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Predicting tipping points of dynamical systems during a period-doubling route to chaos

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Classical indicators of tipping points have limitations when they are applied to an ecological and a biological model. For example, they cannot correctly predict tipping points during a period-doubling route to chaos. To counter this limitation, we here try to modify four well-known indicators of tipping points, namely the autocorrelation function, the variance, the kurtosis, and the skewness. In particular, our proposed modification has two steps. First, the dynamic of the considered system is estimated using its time-series. Second, the original time-series is divided into some sub-time-series. In other words, we separate the time-series into different period-components. Then, the four different tipping point indicators are applied to the extracted sub-time-series. We test our approach on an ecological model that describes the logistic growth of populations and on an attention-deficit-disorder model. Both models show different tipping points in a period-doubling route to chaos, and our approach yields excellent results in predicting these tipping points. *Published by AIP Publishing.* https://doi.org/10.1063/1.5038801

Predicting tipping points in biological and ecological systems is a fashionable topic. Tipping points are important because they can cause a system to experience an unknown, unwanted, or desired huge change. There are many studies that propose predictor indices. The four well-known indicators of tipping points are the autocorrelation function, the variance, the kurtosis, and the skewness. They provide excellent results in predicting tipping points of period-one attractors. However, in more complex transitions, those indicators fail to predict tipping points. To improve these four indicators, we apply them in two steps. In the first step, we estimate the dynamic of the system. In the second step, based on the estimated dynamic, the original time-series is divided into some sub-time-series. Then, these four well-known indicators are applied to the sub-time-series, which are the different period-components of its attractor. We believe that the improved indicators can deal with more complex transitions than period-one attractors. The proposed tipping point indicators are applied to an ecological and a biological system, and they give excellent results. The improved indicators can predict complex transitions in a period-doubling route to chaos.

I. INTRODUCTION

Different kinds of bifurcation have been observed in many real world dynamical systems such as dynamical disease, brain response to flickering light, climate, and financial markets.[1–4] In bifurcation points, a critical transition occurs, and the dynamic of the system changes from one regime to another. The condition in which a bifurcation from one dynamical behavior to another one occurs is called tipping point (TP).[5] The occurrence of TPs is unexpected in many cases. Thus, prediction of the TP conditions is an important challenge. To date, many methods have been proposed to predict TPs using the time-series extracted from the systems.[6,7] Near a TP, the basin of attraction of the system's attractor becomes shallower. In such a case, a perturbation that drives the system away from the attractor is followed by a slower return to the attractor. This is an important phenomenon which is called critical slowing down.[8] TPs can cause an unwanted transition or a desired one.[8] Till now, some empirical indicators are used to predict upcoming transitions using the system's time-series.[6] Many studies show that critical slowing down causes an increase in the variance and temporal autocorrelation of fluctuations in the system states.[9] Near a TP, the standard deviation increases, and the autocorrelation at lag-1 approaches its maximum value (unity).[8] TP predictors have been called early warning signals of critical transitions.[8] They can be grouped into two categories: metric based and model based indicators. Metric based indicators quantify changes in the system's behavior without associating

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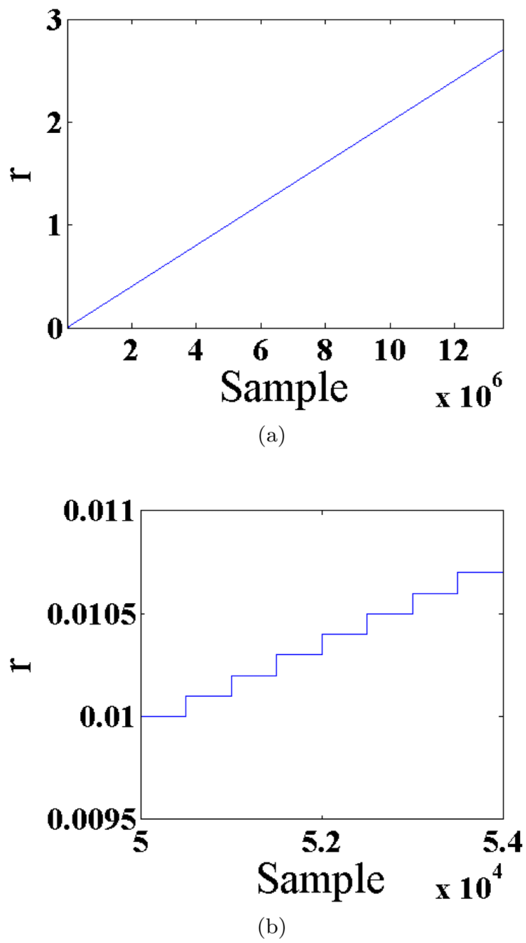


FIG. 1. (a) Variations of parameter r in the interval $r \in [0, 2.7]$ and (b) a zoomed view of part (a). The figure shows variations of parameter r with stepwise changes.

the data to a specific model, while model based early warnings fit a specific model to the data.[10] Some early warning indicators such as autocorrelation at lag-1,[11–14] variance,[10] detrended fluctuation analysis,[10] skewness,[12] kurtosis,[15,16] and complexity measure[17] are metric based indicators. On the other hand, indicators such as nonparametric drift-diffusion-jump models[8] and time-varying AR(p) models[18] are model based indicators. The robustness of these methods in dealing with noise and external disturbances is very important. A proper indicator for predicting TPs should be sensitive to the proximity of the TP. Such an indicator can help to anticipate whether the bifurcations are near. Recently, Lyapunov exponent was proposed as an early warning for predicting different types of TPs.[19]

In this paper, several modified indicators are proposed to predict different TPs. The failure of some applicable indicators encountering different types of TPs is discussed in Section II. In Section III, we propose some modified indicators of TPs and a way to overcome limitations of pervious indicators. In Section IV, quantification of the proposed indicators is discussed. Finally, the paper is concluded in Section V.

