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Rewarding endowments lead to a win-win in the evolution of public cooperation and the accumulation of common resources

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ABSTRACT

We consider different resource allocations of rewarding endowments in the collective-risk social dilemma, and we study their impact on the evolution of public cooperation and the accumulation of common resources in structured populations. We assume that if the accumulated resources in the common pool meet the basic demands of everybody in the group, then each group member obtains an equal basic endowment. However, if the resources in the group exceed this sum, then each group member can get an additional rewarding endowment from the common-pool resource. By means of Monte Carlo simulations, we find that the consideration of rewarding endowment is favorable for the evolution of cooperation. But the common resources may be exhausted if the rewarding is too frequent or too generous. Interestingly, we do find a parameter region in the basic endowment and the reward intensity in which cooperation is promoted whilst the common resources are maintained. We introduce a quantitative index to precisely identify this parameter region, and find that such win-win situations for the evolution of cooperation and the maintenance of common resources occur when the basic endowment is low and the reward intensity is intermediate.

1. Introduction

With the rapid development of human society, ensuring the effective maintenance of common resources has become a global challenge [1,2]. The key to solve the challenge is to promote public cooperation among involved individuals. As a typical paradigm for studying how to promote public cooperation, the public goods game (PGG) has received considerable attention in recent years [3–11]. In the game, individuals are best off by contributing nothing to the common pool. But if nobody contributes, then the community in the game will fail to harvest the benefits, which leads to the collapse of cooperation and the tragedy of the commons finally [2]. In order to promote public cooperation in the game, many mechanisms, such as punishment [12–22], reward [23–28], and network reciprocity [29–38] have been proposed and studied in the last decades.

However, it is worth mentioning that these mechanisms mentioned above are investigated on the basis of the classical PGG. In fact, the classical PGG does not consider the feedback between individual behaviors and the environment, which is an unignored factor reported by some recent works [39–48]. Thus the traditional PGG may not capture the essence of the evolution of public cooperation driven by the proposed mechanisms. Alternatively, the game with environmental feedback, which links the evolution of public cooperation with the governance of common resources, can be used to study the coevolutionary outcomes of public cooperation and the accumulation of common resources and could be more meaningful.

On the other hand, providing additional rewarding endowment from the common pool for prosocial individuals is often used in human society [27,49]. In an enterprise, for instance, in addition to the basic wage an employee from a department will be provided with an extra bonus according to his/her working performance. Such regime of resource allocation may not only make employees work more efficiently, but also increase the enterprise's

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whole profit. It is found that this kind of resource allocation can solve the second-order free-rider problem in the classical PGG [28]. However, this kind of resource allocation for the common pool has not yet been considered into the PGG with environmental feedback, and thus it is still unclear how the rewarding endowment influences the evolution of cooperation and the accumulation of common resources.

Inspired by the above consideration, in this work we consider the resource allocation with the rewarding endowment into the PGG with environmental feedback proposed by Chen and Perc [41], which is a collective-risk social dilemma game where the group's common-pool resource is adjustable based on the group members' behaviors. We assume that if the common-pool accumulated resource can meet the group's basic demand, then each group member can obtain a basic endowment. Otherwise, all the common-pool resource is equally allocated to the group members. Furthermore, if contributing resource in the group is beyond the sum of all members' basic demands, then each group member can get an additional rewarding endowment from the common-pool resource. By means of Monte Carlo simulations, we find that the consideration of rewarding endowment can promote the evolution of cooperation in spatially structured populations, and the fraction of cooperators increases with the rewarding intensity. In addition, we find that high value of basic endowment can lead to the enhancement of cooperation, but it is detrimental to the maintenance of the common resources. Interestingly, when the basic endowment is low and the rewarding intensity is intermediate, a win-win situation where the common resource can be greatly accumulated and meanwhile cooperation can be effectively promoted can be achieved. Furthermore, we demonstrate that these observations are robust against our model parameters.

