Chaos, Solitons & Fractals 80 (2015) 1–6

Contents lists available at ScienceDirect

Chaos, Solitons & Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Networks of networks – An introduction

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ARTICLE INFO

ABSTRACT

This is an introduction to the special issue titled "Networks of networks" that is in the making at Chaos, Solitons & Fractals. Recent research and reviews attest to the fact that networks of networks are the next frontier in network science [1–7]. Not only are interactions limited and thus inadequately described by well-mixed models, it is also a fact that the networks that should be an integral part of such models are often interconnected, thus making the processes that are unfolding on them interdependent. From the World economy and transportation systems to social media, it is clear that processes taking place in one network might significantly affect what is happening in many other networks. Within an interdependent system, each type of interaction has a certain relevance and meaning, so that treating all the links identically inevitably leads to information loss. Networks of networks, interdependent networks, or multilayer networks are therefore a much better and realistic description of such systems, and this Special Issue is devoted to their structure, dynamics and evolution, as well as to the study of emergent properties in multi-layered systems in general. Topics of interest include but are not limited to the spread of epidemics and information, percolation, diffusion, synchronization, collective behavior, and evolutionary games on networks of networks. Interdisciplinary work on all aspects of networks of networks, regardless of background and motivation, is very welcome.

1. Networks of networks

Because current methods deal almost exclusively with individual networks treated as isolated systems, many challenges remain [3]. In most real-world systems an individual network is one component within a much larger complex multi-level network (is part of a network of networks). As technology has advanced, coupling between networks has become increasingly strong. As the study of individual particles has enabled physicists to understand the properties of a gas, but in order to understand and describe a liquid or a solid the interactions between the particles also need to be understood. So also in network theory, the study of isolated single networks brings extremely limited results—real-world noninteracting systems are extremely rare in both classical physics and network study. Most real-world network systems continuously interact with other networks, especially since modern technology has accelerated network interdependency.

To adequately model most real-world systems, understanding the interdependence of networks and the effect of this interdependence on the structural and functional behavior of the coupled system is crucial. Introducing coupling between networks is analogous to the introduction of interactions between particles in statistical physics, which allowed physicists to understand the cooperative behavior

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http://dx.doi.org/10.1016/j.chaos.2015.03.016
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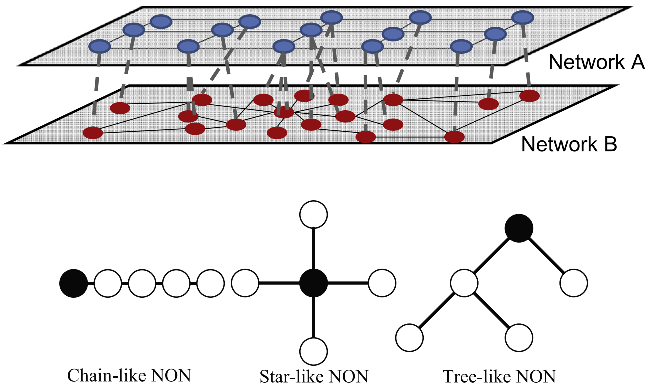

Fig. 1. (Top) Example of two interdependent networks. Nodes in network B (communications network) are dependent on nodes in network A (power grid) for power; nodes in network A are dependent on network B for control information. (Bottom) Three types of loopless networks of networks (NON) composed of five coupled networks. All have same percolation threshold and same giant component. The dark node is the origin network on which failures initially occur. After [9].

of such rich phenomena as phase transitions. The results of recent studies [1,8–13] show that analyzing complex systems as a network of coupled networks may alter the basic assumptions that network theory has relied on for single networks.

In order to model interdependent networks, one can consider the most simple example of two networks, A and B, in which the functionality of a node in network A is dependent upon the functionality of one or more nodes in network B (see Fig. 1, top), and vice versa: the functionality of a node in network B is dependent upon the functionality of one or more nodes in network A. The networks can be interconnected in several ways. In the most general case one can specify a number of links that arbitrarily connect pairs of nodes across networks A and B. The direction of a link specifies the dependency of the nodes it connects, i.e., link $A_i \rightarrow B_j$ provides a critical resource from node A_i to node B_j. If node A_i stops functioning due to attack or failure, node B_j stops functioning as well but not vice versa. Analogously, link $B_i \rightarrow A_j$ provides a critical resource from node B_i to node A_j.