II. PROBLEM DEFINITION

Evidence shows that many different bifurcations can occur in biological systems. An experiment has shown the existence of a period-doubling route to chaos in the flickering vision of a salamander.[20] In that experiment, the electroretinogram (ERG) signal of the salamander as a function of flash frequency and the contrast of emitted light to its eye were recorded.[20] In another experiment, aggregates of embryonic chick cardiac cells induced by potassium channel block can show different dynamics such as irregular dynamics, bursts, and doublets.[21] Another example is bifurcations that occur in neural systems. Bifurcation scenarios such as the period-adding route to chaos are reported in some studies.[22–29] Experimental records emphasize the existence of such bifurcations using a chronic constriction injury model of the sciatic nerve.[30] Also, bifurcations can be seen with respect to changing extracellular potassium or calcium concentrations.[26,31–33] Thus, it is important to predict the occurrence of such bifurcations in biological systems.

Benchmarks are needed to investigate the efficiency of leading indicators[34,35] in predicting different types of tipping points. We use an ecological and a biological system which has rich dynamical behaviors and they are well-known in tipping point studies. The formulation of these two models is described in Section II A.

A. Simulated data

The first model used here is a discrete Ricker-type model, which is an ecological model that describes the logistic growth of a population N with an extra loss term. This model is used to depict the dynamics of different organisms such as fish and birds.[36–38] The Ricker-type model is

$$N_{t+1} = N_t e^{r - bN_t + \sigma_E \varepsilon_t} - F\frac{N_t^p}{N_t^p + h^p}, \tag{1}$$

where N_t, r, and b are population biomass, intrinsic growth rate, and the density-dependence $b = r/K$ with a carrying capacity K, respectively. The exploitation is a sigmoid function with half-saturation h and a maximum harvesting rate F. To mimic the effect of the environment in the Ricker-type model, stochasticity is applied with zero mean and standard deviation σ_E. In this paper, we consider parameters $K = 10$, $p = 2$, and $h = 0.75$. We ignore the stochastic term ($\sigma_E = 0$) in investigating the dynamics of model (1). The system has qualitatively different solutions depending on the parameter r, including stable fixed points, periodicity, and chaos.

The second system is a model proposed for attention deficit disorder (ADD).[39] Dopamine deficiency is one of the causes of this disorder. This model involves a nonlinear neuronal network which describes the interactions of inhibitory and excitatory parts of brain action. The ADD model is given by

$$x_{k+1} = B\tanh(w_1 x_k) - A\tanh(w_2 x_k), \tag{2}$$

where $B = 5.821$, $w_1 = 1.487$, and $w_2 = 0.2223$ are constant parameters, and A is considered as the bifurcation parameter. This equation is a behavioral model of a neuronal network. x is the electrical activity of this network.

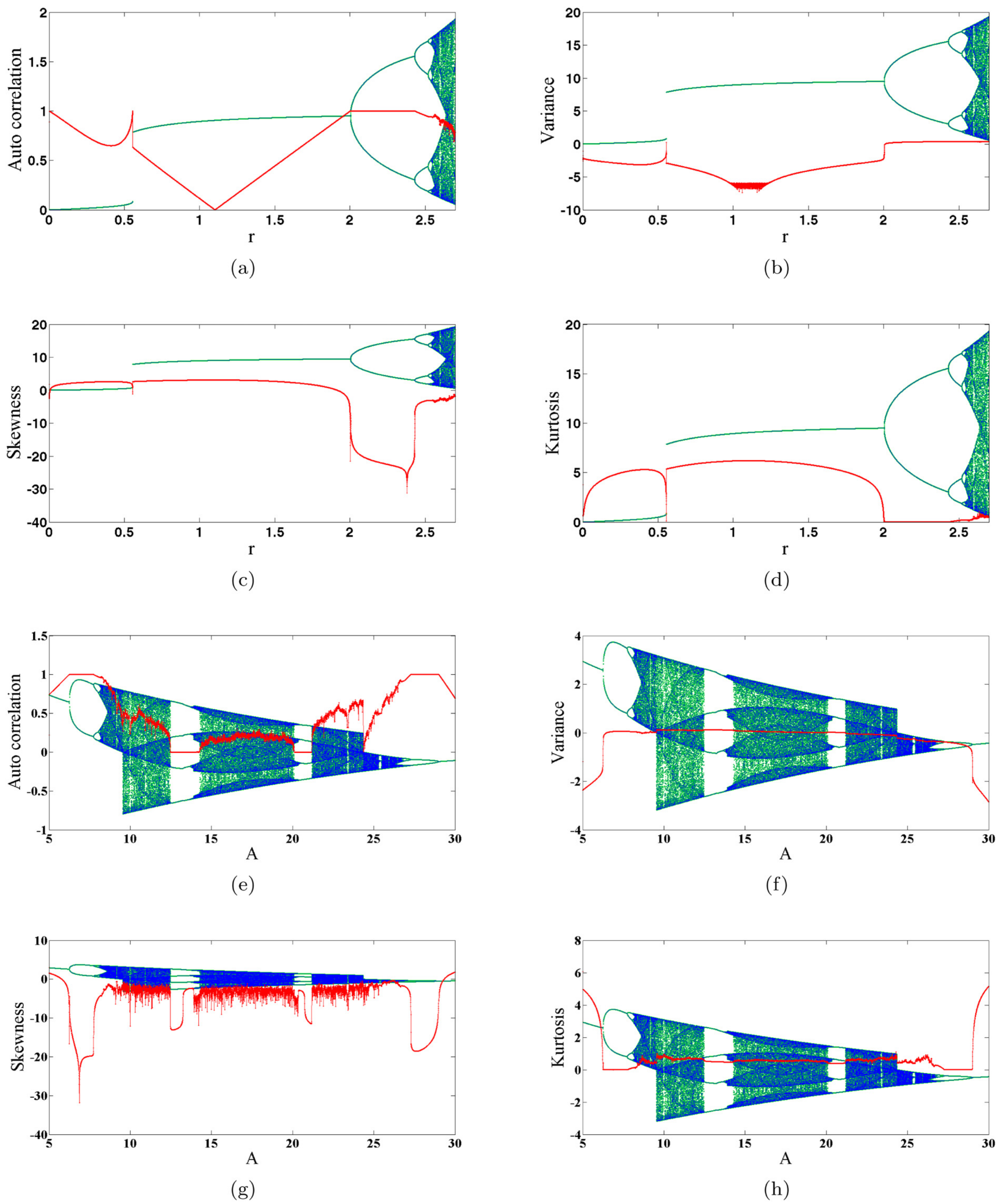
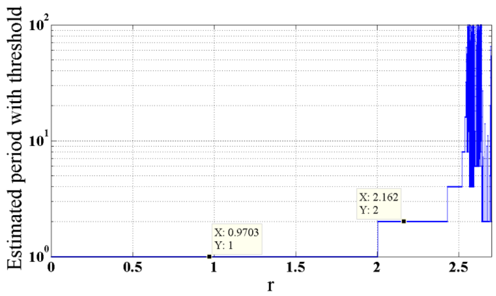
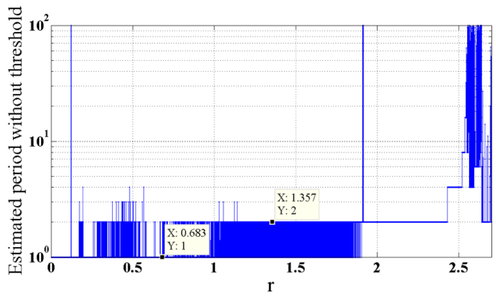