2. Model

We consider that the game is played on a square lattice of size $L \times L$ with periodic boundary conditions. Each player on site x with von Neumann neighborhood is a member of five overlapping groups of size $G = 5$. Besides, each player is initially designated either as a cooperator ($s_x = 1$) or defector ($s_x = 0$) with equal probability. Cooperators contribute a fixed amount c to the group, while defectors contribute nothing. Subsequently, the sum of all contributions in each group i is multiplied by the synergy factor α [41,46], which represents the contributing resources of cooperators to the group. As noted before, we assume that every player will respectively receive an assigned endowment and an additional rewarding endowment based on the amounts of common resources and contributing resources. In addition, we assume that the assigned endowment is endowed for players with a priority in the regime of resource allocation. To be specific, we assume that at time t the assigned endowment $b_x^i(t)$ from group i is given as

$$b_x^i(t) = \begin{cases} b & \text{if} \quad R^i(t-1) + \sum_{x \in i} s_x \alpha c \geq Gb, \\ [R^i(t-1) + \sum_{x \in i} s_x \alpha c]/G & \text{if} \quad R^i(t-1) + \sum_{x \in i} s_x \alpha c < Gb, \end{cases} \tag{1}$$

where $R^i(t-1)$ is the amount of common resources (public goods) available to the group i at time $t-1$ and b represents the basic endowment assigned to a player from the group's common resource [41,46].

Furthermore, we consider that group members will be endowed with an additional reward endowment if the contributing resources at time t by cooperators in the group is productive. To be specific, we assume that at time t the rewarding endowment $e_x^i(t)$ which player x can receive from group i is given as

$$e_x^i(t) = \begin{cases} \sum_{x \in i} (s_x \alpha c - b)\delta / G & \text{if} \quad \sum_{x \in i} s_x \alpha c \geq Gb, \\ 0 & \text{if} \quad \sum_{x \in i} s_x \alpha c < Gb, \end{cases} \tag{2}$$

where δ ($0 \leq \delta \leq 1$) is the rewarding intensity. Thus, at time t the total endowment $a_x^i(t)$ of player x from group i is given as

$$a_x^i(t) = b_x^i(t) + e_x^i(t). \tag{3}$$

Consequently, the payoff of player x from group i at time t is $P_x^i(t) = a_x^i - s_x c$. Due to the overlapping groups, the total income $P_x(t)$ of player x is simply the sum over all $P_x^i(t)$ received from five overlapping groups where x is a member.

Starting with $R^i(0) = R_0$ in all groups, the amount of accumulated common resources in each group i is updated according to

$$R^i(t) = R^i(t-1) + \sum_{x \in i} [s_x \alpha c - a_x^i(t)], \tag{4}$$

where $R^i(t)$ is the amount of common resource available to group i at time t. For simplicity, we set $c = 1$ in this study.

After each round of the game, player x is given the opportunity to imitate the strategy of one randomly selected neighbor y. The strategy transfer occurs with the probability

$$q = \frac{1}{1 + exp[(P_x(t) - P_y(t))/K]}, \tag{5}$$

where K is the uncertainty by strategy adoptions [50]. Without losing generality, we use $K = 0.5$, so that it is very likely that better performing players will be imitated.

As one key quantity, we measure the fraction of cooperators f_c at the equilibrium state in the population for characterizing the cooperative behavior of our computational system. Another key quantity which we focus on is the amount of common resources R over all the interaction groups when the system reaches the equilibrium state. In our study, synchronous updating protocol is applied, and all the simulation results are averaged over 100 different realizations of initial conditions.

3. Results

We begin by presenting the stationary fraction of cooperators f_c and the amount of common resources R in dependence on both the reward intensity δ and the basic endowment b, respectively, as shown in Fig. 1. In Fig. 1(a), we can find that the fraction of cooperators f_c increases with increasing δ (from left to right) when the value of b is less than 7.8. And full cooperation can be always achieved when $b > 7.9$, no matter what value of δ it is. This suggests that the reward intensity has no obvious effect on the evolution of cooperation when the basic endowment is high. We can also find that the f_c value increases with increasing b (from bottom to top) when δ is low ($\delta < 0.52$). This suggests that the potential way to maintain cooperation is to increase the basic endowment when the reward intensity is not large enough. In addition, f_c can always reach one when $\delta > 0.94$. However, the impacts of b on f_c is not straightforward when δ is in the range of $0.52 < \delta < 0.94$. To be specific, with the increase of basic endowment b, the stationary fraction of cooperators f_c first decreases from one until reaching the minimal value, and then increases to one. This indicates that the influence of the basic endowment on the evolution of cooperation is dependent on the reward intensity.

