Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Chaos, Solitons and Fractals 31 (2007) 64–69
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Spatial coherence resonance in neuronal media with discrete local dynamics

Matjaž Perc *

Department of Physics, Faculty of Education, University of Maribor, Koroška cesta 160, SI-2000 Maribor, Slovenia

Accepted 7 September 2005

Abstract

We study effects of spatiotemporal additive noise on the spatial dynamics of excitable neuronal media that is locally modelled by a two-dimensional map. We focus on the ability of noise to enhance a particular spatial frequency of the media in a resonant manner. We show that there exists an optimal noise intensity for which the inherent spatial periodicity of the media is resonantly pronounced, thus marking the existence of spatial coherence resonance in the studied system. Additionally, results are discussed in view of their possible biological importance.

1. Introduction

It is a well-documented fact that noise can enhance the response of a system to weak external stimuli in a resonant manner [1,2], whereby this phenomenon has been termed accordingly as stochastic resonance. Remarkably, constructive effects of noise can be observed also in the absence of any deterministic external inputs in systems with no explicit time scales [3,4]. This striking phenomenon, on the other hand, has been termed coherence resonance [5].

Following advances in the study of constructive effects of noise on temporal systems, noise effects on system with spatial degrees of freedom [6] have also gained a lot of attention during the past decade. Spatiotemporal stochastic resonance has been first reported in [7] for excitable systems, while spatial coherence resonance has been introduced in [8] for systems near pattern forming instabilities. Moreover, there exist studies reporting noise-induced spiral growth and enhancement of spatiotemporal order [9–14], noise sustained coherence of space–time clusters and self-organized criticality [15], noise enhanced and induced excitability [16,17], noise induced propagation of harmonic signals [18], as well as noise sustained and controlled synchronization [19] in space extended systems. Remarkably, stochastic resonance can also be observed in a noise-free environment in systems with spatiotemporal on–off intermittency [20], whereby the term dynamical stochastic resonance was proposed to described the phenomenon.

Until now, however, little attention has been devoted to the explicit analysis of characteristic spatial frequencies of nonlinear media. Following the seminal work of Carrillo et al. [8], we recently showed that additive or multiplicative spatiotemporal noise is able to enhance a particular spatial periodicity of excitable media in a resonant manner [21,22].

* Tel.: +386 2 2293643; fax: +386 2 2518180.
E-mail address: matjaz.perc@uni-mb.si
URL: http://fizika.uni-mb.si/~matjaz

0960-0779/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.chaos.2005.09.021

Thereby, we were able to extend results reporting spatial coherence resonance published previously in [8] for system near pattern-forming Turing instabilities also to excitable media.

In the present study, we provide first evidences for spatial coherence resonance in excitable neuronal media with discrete local dynamics. Map-based media models are computationally more efficient than their time-continuous counterparts, and thus present a more appropriate environment for large-scale network modelling, as advocated in [23]. However, although being able to replicate complex experimentally observed neuronal behaviour, it is of interest to verify if discrete systems also possess similar properties with respect to noisy perturbations. Recently, Jiang [24] has shown that a discrete temporal neuronal model is able to exhibit multiple resonances in response to periodic stimuli and additive or parametric noise. Here, on the other hand, we focus on the spatial dynamics of noise-induced patterns in locally discrete neuronal media.

In order to evidence spatial coherence resonance in the system, we analyse spatial frequency spectra in dependence on different levels of additive spatiotemporal noise. Note that although stochastic [25] and coherence resonance [26–29] phenomena have been extensively studied in arrays of dynamical systems, our work focuses explicitly on the spatial [8,21,22] rather than temporal or spatiotemporal system scale. In particular, we show that there exists an optimal level of additive noise for which a particular spatial frequency of the system is resonantly pronounced. We emphasize that no additional deterministic inputs are introduced to the system, and the latter is locally initiated from steady state initial conditions. Hence, the studied spatial structures are induced solely by additive noise and reflect an inherent spatial periodicity of the media.

The excitable media under study is locally modelled by a discrete two-dimensional map that has been recently proposed by Rulkov [30] to describe the regularization of an array of chaotically oscillating neurons. Since neurons are known to be noisy analog units, which only if coupled can carry out highly complex and advanced computations with cognition and reliability [31], it is evident that excitable neuronal tissue combines features of being both noisy and spatially extended. Therefore, our study is strongly motivated also from the biomedical point of view, and hopefully outlines some possibilities for future experimental work, especially in the field of neuroscience, where excitability and noise in space extended systems appear to be universally present.