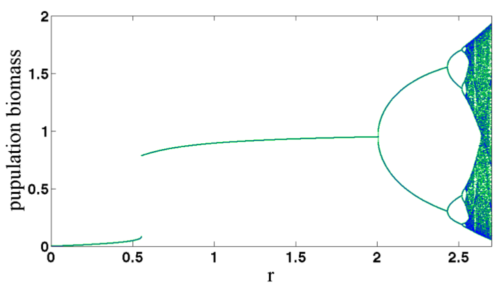
FIG. 2. (a) Bifurcation diagram of the Ricker-type model with respect to changing parameter r in the interval [0,2.7] multiplied by 0.1 for better observation in green-blue color and the absolute value of the autocorrelation at lag-1 in red, (b) bifurcation diagram of the Ricker-type model with respect to changing parameter r in the interval [0,2.7] multiplied by 0.1 for better observation in green-blue color and the logarithm of the variance in red, (c) bifurcation diagram of the Ricker-type model with respect to changing parameter $r \in [0, 2.7]$ multiplied by 0.1 for better observation in green-blue color and the logarithm of the skewness in red, (d) bifurcation diagram of the Ricker-type model with respect to changing parameter $r \in [0, 2.7]$ multiplied by 0.1 for better observation in green-blue color and the logarithm of the kurtosis in red, (e) bifurcation diagram of the ADD model with respect to changing parameter $A \in [5, 30]$ (multiplied by 0.25 for better observation) in green-blue color and the autocorrelation method in red, (f) bifurcation diagram of the ADD model with respect to changing parameter$A \in [5, 30]$ (multiplied by 0.25 for better observation) in green-blue color and the logarithm of the variance in red, (g) bifurcation diagram of the ADD model with respect to changing parameter $A \in [5, 30]$ (multiplied by 0.25 for better observation) in green-blue color and the logarithm of the skewness in red, and (h) bifurcation diagram of the ADD model with respect to changing parameter $A \in [5, 30]$ (multiplied by 0.25 for better observation) in green-blue color and the logarithm of the kurtosis in red. The figure shows that the four well-known indicators have not a proper trend in approaching a tipping point and receding from it except for period-one dynamic.



(a)



(b)



(c)

FIG. 3. (a) Estimated period of the Ricker-type model with a threshold on autocorrelation and variance, (b) estimated period of the Ricker-type model without variance threshold, and (c) bifurcation diagram of the Ricker-type model with respect to changing parameter r in two green-blue colors. It can be observed from this figure that the threshold is necessary to have a proper prediction of the period of system.

B. Tipping point predictors

Much research has been done on predicting those tipping points that cause an overexploitation bifurcation in the system's state.[5,8,16,40–42] In this type of bifurcation, there exist two fixed points in the system. By changing a parameter, a stable fixed point of the system becomes unstable, while another one which was unstable becomes stable. Also, in some intervals of some parameters, two fixed points are stable simultaneously and exhibit hysteresis with changing parameters. In other words, there are two coexisting attractors in this situation, and the system can jump between them by perturbations. Thus, the system can have an abrupt shift between these alternate attractors. Recently, some studies considered different transitions in the dynamics of populations.[43,44] They use a Ricker-type model to simulate transitions from a stable equilibrium to cyclic and chaotic behaviors. They attempt to predict these tipping points using a nonlinearity measure and compare its results by autocorrelation at lag-1 and variance.[44] Figure 1 of Ref. 44 shows population abundance trajectories with respect to changing conditions and the three indicators. This figure shows that the three indicators cannot predict different tipping points such as "period-one to period-two" or "periodic to chaotic."[44]

A desirable method should have several features in predicting tipping points. One of these features is that it should have a specific similar value in the occurrence of different tipping points to indicate how close the system is to a tipping point. Also, the method should have an appropriate trend when approaching a tipping point (a proper indicator should have an extremum in the tipping point). Such a trend helps prediction of tipping points before they happen (especially in cases where the bifurcation parameter is varying in time). To test the results of classic tipping point indicators on the Ricker-type model, these indicators are calculated for each value of the parameter r using 500 samples of its time-series. After 500 iterations, the parameter r is changed to $r + \Delta r$ and the system is followed for another 500 iterations. So, parameter r changes step by step, and each step is 500 iterations. In other words, the bifurcation parameter changes with a step function. The bifurcation parameter is constant for a window with length 500 and then changes to a new value. Figure 1 shows the variations of parameter r, which is changed in the interval $r \in [0, 2.7]$ and $\Delta r = 0.0001$. Part (b) shows a zoomed view of part (a) with its stepwise changes.