We further find that the common resources cannot be effectively maintained for any value of δ when the basic endowment is large as shown in Fig. 1(b), even if full cooperation can be achieved in the parameter region. This is because large b is detrimental to the accumulation of common resource in the pool [41]. Whereas

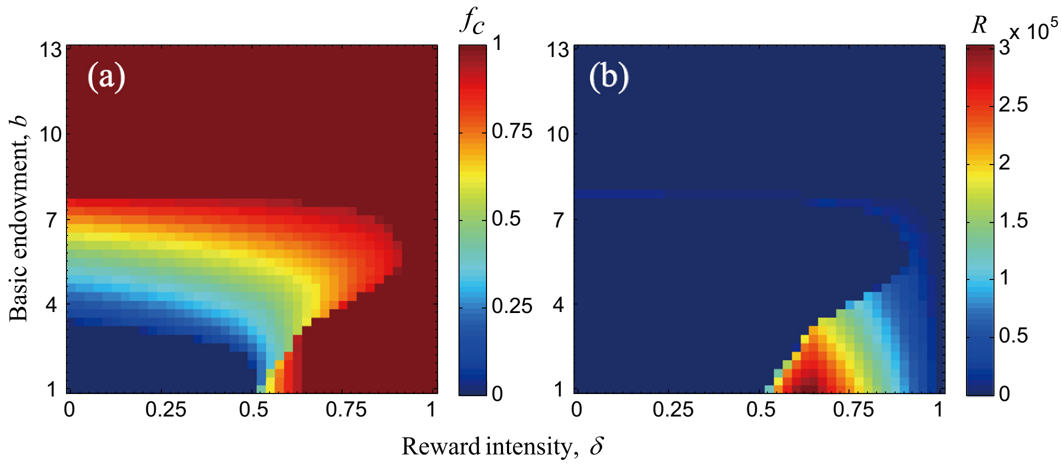

Fig. 1. Panel (a) shows the fraction of cooperators in dependence on the reward intensity δ and the basic endowment b. Panel (b) shows the amount of common resources in dependence on the reward intensity δ and the basic endowment b. Here, $\alpha = 8$, $R_0 = 20$, and $L = 100$.

when the basic endowment is small, we find that the common resources can only be accumulated at an intermediate range of the reward intensity, which can indeed lead to high fraction of cooperators for the sufficient contributions to the common pools. Together with Fig. 1(a) and (b), we can realize that there exists a parameter region of the basic endowment and the reward intensity in which the common resource can be greatly accumulated and meanwhile cooperation can be effectively maintained. In other words, a win-win situation for the promotion of cooperation and the accumulation of common resources can achieved in this parameter region.

In order to precisely identify the parameter region of the basic endowment and the reward intensity for the win-win situation, we herein introduce a quantitative index $I_{b,\delta}$ to characterize both the cooperation level and the accumulation of common resources, which is defined as

$$I_{b,\delta} = \eta \frac{f_c^{b,\delta}}{f_c^{max}} + (1-\eta)\frac{R_{b,\delta}}{R^{max}}, \tag{6}$$

where η $(0 < \eta < 1)$ is a parameter characterizing the relative weight between the cooperation level and the accumulation of common resources in the index $I_{b,\delta}$. $f_c^{b,\delta}$ represents the stationary fraction of cooperators in dependence on the basic endowment b and the reward intensity δ. $R_{b,\delta}$ represents the amount of common resources in dependence on the basic endowment b and the reward intensity δ. Here, $f_c^{\max}$ and $R^{\max}$ are respectively the maximal values of the fraction of cooperators and the amount of common resources in the parameter region b and δ. Without loss of generality, $f_c^{b,\delta}$ and $R_{b,\delta}$ are respectively divided by $f_c^{\max}$ and $R^{\max}$, respectively, and accordingly normalized. Accordingly, through these proper normalizations, the $I_{b,\delta}$ value is constrained between zero and one.