2. Mathematical model

The studied mathematical model of locally discrete excitable media is given by

$$u_{i+1} = \alpha/(1+u_i^2) + v_i + D\nabla^2 u_i + \xi, \tag{1}$$

$$v_{i+1} = v_i - \beta u_i - \gamma, \tag{2}$$

where the neuron cell membrane voltage $u_i(x,y)$ and the variation of ion concentration near the neuron membrane $v_i(x,y)$ are considered as dimensionless two-dimensional scalar fields on a $n \times n$ square lattice with mesh size $\Delta x = \Delta y$, whereby the local dynamics of u_i is much faster ($\beta, \gamma \ll 1$) than that of v_i, whose diffusive spread is also neglected. Moreover, ξ is spatiotemporal additive Gaussian noise with zero mean, white in space and time, and variance σ^2 [6]. The Laplacian $D\nabla^2 u_i$, D being the diffusion coefficient, is integrated into the numerical scheme via a five-point finite-difference formula as described by Barkley [32] with no-flux boundary conditions. For parameter values $a < 2$ and $\beta = \gamma = 0.001$ the local dynamics is governed by an excitable steady state $(u^*, v^*) = (-1, -1 - \alpha/2)$, whilst for $\alpha > 2$ the discrete model exhibits various oscillatory behaviour ranging from simple spike-like to chaotic bursting oscillations [30]. For a more detailed analysis of the local dynamics of the model we refer the reader to the original article [30], as well as to Refs. [23,33,34], where a slightly more elaborate but similar map-based neuronal model is analysed. In what follows, we will show that there exists an optimal intensity of additive spatiotemporal noise ξ for which a particular spatial frequency of the media is resonantly pronounced, thus providing evidences for spatial coherence resonance in the studied system.

3. Spatial coherence resonance

To quantify effects of various noise intensities on the spatial scale of the studied system we calculate the structure function according to the equation

$$P(k_x, k_y) = \langle H^2(k_x, k_y) \rangle, \tag{3}$$

where $H(k_x, k_y)$ is the spatial Fourier transform of the u_i-field at a particular i, and $\langle \cdots \rangle$ is the ensemble average over noise realizations. Note that $P(k_x, k_y)$ can also be interpreted as the spatial power spectrum of the system. Fig. 1 shows

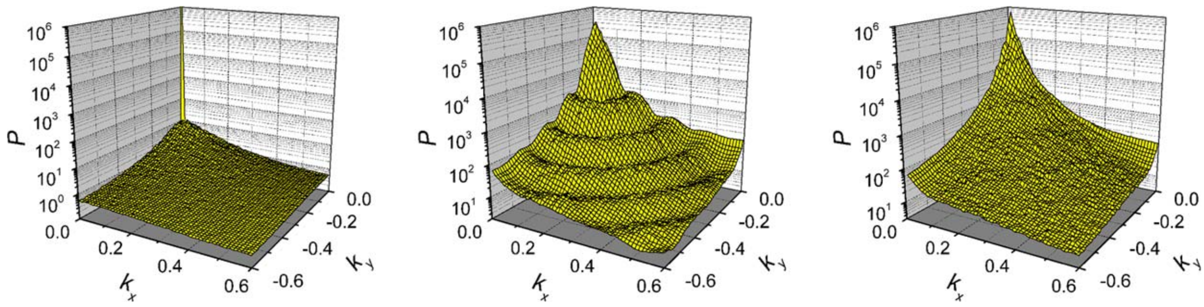

Fig. 1. Two-dimensional power spectra of the spatial profile of u_i for $\sigma = 0.0033$ (left panel), $\sigma = 0.0038$ (middle panel), and $\sigma = 0.0048$ (right panel). Parameter values used for the calculations where $\alpha = 1.99$, $\beta = \gamma = 0.001$, $D = 0.02$, $n = 128$, and $\Delta x = 1.0$, whereby the system was initiated from steady state initial conditions from all lattice sites.

three spatial power spectra for various additive noise levels. It can be well observed that for small noise levels the presented spectrum shows no particularly expressed spatial frequency. For somewhat larger σ, however, the spectrum develops waterfall-like well-expressed circularly symmetric rings, indicating the existence of a preferred spatial frequency induced by additive noise. As the noise level is further increased random fluctuations start to dominate the spatial scale and thus, similarly as by small noise levels, no preferred spatial frequency can be inferred.