In this paper, two colors are used to obtain a more beautiful bifurcation diagram (green is the shadow of blue). Part (a) of Fig. 2 shows the absolute value of the autocorrelation at lag-1 for the bifurcation of model (1). The bifurcation diagram depicts the final state of N_t by scanning the parameter upward. As the figure shows, autocorrelation at lag-1 has a good trend in approaching a tipping point of period-one behavior and receding from it. But in higher periods, autocorrelation at lag-1 does not give good results. Another well-known tipping point indicator is variance. Part (b) of Fig. 2 shows the logarithm of the variance with respect to changing parameter r of model (1). The logarithm function provides a better observation. Similar to the autocorrelation at lag-1, the results show that the variance has a good trend in approaching and receding from the tipping point of the period-one attractor, but it does not give a good result in more complex behaviors. Parts (c) and (d) of Fig. 2 show the logarithm of the skewness and kurtosis of model (1) with respect to changes in the parameter r. Parts (e), (f), (g), and (h) of Fig. 2 present the results of autocorrelation at lag-1, logarithm of variance, skewness, and kurtosis of model (2) with respect to changing parameter A. The results show that these methods cannot predict different tipping points of ADD model except the period-one transitions. In other words, Fig. 2 shows that these leading indicators sometimes have proper performance in transitions between some period-one attractors, but they do not show good results in transitions between more complex behaviors. In this paper, we modify these well-known "tipping point indicators" and make them more efficient to predict various types of bifurcations in biological systems.

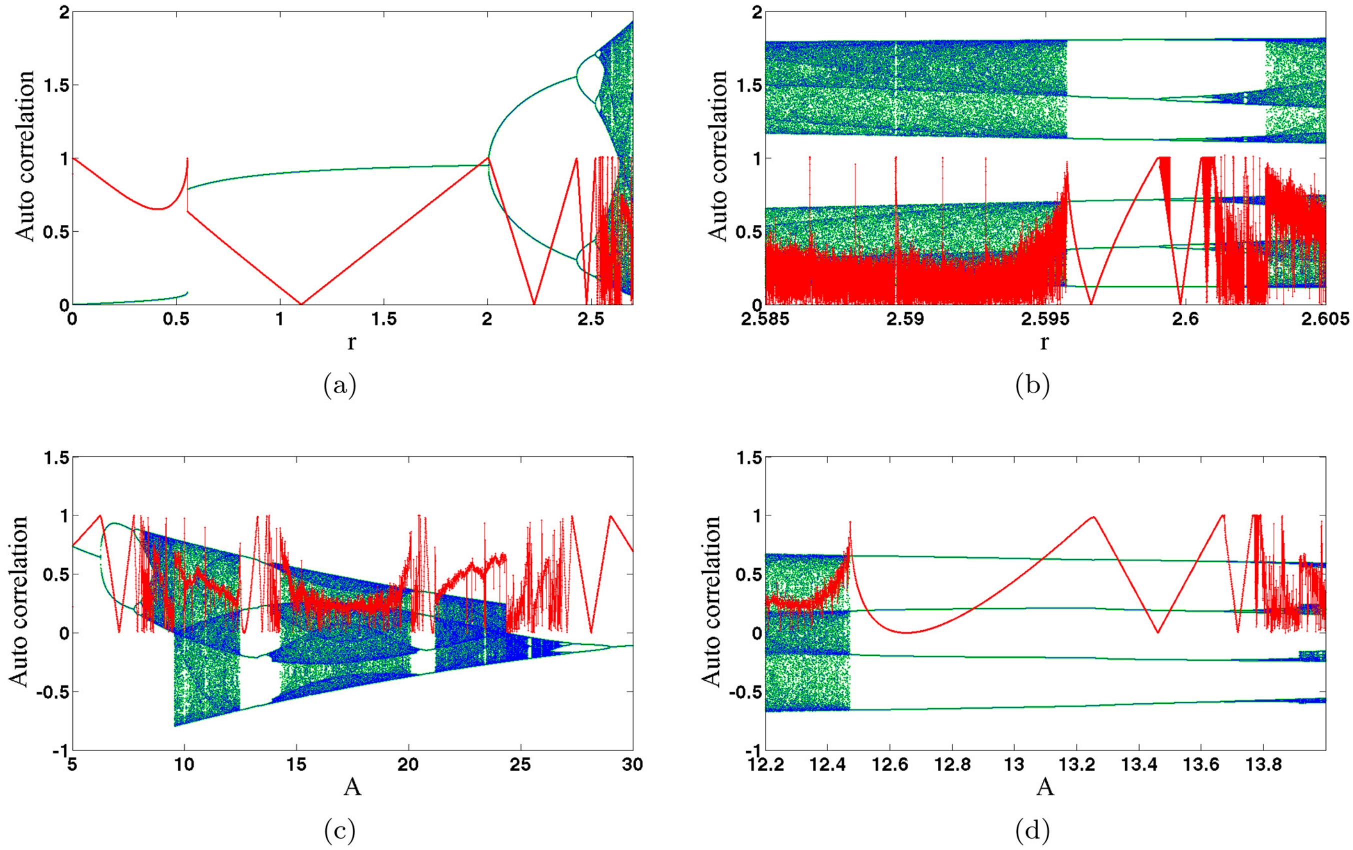


FIG. 4. (a) The rescaled bifurcation diagram of the Ricker-type model and the absolute value of the improved autocorrelation method in red with respect to changing parameter r in the interval [0,2.7], (b) the rescaled bifurcation diagram of the Ricker-type model and the absolute value of the improved autocorrelation method in red with respect to changing parameter r in the interval [2.585,2.605], (c) bifurcation diagram of the ADD model (multiplied by 0.25) and the improved autocorrelation method in red with respect to changing parameter A in the interval [5,30], and (d) bifurcation diagram of the ADD model (multiplied by 0.25) and the improved autocorrelation method in red with respect to changing parameter A in the interval [12.2,14]. The improved autocorrelation has a proper trend close to and far from the TPs. The index has a proper trend when approaching the edge of the periodic window and also shows the tipping points of the pitchfork bifurcations.