We then present the index $I_{b,\delta}$ in dependence on both the basic demand b and the reward intensity δ as shown in Fig. 2. We can clearly find that when the basic endowment b is larger than 7.9, the $I_{b,\delta}$ value cannot reach the maximal value for any values of δ, even if full cooperation can be achieved. Instead, the $I_{b,\delta}$ value is around 0.5 in this parameter region. When the basic endowment b is intermediate ($4 < b < 7.9$), the $I_{b,\delta}$ value increases with increasing δ. And when the basic endowment is less than 4, we find that the $I_{b,\delta}$ value first increases until reaching the maximal value, and then decreases with increasing the reward intensity δ. In addition, for low δ, $I_{b,\delta}$ increases with increasing the basic endowment b; for moderate δ, $I_{b,\delta}$ decreases from the maximal value with increasing b; for large δ, the $I_{b,\delta}$ value only has tiny changes with increasing b. We thus can conclude that there exists an optimal parameter region of $(b,\ \delta)$ in which the b value is small and the δ value is intermediate, which can lead to the maximal value of $I_{b,\delta}$. Furthermore, we emphasize that in Fig. 2, η is fixed at 0.5. However, even if the η value is appropriately adjusted, the optimal region of $(b,\ \delta)$ still exists for the occurrence of win-win situation.

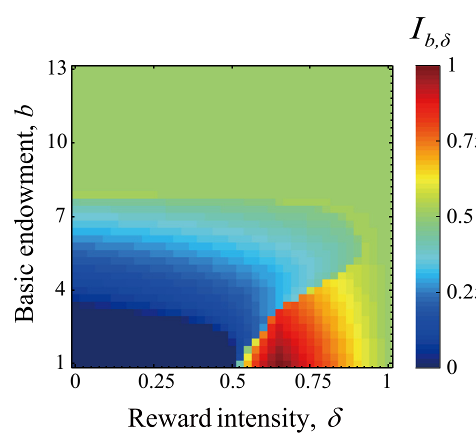

Fig. 2. The index $I_{b,\delta}$ in dependence on the reward intensity δ and the basic endowment b. Parameters: $\alpha = 8$, $R_0 = 20$, $L = 100$, and $\eta = 0.5$.

In what follows, in order to intuitively understand the nontrivial impact of basic demand b on the evolution of cooperation for an intermediate δ, which is different from the finding in Ref. [41], we show some typical snapshots as shown in Fig. 3. To do that, we use different colors not just for cooperators and defectors, but also depending on the amount of common resources. To be specific, blue (yellow) denote cooperators (defectors) that are central to groups where $R_i(t-1) + \sum_{x\in i}(s_x\alpha c - a_x^i(t)) \geq Gb$. Green (red) denote cooperators (defectors) that are central to groups where $0 < R_i(t-1) + \sum_{x\in i}(s_x\alpha c - a_x^i(t)) < Gb$. Grey are defectors where $R_i(t-1) + \sum_{x\in i}(s_x\alpha c - a_x^i(t)) = 0$. Besides, cooperators and defectors are initially distributed uniformly at random.

For low b (top row of Fig. 3), cooperation can be effectively maintained. At first, yellow defectors expand rapidly by exploiting the resource. However, blue cooperators can form clusters on a small-scale. Accordingly, cooperators in the clusters have an evolutionary advantage because they can get more rewarded resources due to the low b value and subsequently these blue clusters can expand (see Fig. 4). On the contrary, due to the exhaustion of com-