To study results presented in Fig. 1 in more detail, we exploit the circular symmetry of the presented spatial power spectra as proposed in [8]. In particular, we calculate the circular average of the structure function according to the equation

$$p(k) = \int_{\Omega_k} P\left(\vec{k}\right) \mathrm{d}\Omega_k, \tag{4}$$

where $\vec{k} = (k_x, k_y)$, and Ω_k is a circular shell of radius $k = \left|\vec{k}\right|$. Left panel of Fig. 2 shows results for three different σ. It can be observed that there indeed exists a particular spatial frequency, marked with the thin solid line at $k = k_{\max}$, that is resonantly enhanced for some intermediate level of additive noise. To quantify the ability of each particular noise level to extract the characteristic spatial periodicity in the system more precisely, we calculate the signal-to-noise ratio (SNR) as the peak height at $k = k_{\max}$ normalized with the background fluctuations existing in the system. This is the spatial counterpart of the measure frequently used for quantifying constructive effects of noise on the temporal domain of dynamical systems [35], whereas a similar measure for quantifying effects of noise on the spatial scale of space extended systems was also used in [8]. Right panel of Fig. 2 shows how the SNR varies with σ for three different diffusion constants D. It is evident that there always exists an optimal level of additive noise for which the peak of the circularly averaged structure function is best resolved, thus indicating the existence of spatial coherence resonance in the studied locally discrete excitable media.

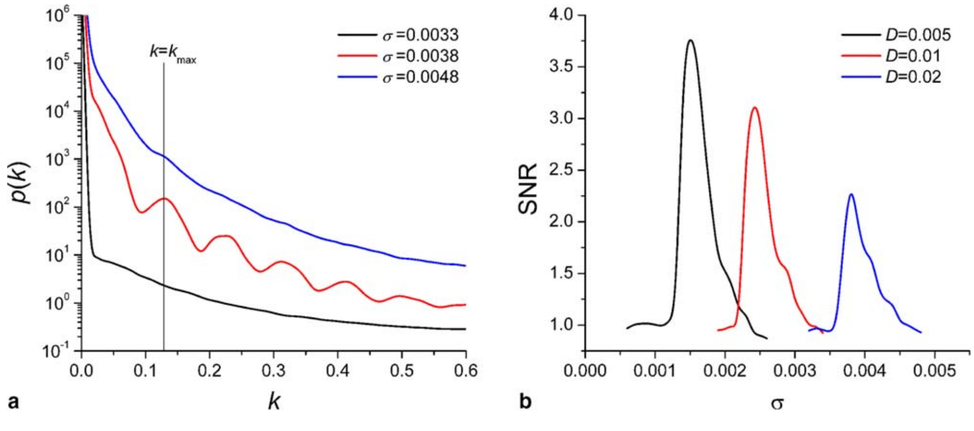

Fig. 2. Spatial coherence resonance in the studied map-based excitable media. (a) Circular average of the structure function for three different σ at $D = 0.02$. (b) SNR in dependence on σ for various diffusion constants. Other parameter values are the same as in Fig. 1.

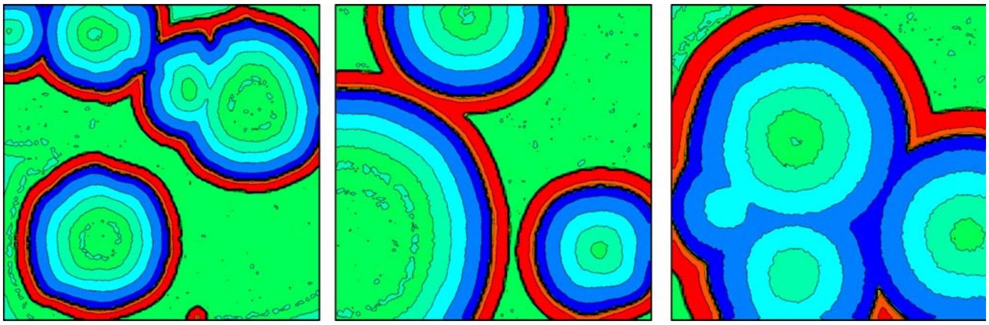

Fig. 3. Characteristic snapshots of the spatial profile of u_i at optimal σ for $D = 0.005$ (left panel), $D = 0.01$ (middle panel), and $D = 0.02$ (right panel). Note that all figures are depicted on 128×128 square grid with a linear colour profile, blue marking -1.6 and red 0.0 values of u_i. (For interpretation of colours in this figure legend, the reader is referred to the web version of the article).

