-Chief in an undisclosed review process. We have no conflicts of interest.

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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