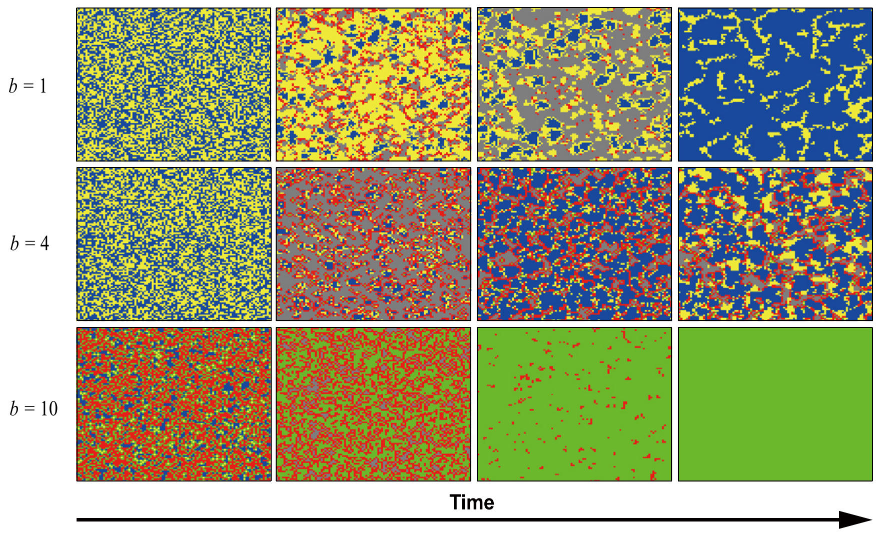

Fig. 3. Blue (yellow) are cooperators (defectors) that are central to groups in which each player can obtain the basic endowment, while green (red) are cooperators (defectors) that are central to groups in which each player can only obtain a part of basic endowment. Grey are defectors that are central to groups in which no resource can be allocated. Top row shows the time evolution of spatial patterns for $b = 1$. Middle row shows the time evolution of spatial patterns for $b = 4$. Bottom row shows the time evolution of spatial patterns for $b = 10$. Other parameters: $\delta = 0.6$, $R_0 = 20$, and $L = 100$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

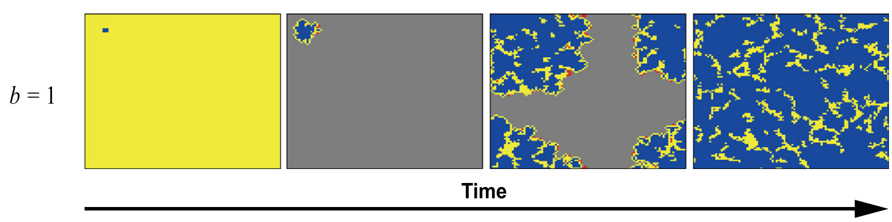

Fig. 4. Pattern formations as observed from a prepared initial state in which a domain of nine blue cooperators are present in a sea of yellow defectors for $b = 1$. Parameters: $\delta = 0.6$, $R_0 = 20$, and $L = 100$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

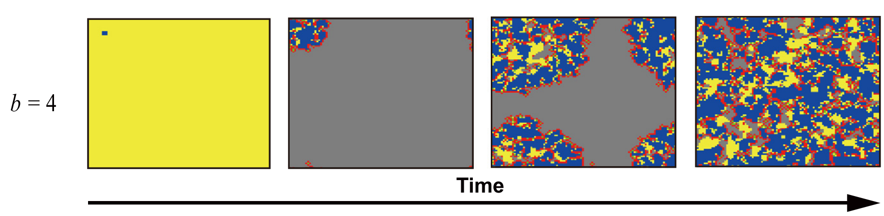

Fig. 5. Pattern formations as observed from a prepared initial state in which a domain of nine blue cooperators are present in a sea of yellow defectors for $b = 4$. Parameters: $\delta = 0.6$, $R_0 = 20$, and $L = 100$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

mon resources yellow defectors first turn to red, and then grey patches occur. These defectors become easy targets once being exposed to blue cooperators. Hence blue cooperators are able to further spread. On the other hand, defectors who are adjacent to these blue cooperators in the expanded clusters will turn to yellow since their neighboring cooperators are able to provide enough resources for them. And they can survive even against blue domains of cooperators. However, due to the emergence of large compact cooperative clusters, cooperators can prevail in this situation.

For intermediate b (middle row of Fig. 3), the scenario is different. At first, yellow defectors can get more resources due to the increase of b. However, the common resources in yellow domains are depleted faster. Then yellow defectors turn to red and then grey patches emerge. On the other hand, blue cooperators cannot get more rewarded resources due to the increase of b. Hence blue cooperators cannot form largely compact clusters. On the contrary, blue cooperators become separated by strips of red defectors (see Fig. 5). And they do not have too much evolutionary advantages over their defective neighbors since the latter can benefit from these neighboring blue domains. Subsequently, yellow defectors emerge in large numbers, which can be detrimental to the evolution of blue cooperators. Accordingly, cooperation cannot be greatly maintained.

For large b (bottom row of Fig. 3), the situation changes again. Note that the resources have always been consumed because the common resource in common pool always do not meet every member's large basic demand b. Hence, green cooperators and red defectors emerge quickly. Correspondingly, each member of the group receives an equal share of the public good, and cooperators have an evolutionary advantage for the large b value [41]. Hence green cooperators can dominate the whole population, although the common resource in each group cannot be sustained.