The existence of a characteristic spatial periodicity in the studied media at the optimal σ can be well corroborated by studying snapshots of typical u_i-field configurations for various D, as presented in Fig. 3. It is evident that for all D there exists an inherent spatial scale that is resonantly enhanced by additive noise, thus providing visible evidences that support results presented in Figs. 1 and 2. Importantly, note how the characteristic scale, i.e. the width of noise-induced patterns, increases with increasing D from left towards the right panel in Fig. 3. As we will argue next, this observation holds the key to explaining the above-reported spatial coherence resonance in locally discrete excitable media.

It is an establish fact that coherence resonance phenomena in temporal excitable systems with no explicit time scales can be attributed to different noise dependencies of the activation (t_a) and excursion time (t_e) [5]. In particular, since t_e is much more robust to noisy perturbations [36] than t_a, there exists an optimal noisy intensity where $t_a \ll t_e$ but still fluctuations of t_e are moderate, thereby marking the most coherent response of the system in dependence on σ. Jiang [24] has recently advocated that for discrete systems the coherence resonance, both in excitable as well as oscillatory states, must also be attributed to the very noise susceptible initial stage of oscillations. Here we extend this reasoning to systems with spatial degrees of freedom by taking into account the rate of diffusive spread in each particular space direction that is proportional to $\sqrt{D}$ [37]. We argue that during t_e a particular lattice site acts like a circular (after local initialization all directions for spreading are equally probable) front initiator. After initialization the front starts to spread through the media with a rate proportional to $\sqrt{D}$. When embarking on neighbouring sites the front can, depending on the level of additive noise, cause new excitation or eventually die out. In particular, if σ is large enough, i.e. t_a short enough, neighbouring sites have a large probability to also become excited, which eventually nucleates a wave that propagates through the media. Since larger D constitute faster diffusive spread, it is understandable that the characteristic spatial scale of coherent structures induced by increasing D increases (see Fig. 3). However, since

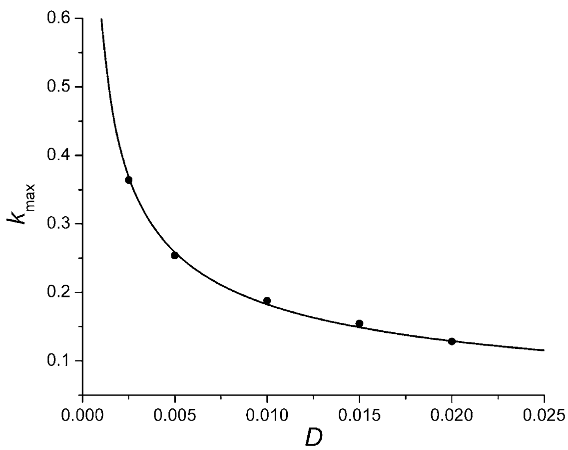

Fig. 4. Dependence of the optimal spatial wave number k_{max}, corresponding to the maximum of $p(k)$ at the optimal σ, on different values of D. Dots indicate numerically obtained values, whereas the solid line indicates the predicted dependence $k_{max} = 1/\sqrt{\tau D}$.

for larger D local excitations tend to die out more quickly, and larger coherent structures also require a higher rate of local excitations to propagate through the media, it is evident that shorter t_a (larger σ) are required to produce sustained waves. This explains the increasing σ that is required for the optimal response at ever-larger D, as shown in the right panel of Fig. 2. Furthermore, since larger σ blur local excursion phases (t_e) as well, the maximal spatial coherence (SNR) that can be achieved by additive noise decreases with increasing D. In summary, we emphasize that the noise robust excursion time t_e, combined with the diffusive spread rate proportional to $\sqrt{D}$, marks an inherent spatial scale of the media that is indicated by the resonantly enhanced spatial wave number $k_{\max}$. Since the characteristic spatial scale is determined by the inverse of the resonantly enhanced spatial wave number, our reasoning thus predicts the dependence $k_{\max} = 1/\sqrt{\tau D}$, whereby $\tau \propto t_e \approx$ constant. Fig. 4 shows numerically obtained $k_{\max}$ for different D. It is evident that obtained values are in excellent agreement with the inverse square root function, thereby validating our above explanation. Nevertheless, an open question remains how the constant τ is explicitly linked to t_e, which is left as a problem to be solved in future studies. The main point is that the inverse square root function fits to the numerically obtained values with a constant τ, which reflects a noise robust t_e that is characteristic for discrete and continuous excitable systems [5,24]. Together with a given D, this property of excitable systems constitutes an inherent spatial scale that can be resonantly enhanced by additive noise, thus in our case explaining the existence of spatial coherence resonance in locally discrete excitable media.