III. MODIFIED EARLY WARNING INDICATORS

In this section, we design an algorithm to extract the type of system's behavior. Then, we add this algorithm to the well-known early warning indicators. In other words, the proposed modified early warning indicators contain two steps. Consider a time-series x with N samples. In the first step, the autocorrelation at lag-m of the signal is calculated for all $m < m_{threshold}$, where $m_{threshold}$ is a threshold to estimate the period of the time-series. Then, the minimum value of m for which the autocorrelation at lag-m is maximized is selected as the estimated period of the system (we call that m^*). When the estimated period of the system is m^*, it means that the system has a period-m^* cycle. The period of the time-series is estimated as $m_{threshold}$ when its real period is more than $m_{threshold}$ or if it is chaotic. In the second step, m^* vectors are created from the signal

$$\begin{aligned}
V_1 &= (x_1, x_{m^*+1}, x_{2m^*+1}, \ldots, x_{\left[\left(\frac{N}{m^*}\right)-1\right]m^*+1)},\\
V_2 &= (x_2, x_{m^*+2}, x_{2m^*+2}, \ldots, x_{\left[\left(\frac{N}{m^*}\right)-1\right]m^*+2)},\\
&\vdots\\
V_{m^*} &= (x_{m^*}, x_{m^*+m^*}, x_{2m^*+m^*}, \ldots, x_{\left[\left(\frac{N}{m^*}\right)-1\right]m^*+m^*)},
\end{aligned} \tag{3}$$

where [.] is the floor function. The classical early warning indicators are calculated for each of the m^* vectors. The average of these m^* calculated indicators is the proposed improved early warning index.

A. First step: Extracting the system's dynamic type

The period of the time-series is assumed to be the first nonzero maximum of the autocorrelation function among different lags. In this step, we use two thresholds which set the autocorrelation to unity if it is greater than the threshold or the variance of the time-series is less than a threshold. These thresholds help the algorithm deal with transients in the observed time-series. In this paper, $m_{threshold}$ is taken as 100. The threshold value of the autocorrelation is 0.99999999, and the threshold value of the variance is 0.001. Part (a) of Fig. 3 shows the estimated periods of the Ricker-type model using autocorrelation as the parameter r is changed. Two data cursors show the estimated period for parameters $r = 0.9703$ and 2.162. The transient parts are not a relevant dynamic of the system and cause errors in extracting the type of dynamic. Therefore, these parts of the signal are removed by observing a long time-series, or some thresholds in the autocorrelation and variance can help the algorithm cope with it. If the value of the autocorrelation is larger than a threshold or the variance

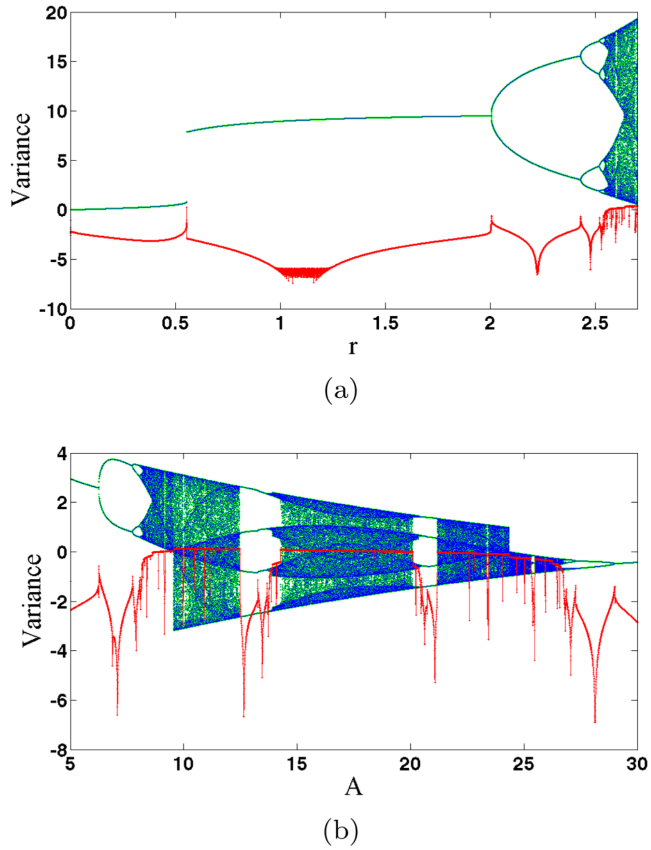


FIG. 5. (a) Bifurcation diagram of the Ricker-type model and one tenth of the logarithm of the improved variance method in red with respect to changing parameter r in the interval [0,2.7] and (b) bifurcation diagram of the ADD model and one tenth of the logarithm of the improved variance method in red with respect to changing parameter A in the interval [5,30]. The improved variance method has a proper trend when close to or far from the tipping points, and it does not have the same value at different tipping points.