Finally, we are interested in investigating whether the impacts of basic demand b and reward intensity δ on the fraction of cooperators f_c still exist when the synergy factor α is changed. Accordingly, we first show the fraction of cooperators f_c as a function of the basic endowment b for different values of α as shown in Fig. 6(a). We can find that f_c first decreases until reaching the

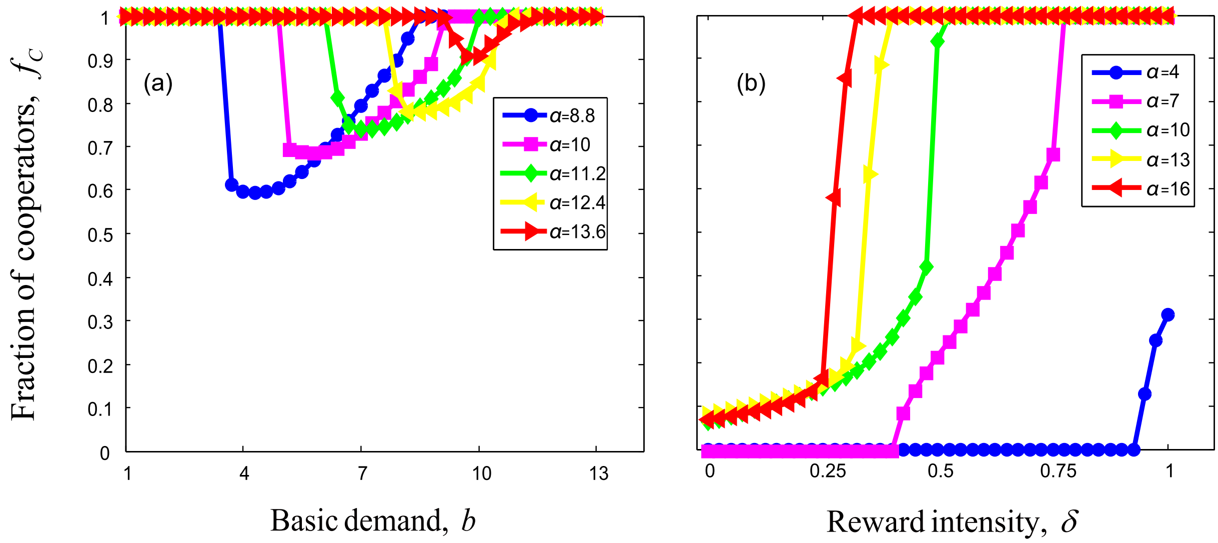

Fig. 6. Panel (a) shows the fraction of cooperators f_c as a function of b for fixed reward intensity $\delta = 0.6$ and different synergy factor α values. Panel (b) shows the fraction of cooperators f_c as a function of δ for fixed basic demand $b = 3$ and different synergy factor α values. Here, $R_0 = 20$ and $L = 100$.

minimal value and then gradually increases to one with increasing b for each value of α. And the corresponding minimal value of f_c increases with increasing α. These results indicate that the nontrivial impact of b on f_c is still in existence when α is in a certain range. In Fig. 6(b), we show the fraction of cooperators as a function of the reward intensity δ for different values of the synergy factor α. We can find that for each α value, f_c increases with increasing δ. It indicates that large reward intensity can still promote cooperation even if the α value is changed.

4. Discussion

To summarize, we have considered the resource allocation with the rewarding endowment into the collective-risk social dilemma and studied its impacts on the evolution of cooperation and common resources in spatially structured populations. We find that the consideration of rewarding endowment is favorable for the evolution of cooperation. And the fraction of cooperators in the population increases with increasing the rewarding intensity. However, the common resources will be exhausted if the reward intensity is too large, even full cooperation can be achieved. In addition, we find that when the basic endowment is high, full cooperation can be achieved no matter how much the reward intensity is. However, the common resources cannot be effectively sustained for high basic endowment. Interestingly, we find that there exists a parameter region of the basic endowment and the reward intensity in which cooperation can be promoted and meanwhile the common resources can be maintained. We further introduce the quantitative index to precisely identify this parameter region, and find that such win-win situation for the evolution of cooperation and the maintenance of common resources can happen when the basic endowment is low and the reward intensity is intermediate. We further check that these observations can be still found when the values of other model parameters are changed.