4. Discussion and outlook

We show that spatiotemporal additive noise is able to extract an inherent spatial scale of map-based excitable media in a resonant manner. In particular, there exist an optimal level of additive noise for which the spatial periodicity of the system is resonantly pronounced. Thus, the presented results offer convincing evidence for the existence of spatial coherence resonance in the studied media. We argue that the observed phenomenon occurs as a consequence of different noise dependencies of the activation and excursion times of the local map whereby the diffusion constant, representing the rate of diffusive spread, determines the actual resonant spatial frequency, which decreases with increasing D.

The present study supports the appropriateness of map based neuronal models for simulating complex large-scale neurobiological networks, as previously advocated in [23]. Particularly, in conjunction with studies regarding the temporal dynamics of noise-induced behaviour [24], our results show that discrete models possess identical noise-response properties as their continuous counterparts. Thus, given the computational efficiency of map-based models, combined with their ability to fully replicate complex autonomous as well as noise-induced temporal and spatial behaviour of continuous models, we argue that they have a bright future in mathematical modelling of entangled biological systems.

Moreover, since it has been discovered that excitable systems guarantee robust signal propagation through the neuronal tissue in a substantially noisy environment [38], and studies evidencing the existence of stochastic resonance in the human brain have recently been mounting [39–42], it would be very interesting to elucidate if spatial coherence resonance in the nervous system can be confirmed also experimentally. In conjunction, it would be extremely interesting to elucidate if, similarly as in temporal systems, in space extended systems also the so-called spatial stochastic resonance can be observed. Since given the omnipresence of wireless communication techniques nowadays, deterministic-like external influences like electromagnetic radiation also affect the functioning of neuronal tissue, it is of outstanding importance to provide insights into how such spatially periodic deterministic signals, in conjunction with stochastic fluctuations, might affect the brain functioning as well.

References

[1] Gammaitoni L, Hänggi P, Jung P, Marchesoni F. Stochastic resonance. Rev Modern Phys 1998;70:223–87.
[2] Lindner B, García-Ojalvo J, Neiman A, Schimansky-Geier L. Effects of noise in excitable systems. Phys Rep 2004;392:321–424.
[3] Hu G, Ditzinger T, Ning CZ, Haken H. Stochastic resonance without external periodic force. Phys Rev Lett 1993;71:807–10.
[4] Rappel WJ, Strogatz SH. Stochastic resonance in an autonomous system with a nonuniform limit cycle. Phys Rev E 1994;50:3249–50.
[5] Pikovsky AS, Kurths J. Coherence resonance in a noise-driven excitable system. Phys Rev Lett 1997;78:775–8.
[6] García-Ojalvo J, Sancho JM. Noise in spatially extended systems. New York: Springer; 1999.
[7] Jung P, Mayer-Kress G. Spatiotemporal stochastic resonance in excitable media. Phys Rev Lett 1995;74:2130–3.
[8] Carrillo O, Santos MA, García-Ojalvo J, Sancho JM. Spatial coherence resonance near pattern-forming instabilities. Europhys Lett 2004;65:452–8.
[9] Jung P, Mayer-Kress G. Noise controlled spiral growth in excitable media. Chaos 1995;5:458–62.
[10] Jung P, Cornell-Bell A, Moss F, Kadar S, Wang J, Showalter K. Noise sustained waves in subexcitable media: from chemical waves to brain waves. Chaos 1995;8:567–75.