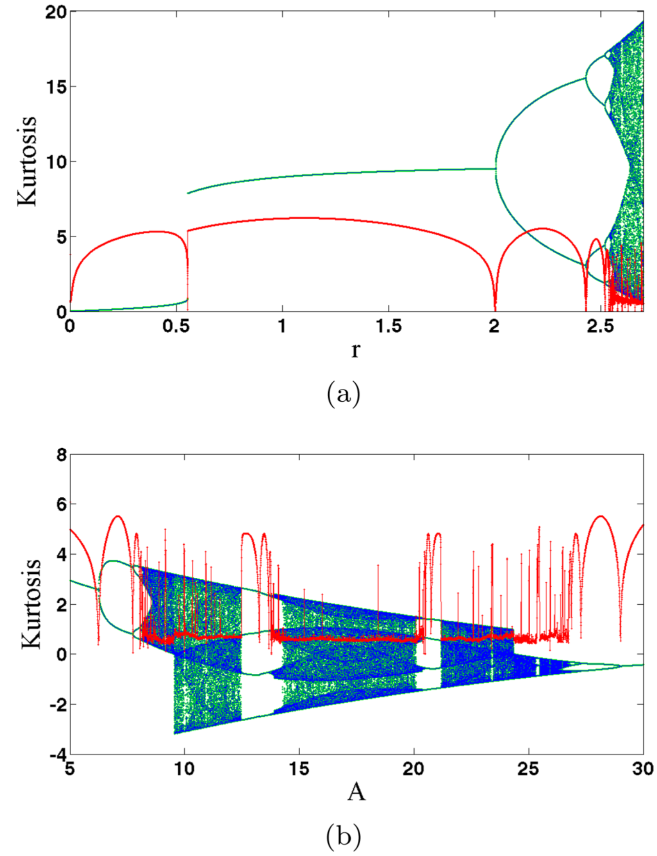


FIG. 6. (a) Bifurcation diagram of the Ricker-type model and the logarithm of the improved kurtosis method in red with respect to changing parameter r in the interval [0,2.7] and (b) bifurcation diagram of the ADD model and the logarithm of the improved kurtosis method in red with respect to changing parameter A in the interval [5,30]. The improved kurtosis decreases near the tipping point because the probability distribution is close to a Gaussian distribution. It can predict pitchfork bifurcations as well as period-doubling bifurcations.

of the time-series is less than a threshold, the autocorrelation is set to unity. Without these thresholds, the transients in the time-series cause incorrect estimated periods. Part (b) of Fig. 3 shows the estimated periods of the Ricker-type model with respect to changing the parameter r without the variance threshold. Two data cursors show the estimated period for parameters $r = 0.6083$ and 1.357. In part (c) of this figure, the bifurcation diagram of the Ricker-type model with respect to changing parameter r is shown.

B. Second step: Calculating the proposed modified indicators

After estimating the period of the signal for each value of the parameter, the indicator is calculated for each vector of Eq. (3). Then, its average is taken as the improved indicator. More details are discussed in Sections III C–F.

C. Modified autocorrelation method

The first early warning indicator which is discussed in this paper is autocorrelation. In this section, the modified autocorrelation method is investigated using the two mentioned models, the Ricker-type and the ADD model.

In the Ricker-type model, after estimating the period of the time-series for each value of the parameter, the autocorrelation is calculated for each V_i vector of Eq. (3). Then, their average is taken as the improved autocorrelation. For example, when the estimated period is four, we calculate the autocorrelation for each V_i vector where $i = 1, 2, 3, 4$. Finally, the average of these four autocorrelation values is taken as the new early warning indicator. Part (a) of Fig. 4 shows the absolute value of this early warning for the Ricker-type model with respect to changing parameter r. The result shows that the improved early warning can predict different TPs and has a proper trend when close to or far from the TPs. Also, the value of the early warning index is unity in the occurrence of different types of tipping point. Part (b) of Fig. 4 shows the improved early warning near a periodic window. As the figure shows, it has a proper trend before the occurrence of the periodic window which can allow its prediction. Also, it reveals the thin periodic windows between chaotic domains. Parts (c) and (d) of Fig. 4 show the improved autocorrelation index of the ADD model with respect to changing parameter A. The improved autocorrelation index has a proper trend when approaching the edge of the periodic window and also shows

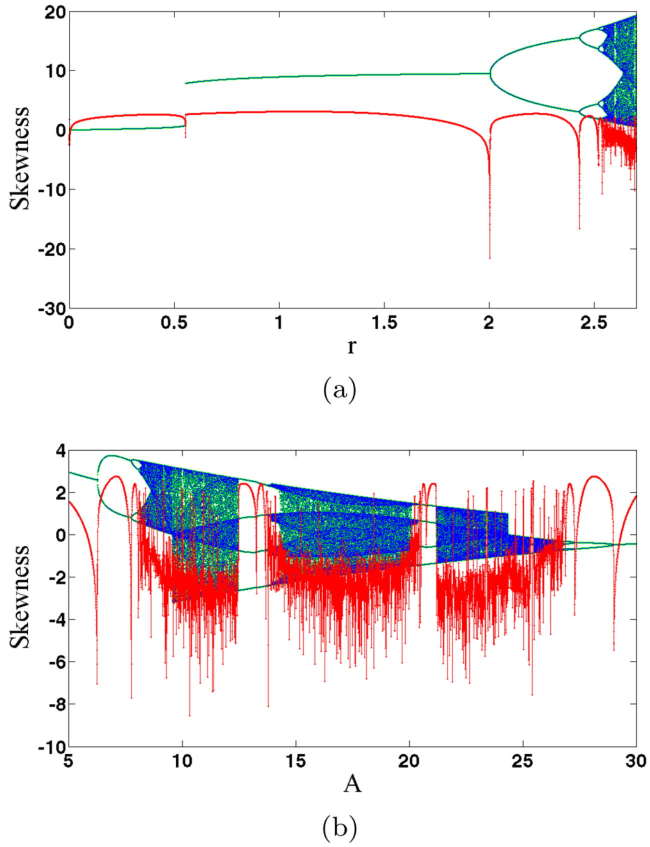


FIG. 7. (a) Bifurcation diagram of the Ricker-type model and the logarithm of the absolute value of the improved skewness method in red with respect to changing parameter r in the interval [0,2.7] and (b) bifurcation diagram of the ADD model and the logarithm of the absolute value of the improved skewness method in red with respect to changing parameter A in the interval [5,30]. The improved skewness has a proper trend close to and far from the tipping points, but it does not have a constant value at different tipping points.