In this work, we consider the resource allocation in a population of individuals who play the PGG with environmental feedback, motivated by the regimes in realistic resource management systems. Accordingly, in our proposed model individual's payoff is not only influenced by the behavior choice, but also influenced by the accumulation of common resources. In addition, it is dependent on the regime of resource allocation in which the basic endowment and rewarding endowment are both considered. Furthermore, we not only focus on the evolution of cooperation, but also focus on the accumulation of common resources. Hence, different from previous works [28,41], our work is further strengthened on the study of the governance of the commons. We find that there exists the win-win situation, which can be present by properly adjusting the basic endowment and the rewarding intensity. Hence our work may provide some insights into the design of allocation scheme for the maintenance of common resources, and we also hope that our study will inspire further research aimed at studying the evolution of cooperation and the maintenance of common resources.

Conflict of Interest

Matjaz Perc is Editor of Chaos, Solitons & Fractals. In keeping with Elsevier's guidelines on potential editorial conflicts of interest, manuscripts coauthored by one of the Editors will be handled fully by other Editors or the Editor-in-Chief in an undisclosed review process. We have no conflicts of interest.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Brito L. Analyzing sustainable development goals. Science 2012;336:1396.
[2] Hardin G. The tragedy of the commons. Science 1968;162:1243–8.
[3] Smith JM. Evolution and the theory of games. Cambridge University Press; 1982.
[4] Weibull JW. Evolutionary game theory. Curr Biol 1995;9:503–5.
[5] Nowak MA. Evolutionary dynamics: exploring the equations of life. Harvard University Press; 2006.
[6] Gintis H. Game Theory Evolving. Princeton University Press; 2009.