[11] García-Ojalvo J, Schimansky-Geier L. Noise-induced spiral dynamics in excitable media. Europhys Lett 1999;47:298–303.
[12] Hempel H, Schimansky-Geier L, García-Ojalvo J. Noise-sustained pulsating patterns and global oscillations in subexcitable media. Phys Rev Lett 1999;82:3713–6.
[13] Alonso S, Sendiña-Nadal I, Pérez-Muñuzuri V, Sancho JM, Sagués F. Regular wave propagation out of noise in chemical active media. Phys Rev Lett 2001;87:078302.
[14] Busch H, Kaiser F. Influence of spatiotemporally correlated noise on structure formation in excitable media. Phys Rev E 2003;67:041105.
[15] Jung P. Thermal waves, criticality, and self-organization in excitable media. Phys Rev Lett 1997;78:1723–6.
[16] García-Ojalvo J, Sagués F, Sancho JM, Schimansky-Geier L. Noise-enhanced excitability in bistable activator-inhibitor media. Phys Rev E 2001;65:011105.
[17] Ullner E, Zaikin AA, García-Ojalvo J, Kurths J. Noise-induced excitability in oscillatory media. Phys Rev Lett 2003;91:180601.
[18] Zaikin AA, García-Ojalvo J, Schimansky-Geier L, Kurths J. Noise induced propagation in monostable media. Phys Rev Lett 2002;88:010601.
[19] Zhou CS, Kurths J. Noise-sustained and controlled synchronization of stirred excitable media by external forcing. New J Phys 2005;7:18.
[20] Stepień L, Krawiecki A, Kosiński RA. Stochastic resonance in spatiotemporal on–off intermittency. Chaos, Solitons & Fractals 2004;19:1243–50.
[21] Perc M. Noise-induced spatial periodicity in excitable chemical media. Chem Phys Lett 2005;410:49–53.
[22] Perc M. Spatial coherence resonance in excitable media. Phys Rev E 2005;72:016207.
[23] Rulkov NF, Timofeev I, Bazhenov M. Oscillations in large-scale cortical networks: map-based model. J Comput Neurosci 2004;17:203.
[24] Jiang Y. Multiple dynamical resonances in a discrete neuronal model. Phys Rev E 2005;71:057103.
[25] Lindner JF, Meadows BK, Ditto WL, Inchiosa ME, Bulsara AR. Array enhanced stochastic resonance and spatiotemporal synchronization. Phys Rev Lett 1995;75:3.
[26] Wio HS. Stochastic resonance in a spatially extended system. Phys Rev E 1995;54:R3075–8.
[27] Han SK, Yim TG, Postnov DE, Sosnovtseva OV. Interacting coherence resonance oscillators. Phys Rev Lett 1999;83:1771–4.
[28] Neiman A, Schimansky-Geier L, Cornell-Bell A, Moss F. Noise-enhanced phase synchronization in excitable media. Phys Rev Lett 1999;83:4896–9.
[29] Zhou CS, Kurths J, Hu B. Array-enhanced coherence resonance: nontrivial effects of heterogeneity and spatial independence of noise. Phys Rev Lett 2001;87:098101.
[30] Rulkov NF. Regularization of synchronized chaotic bursts. Phys Rev Lett 2001;86:183–6.
[31] Izhikevich EM. Neural excitability, spiking and bursting. Int J Bifurcat Chaos 2000;10:1171–266.
[32] Barkley D. A model for fast computer simulation of waves in excitable media. Physica D 1991;49:61–70.
[33] Rulkov NF. Modeling of spiking-bursting neural behavior using two-dimensional map. Phys Rev E 2001;65:041922.
[34] Shilnikov AL, Rulkov NF. Subthreshold oscillations in a map-based neuron model. Phys Lett A 2004;328:177–84.
[35] Jung P, Hänggi P. Amplification of small signals via stochastic resonance. Phys Rev A 1991;44:8032–42.
[36] Pikovsky AS. On the interaction of strange attractors. Z Phys B 1984;55:149–54.
[37] Pinsky MA. Introduction to partial differential equations with applications. New York: McGraw-Hill; 1984.
[38] Keener J, Snyder J. Mathematical physiology. New York: Springer; 1998.
[39] Simonotto E, Riani M, Seife C, Roberts M, Twitty J, Moss F. Visual perception of stochastic resonance. Phys Rev Lett 1997;78:1186–9.
[40] Hidaka I, Nozaki D, Yamamoto Y. Functional stochastic resonance in the human brain: noise induced sensitization of baroreflex system. Phys Rev Lett 2000;85:3740–3.
[41] Mori T, Kai S. Noise-induced entrainment and stochastic resonance in human brain waves. Phys Rev Lett 2002;88:218101.
[42] Kitajo K, Nozaki D, Ward LM, Yamamoto Y. Behavioral stochastic resonance within the human brain. Phys Rev Lett 2003;90:218103.