2. Spread of epidemics and information

Spreading processes in networks of networks are actually one of the best suited frameworks to uncover the effects that the coupling between networks has on their functioning. For instance, if one just considers the simplest setup, i.e. linear diffusion, the most relevant result is the observation of an abrupt transition between two very different regimes: one in which the networks behave independently, and another that corresponds to coordinated functioning. On the other hand, more refined diffusion processes (such as random walkers) allows deeper investigations, especially for what concerns the proper individuation of tools for ranking the importance of the nodes, and monitoring it as a function of the coupling between networks. Finally, the study of more realistic diffusion processes, such as those modeling the transport of information packets, on interconnected networks has been recently addressed, with emphasis on gathering a better knowledge and understanding of the transition to congestion in information networks.

Of particular importance is the problem of disease spreading on networks of networks, where several related disease dynamics propagate on separated networks, or both social information and infection information simultaneously diffuse and affect each other. When compared with the case of single networks, the novel framework can greatly change the threshold of disease outbreak, which is usually shown in the form of a phase transition between the healthy and epidemic phases. Actually, a flurry of studies on this subject has to be expected, as SIR and SIS models present important qualitative differences. SIR dynamics, for instance, allows system states in which the epidemic regime only occurs in one of the networks, whereas under SIS dynamics the propagation in one network automatically triggers the epidemic in all the other networks.

3. Percolation and diffusion

This rich formalism of network of networks generalization has shows that the percolation on a single network studied for more than 50 years is a limited case of the more general case of network of networks. Node failures in one network will cause the failure of dependent nodes in other networks, and vice versa [1]. This recursive process can lead to a cascade of failures throughout the network of networks system. The percolation properties of a network of networks differ greatly from those of single isolated networks. In particular, networks with broad degree distributions, such as scale free networks, that are robust when analyzed as single networks become highly vulnerable in a network of networks. Moreover, in a network of networks, cascading failures appear due to dependent nodes of different networks. When there is strong interdependent coupling between the networks, the percolation transition is discontinuous (is a first-order transition), unlike the well-known continuous second-order transition in single isolated networks.

To study the robustness of interdependent networks systems illustrated in Fig. 1, one begins by removing a fraction $1 - p$ of network A nodes and all the A-edges connected to these nodes. As an outcome, all the nodes in network B that are dependent on the removed A-nodes by $A \to B$ links are also removed and their B nodes will cause the removal of additional nodes in network A which are dependent on the removed B-nodes by $B \to A$ links. As a result, a cascade of failures that eliminates virtually all nodes in both networks can occur. As nodes and edges are removed, each network breaks up into connected components (clusters). The clusters in network A (connected by A-edges) and the clusters in network B (connected by B-edges) are different since the networks are each connected differently. If one assumes that small clusters not connected to the giant component become non-functional, this may invoke a recursive process of failures that we now formally describe.

The insight based on percolation theory is that when the network is fragmented the nodes belonging to the giant component connecting a finite fraction of the network are still functional, but the nodes that are part of the remaining small clusters become non-functional. Thus in interdependent networks only the giant mutually-connected cluster is of interest. Unlike clusters in regular percolation whose size distribution is a power law with a p-dependent cutoff, at the final stage of the cascading failure process just described only a large number of small mutual clusters and one giant mutual cluster are evident. This is the case because the probability that two nodes that are connected by an A-link and their corresponding two nodes are also connected by a B-link scales as $1/N_B$, where N_B is the number of nodes in network B. So the centrality of the giant mutually-connected cluster emerges naturally and the mutual giant component plays a prominent role in the functioning of interdependent networks. When it exists, the networks preserve their functionality, and when it does not exist, the networks split into fragments so small they cannot function on their own. The percolation processes on other network of networks topologies have been presented and discussed in [7,9,12,14–20].