[7] Sigmund K. The calculus of selfishness. Princeton University Press; 2010.
[8] Zhang Y, Wu T, Chen X, Xie G, Wang L. Mixed strategy under generalized public goods games. J Theor Biol 2013;334:52–60.
[9] Li K, Cong R, Wu T, Wang L. Social exclusion in finite populations. Phys Rev E 2016;91:042810.
[10] Perc M. Phase transitions in modesl of human cooperation. Phys Lett A 2016;380:2803–8.
[11] Perc M, Jordan JJ, Rand DG, Wang Z, Boccaletti S, Szolnoki A. Statistical physics of human cooperation. Phys Rep 2017;687:1–51.
[12] Brandt H, Hauert C, Sigmund K. Punishment and reputation in spatial public goods games. Proc R Soc B 2003;270:1099–104.
[13] Fowler JH. Altruistic punishment and the origin of cooperation. Proc Natl Acad Sci USA 2005;102:7047.
[14] Brandt H, Hauert C, Sigmund K. Punishing and abstaining for public goods. Proc Natl Acad Sci USA 2006;103:495–7.
[15] Hauert C, Traulsen A, Brandt H, Nowak MA, Sigmund K. Via freedom to coercion: the emergence of costly punishment. Science 2007;316:1905–7.
[16] Helbing D, Szolnoki A, Perc M, Szabó G. Evolutionary establishment of moral and double moral standards through spatial interactions. PLoS Comput Biol 2010;6:e1000758.
[17] Short MB, Brantingham PJ, D'Orsogna MR. Cooperation and punishment in an adversarial game: how defectors pave the way to a peaceful society. Phys Rev E 2010;82:66114.
[18] Sigmund K, De Silva H, Traulsen A, Hauert C. Social learning promotes institutions fro governing the commons. Nature 2010;466:861–3.
[19] Szolnoki A, Szabó G, Perc M. Phase diagrams for the spatial public goods game with pool punishment. Phys Rev E 2011;83:036101.
[20] Szolnoki A, Szabó G, Czakó L. Competition of individual and institutional punishments in spatial public goods games. Phys Rev E 2011;84:046106.
[21] A Szolnoki KL, Cong R, Wang L. The coevolution of overconfidence and bluffing in the resource competition game. Sci Rep 2016;6:21104.
[22] Liu L, Chen X, Szolnoki A. Evolutionary dynamics of cooperation in a population with probabilistic corrupt enforcers and violators. Math Models Methods Appl Sci 2019;29:2127–49.
[23] Sigmund K, Hauert C, Nowak MA. Reward and punishment. Proc Natl Acad Sci USA 2001;98:10757–62.
[24] Rand DG, Dreber A, Ellingsen T, Fudenberg D, Nowak MA. Positive interactions promote public cooperation. Science 2009;325:1272–5.
[25] Szolnoki A, Perc M. Reward and cooperation in the spatial public goods game. EPL 2010;92:38003.
[26] Hauert C. Replicator dynamics of reward and reputation in public goods games. J Theor Biol 2010;267:22–8.
[27] Sasaki T, Unemi T. Replicator dynamics in public goods games with reward funds. J Theor Biol 2011;287:109–14.
[28] Wang Q, He N, Chen X. Replicator dynamics for public goods game with resource allocation in large populations. Appl Math Comput 2018;328:162–70.
[29] Santos FC, Santos MD, Pacheco JM. Social diversity promotes the emergence of cooperation in public goods games. Nature 2008;454:213–16.
[30] Szolnoki A, Perc M, Szabó G. Topology-independent impact of noise on cooperation in spatial public goods games. Phys Rev E 2009;80:056109.
[31] Fowler JH, Christakis NA. Cooperative behavior cascades in human social networks. Proc Natl Acad Sci USA 2010;107:5334–8.
[32] Cao X-B, Du W-B, Rong ZH. The evolutionary public goods game on scale-free networks with heterogeneous investment. Phys A 2010;389:1273–80.
[33] Vukov J, Santos FC, Pacheco JM. Escaping the tragedy of the commons via directed investments. J Theor Biol 2011;287:37–41.
[34] Perc M. Success-driven distribution of public goods promotes cooperation but preserves defection. Phys Rev E 2011;84:037102.
[35] Wang Z, Szolnoki A, Perc M. Evolution of public cooperation on interdependent networks: the impact of biased utility functions. EPL 2012;97:48001.
[36] Matsuzawa R, Tanimoto J. A social dilemma structure in diffusible public goods. EPL 2016;116:38005.
[37] Chen M-H, Wang L, Sun S-W, Wang J, Xia CY. Evolution of cooperation in the spatial public goods game with adaptive reputation assortment. Phys Lett A 2016;380:40–7.
[38] Wang C, Wang L, Wang J, Sun S, Xia C. Inferring the reputation enhances the cooperation in the public goods game on interdependent lattices. Appl Math Comp 2017;293:18–29.
[39] Tanimoto J. Environmental dilemma game to establish a sustainable society dealing with an emergent value system. Phys D 2005;200:1–24.
[40] Janssen MA. Introducing ecological dynamics into common-pool resource experiments. Ecol Soc 2010;15:7.
[41] Chen X, Perc M. Excessive abundance of common resources deters social responsibility. Sci Rep 2014;4:4161.
[42] Weitz JS, Eksin C, Paarporn K, Brown SP, Ratcliff WC. An oscillating tragedy of the commons in replicator dynamics with game-environment feedback. Proc Natl Acad Sci USA 2016;113:E7518–25.
[43] Szolnoki A, Chen X. Environmental feedback drives cooperation in spatial social dilemmas. EPL 2017;120:58001.
[44] Hilbe C, Šimsa v, Chatterjee K, Nowak MA. Evolution of cooperation in stochastic games. Nature 2018;559:246–9.
[45] Chen X, Szolnoki A. Punishment and inspection for governing the commons in a feedback-evolving game. PLOS Comp Biol 2018;14:e1006347.
[46] He N, Chen X, Szolnoki A. Central governance based on monitoring and reporting solves the collective-risk social dilemma. Appl Math Comput 2019;347:334–41.
[47] Shao Y, Wang X, Fu F. Evolutionary dynamics of group cooperation with asymmetrical environmental feedback. EPL 2019;126:40005.
[48] Hauert C, Saade C, McAvoy A. Asymmetric games and environmental feedback. J Theor Biol 2019;462:347–60.
[49] Sasaki T, Uchida S, Chen X. Voluntary reards mediate the evolution of pool punishment for maintaining public goods in large populations. Sci Rep 2015;5:8917.
[50] Szabó G, Töke C. Evolutionary prisoner's dilemma game on a square lattice. Phys Rev E 1998;58:69–73.