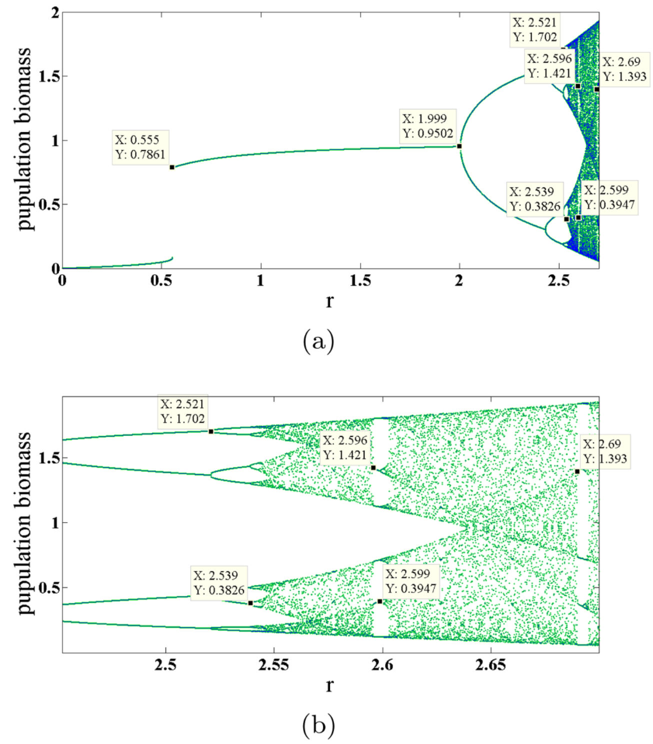


FIG. 8. (a) Seven selected landmarks of the Ricker-type model in the interval $r \in [0, 2.7]$ and (b) selected landmarks of the Ricker-type model in the interval $r \in [2.46, 2.7]$.

the tipping point of the pitchfork bifurcation. In the pitchfork bifurcation which is happened in parameter $A = 13.25$, the attracting period 4-cycle changes into a repelling 4-cycle (as in period doubling) but also splits into two new attracting 4-cycles, each with its own basin of attraction, which change as parameter A changes.

D. Modified variance method

Another well-known early warning is variance. As stated in Section II, this early warning cannot predict transitions between complex dynamics. In this section, the improved variance method is studied in the Ricker-type and the ADD model.

Near a tipping point, the state of a system under a small perturbation returns slowly to its stable point. Thus, the variance increases near the tipping points. But the classical variance can only predict tipping points which are transitions between period-one attractors. For improving the variance method, the same algorithm as autocorrelation is applied to it. The improved early warning indicator is calculated in two steps. In the first step, period of the time-series is calculated, while in the second step the variance of each V_i vector of Eq. (3) is calculated ($i = 1, 2, \ldots, m^*$). The average of these variances is taken as the new early warning index. Part (a) of Fig. 5 shows the logarithm of the improved variance in the Ricker-type map. As the figure shows, the proposed method has a proper trend close to and far from the tipping points. However, this early warning does not have the same value for different tipping points. Thus, the method can just predict approaching the tipping points or receding from them, while it cannot determine when the tipping points occur. As another example, the proposed variance method is applied to the ADD model. Part (b) of Fig. 5 shows the improved variance method. The proposed method has a proper trend to predict tipping points. Part (b) of Fig. 5 shows that this method also can predict pitchfork bifurcations.

E. Modified kurtosis method

The variance of the state increases near tipping points. Thus it enhances the tail of the distribution. The tailedness of the distribution is measured using the kurtosis, and it is used as an early warning signal for predicting tipping points. The classical kurtosis failed to predict tipping points of higher-order transitions. Thus, a new kurtosis method is proposed in this section. To improve the kurtosis early warning indicator, the two step algorithm is used. First, the period of the time-series is calculated, and then the kurtosis is calculated for each V_i vector of Eq. (3). The average value of calculated kurtosis is taken as the improved kurtosis method. Part (a) of Fig. 6 shows the logarithm of the proposed kurtosis method. The results show that the kurtosis decreases near the tipping points (the probability distribution is close to a Gaussian distribution) and the method has a proper trend near the tipping points. Applying the proposed kurtosis method to the ADD

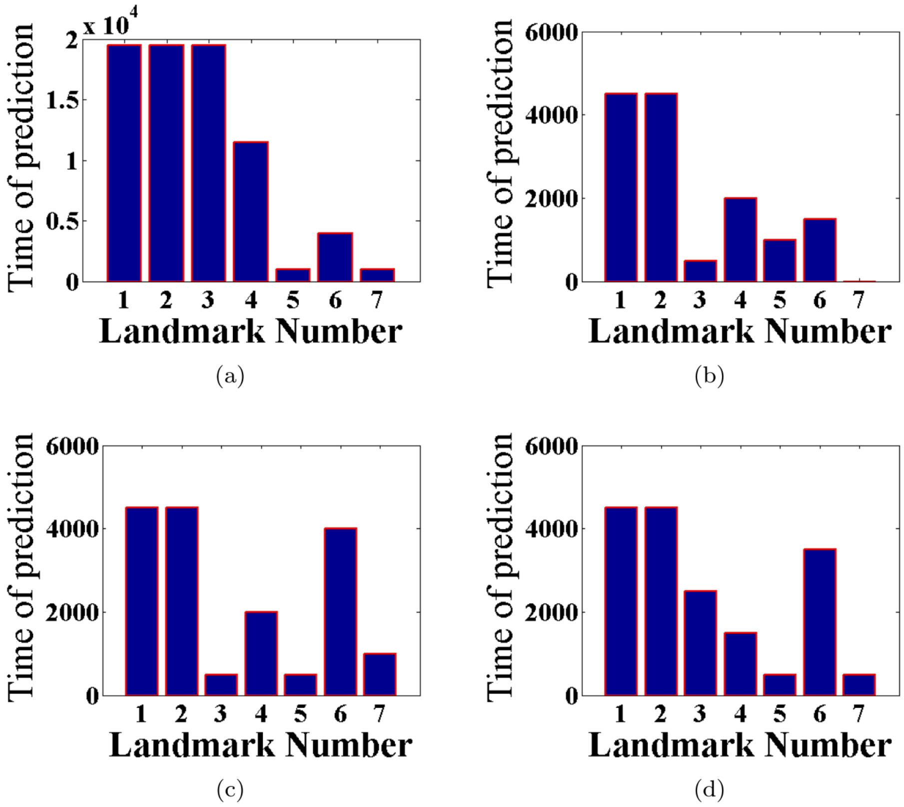


FIG. 9. (a) Sample time interval of prediction of each landmark using the modified autocorrelation method, (b) sample time interval of prediction of each landmark using the modified variance method, (c) sample time interval of prediction of each landmark using the modified kurtosis method, and (d) sample time interval of prediction of each landmark using the modified skewness method. The figure shows the quantification of proposed methods in seven mentioned landmarks.

model shows a proper trend close to and far from the different tipping points [part (b) of Fig. 6]. Thus, this method can be used as a proper predictor of different tipping points. Part (b) of Fig. 6 shows that the method can predict pitchfork bifurcations as well as period-doubling bifurcations.