4. Synchronization

In the past two decades, the emergence of synchronized states in networked dynamical systems has been extensively reported and studied, with the emphasis focusing on how the complexity of the network topology influences the propensity of the coupled units to synchronize. While the literature is indeed extremely large, we refer the interested reader to two recent comprehensive reviews [21,22], whose consultation may help in providing guidance on the major accomplishments, results and related concepts. As soon as the network of networks framework is considered (and especially when the networks are made of the same nodes with different layers of connections), the subject can actually be divided into two main scenarios.

The first case corresponds to having different connectivity configurations that alternate in time. Here, the basic problem is to describe how the existence and stability of synchronization are modified with respect to the classic case, in which the coupling between the networked units is time-independent. The second case, instead, is when the different networks (or layers) are *simultaneously* responsible for the coupling of the network's units, with explicit additional network-network interactions to be accounted for. It is very important to stress that an *inter*-network synchronized state is a different and much more general concept, as it does not necessarily require *intra*-network synchronization. In other words, the dynamics within each network can be out of synchrony, and yet synchronized states may emerge between networks.

In the first case, the major result so far available is the discovery of an enhancement of synchronization that occurs for temporal evolutions along a sequence of commutative networks. Recently, however, it has been demonstrated that one can make much more general rigorous statements, without the need of imposing constraints on the networks and their mutual relations.

As for coexisting networks, the role of symmetry and network-network inter-dependencies on the existence and stability of collective synchronized states is currently almost unexplored. Studies exist describing the importance of topological symmetries for synchronization, but they all deal with single graphs. The interdependence of other networks could lead to substantially different results, which need to be investigated.

5. Collective behavior and evolutionary games

Evolutionary games are another fascinating field of research that can benefit from the theoretical framework of network of networks. At variance with games in single networks [23,24], evolutionary games on networks of networks can induce dependencies via either the utility function or the strategy of the nodes, and therefore the interest is on unveiling the influence of these inter-network couplings on the maintenance, enhancement or suppression of collective states both in pair-wise games (such as the Prisoner's Dilemma) as well as in games that are governed by group interactions [25]. More generally, such collective states are relevant for the explanation of emergent dynamics such as cooperative behavior, species diversity and cyclical dominance [26], and they may also relevantly inform the prevention of climate change [27] and crime [28].

Ever since the discovery of network reciprocity by Nowak and May [29], who showed that in social dilemmas cooperators can survive by forming compact clusters in structured populations, the evolution of cooperation on lattice, networks and graphs has been a vibrant topic across social and natural sciences [23–25,30,31]. The emergence of cooperation and the phase transitions leading to other favorable evolutionary outcomes depend sensitively on the structure of the interaction network and the type of interactions, as well as on the number and type of competing strategies [32–45]. Studies making use of statistical physics and network science have led to significant advances in our understanding of the evolution of

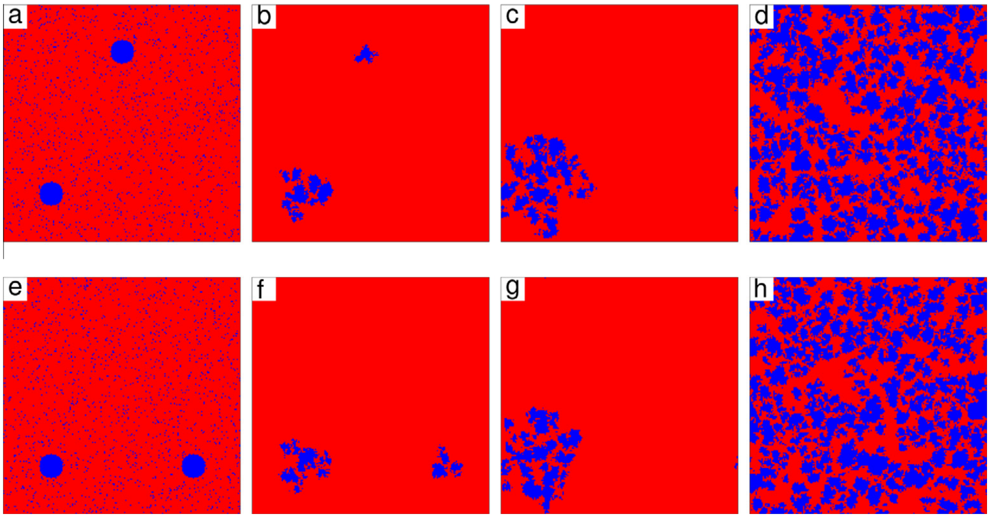

Fig. 2. Demonstration of interdependent network reciprocity. From left to right, panels (a)–(d) depict the evolution of cooperation in one network, while panels (e)–(h) show the same for the other network. Starting from a prepared initial state, only those circular cooperative domains that are initially present on both networks, and which can therefore make immediate use of interdependent network reciprocity, survive and eventually spread across both networks. Cooperators that are initially distributed uniformly at random, as well as clustered cooperators present on one but not the other network, surrender under the evolutionary pressure from defectors already at the early stages of evolution. For details we refer to [50], from where this figure is reproduced.

cooperation, for example by expanding our understanding of the role of heterogeneity of interaction networks [35,46], the dynamical organization of cooperation [38] and population growth [47], the emergence of hierarchy among competing individuals [39], as well as the intriguing role of strategic complexity [41,43], to name just some examples. Most recently, evidence in support of the fact that static networks promote cooperation in human experiments have also been presented [48], in addition to the fact that this has been shown already before for networks with rewiring [49].

Evolution of cooperation in structured populations remains a hot topic to date, yet the attention has recently been shifting increasingly towards interdependent and multilayer networks [43,50–60]. Indeed, several mechanisms have been discovered by means of which the interdependence between different networks or network layers may help to resolve social dilemmas beyond the potency of traditional network reciprocity [29]. A prominent example is interdependent network reciprocity [55], which is capable to maintain healthy levels of public cooperation even under extremely adverse conditions. The mechanism, however, requires simultaneous formation of correlated cooperator clusters on both networks, as demonstrated in Fig. 2. If this does not emerge or if the coordination process is disturbed, network reciprocity fails, resulting in the total collapse of cooperation. Network interdependence can thus be exploited effectively to promote cooperation past the limits imposed by isolated networks, but only if the coordination between the interdependent networks is not disturbed. Other mechanisms that promote the evolution of cooperation and build prominently on networks of networks include non-trivial organization of cooperators across the interdependent network layers [52], probabilistic interconnectedness [54], information transmission between different networks [57], rewarding evolutionary fitness by enabling links between populations [58], as well as self-organization towards optimally interdependent networks by means of coevolution [59].

We hope contributions to this special issue will continue the beautiful tradition of the fruitful collaboration between statistical physics, network science and evolutionary game theory, and indeed, we will very much welcome submissions that make an effort towards this goal.

6. Future research

In terms of future directions for research, we can highlight a few major ones. One direction should continue to expand the analytical and theoretical knowledge on the properties of network of networks and multilayer networks, failure and information spreading processes in network of networks and multilayer networks, and the dynamics of network of networks and multilayer networks. A second direction should be the application of the theoretical and analytical frameworks to real data, ranging from social, financial and economic systems, to infrastructure and technological systems, to biological, brain functioning, and natural systems. A third major direction should be the introduction of recovery mechanisms that will exploit the properties of network of networks and multilayer networks (see for example [61,62]).

More specifically in terms of evolutionary games, for example, it seems that the evolution of cooperation on both interdependent and multilayer networks has reached fruition to a degree that the next step might be to focus more on coevolution between cooperation and interdependence, as has partly already been explored in [59].

To conclude, we note that this special issue is also about to feature future research. In order to avoid delays that are sometimes associated with waiting for a special issue to become complete before it is published, we have adopted an alternative approach. The special issue will be updated continuously from the publication of this introduction onwards, meaning that new papers will be published immediately after acceptance. The issue will hopefully grow in size on a regular basis, with the last papers being accepted no later than August 30th for the special issue to be closed by the end of 2015. The down side of this approach is that we cannot feature the traditional brief summaries of each individual work that will be published, but we hope that this is more than made up for by the immediate availability of the latest research. Please stay tuned, and consider contributing to "Networks of networks".

Acknowledgments

We would like to thank the other two Editors-in-Chief of Chaos, Solitons & Fractals, Paolo Grigolini and Maurice Courbage, for their support to this special issue. We would also like to thank all the authors and referees for their valuable contributions and help.

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