F. Modified skewness method

Close to the tipping points, the state of the system returns to its attractor slowly. The asymmetry of the system by approaching a bifurcation point increases the absolute value of skewness. The discussion in Section II shows that the common skewness can only predict bifurcations with transitions between period-one attractors. In this section, a new skewness measure is proposed which can predict different tipping points.

In the proposed skewness method, the period of the time-series for each parameter of the system is estimated, and the skewness is calculated for each period-component. The average values of these measures are proposed as an improved skewness method. Part (a) of Fig. 7 shows the logarithm of the absolute value of the skewness for the proposed method. It shows that this method has a proper trend close to and far from the tipping points, but it does not have a constant value at different tipping points. By applying the proposed skewness method to the ADD model, the results show that the proposed method showed a good performance in predicting pitchfork bifurcation as well as period-doubling [part (b) of Fig. 7]. The results show that the improved skewness does not have a constant value at different tipping points.

IV. QUANTIFICATION OF THE PERFORMANCE OF INDICATORS

In order to quantify the performance of the proposed indicators, we use seven landmarks on the bifurcation points of the Ricker-type model. Then we compare the application of indicators in predicting these landmarks. The mentioned landmarks are shown in part (a) of Fig. 8. Part (b) of the figure is a zoomed view of part (a).

To quantify the performance of the modified autocorrelation indicator in each landmark, consider the bifurcation diagram and modified autocorrelation indicator of the Ricker-type model in parts (a) and (b) of Fig. 4. We use a threshold on autocorrelation equal to 0.8 to find TPs. Then, we investigate the existence of any warning in the 40 past windows (20 000 past samples) which shows the occurrence of a TP is near. Part (a) of Fig. 9 shows the sample time interval of prediction of each landmark using the modified autocorrelation method. As seen in Fig. 5, the modified variance method increases near the occurrence of TPs. So we follow the ascending of the modified variance in three consecutive samples. Part (b) of Fig. 9 shows the sample time interval of prediction of each landmark using the modified variance method. In the modified kurtosis and skewness (Figs. 6 and 7, they decrease near the occurrence of TPs. So the descending of the modified indicators in three consecutive samples is used to predict TPs. Parts (c) and (d) of Fig. 9 show the sample time interval of prediction

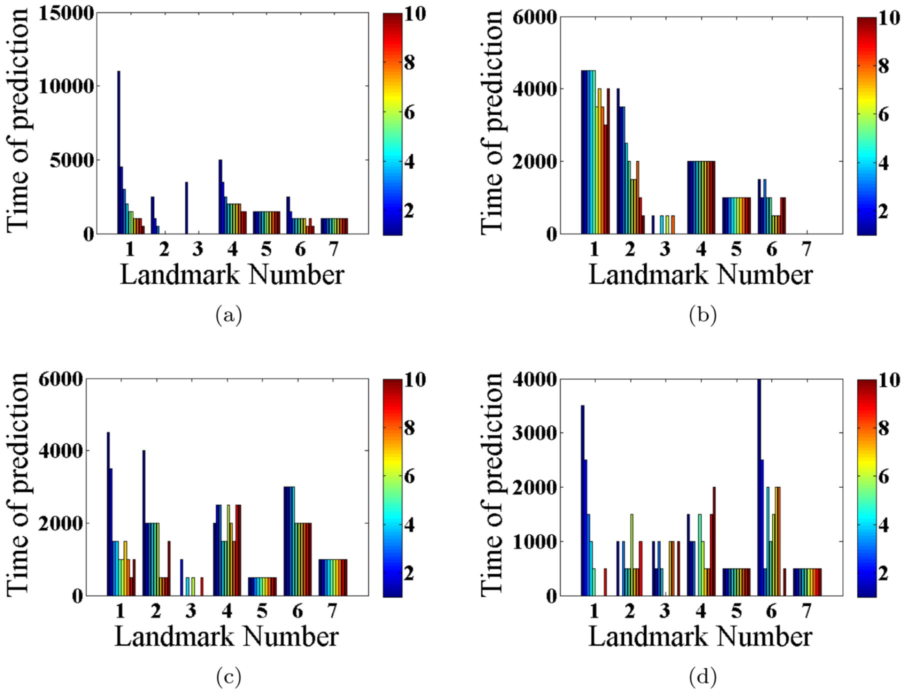


FIG. 10. (a) The sample time interval of prediction of modified autocorrelation, (b) the sample time interval of prediction of modified variance, (c) the sample time interval of prediction of modified kurtosis, and (d) the sample time interval of prediction of modified skewness method in the Ricker-type model, for seven landmarks, and for ten values of $\sigma_n \in [0.0001, 0.001]$ which are depicted by colors as the color bar. The variance method has the most robust result by increasing the power of noise.

of each landmark using the modified kurtosis and skewness method, respectively.

V. CONCLUSION AND DISCUSSION

In this paper, a new perspective has been illustrated to make the old early warning indicators suitable for predicting different types of TPs. In other words, we have proposed several modified indicators to predict different TPs such as bifurcations between different periods, crises, and periodic windows. These types of bifurcations can be seen in a common period-doubling route to chaos which is occurred in many real world dynamical systems. There were two main steps in our proposed procedure. First, the dynamic of the considered system was estimated using its time-series. Second, the original time-series was divided into some sub-time-series. In other words, we separated the time-series into different period-components. Then, the four different tipping point indicators were applied to the extracted sub-time-series. These modified indicators were applied to a biological model and an ecological model. The ecological model (Ricker-type) has different bifurcations such as jumping between equilibrium points, period-doubling with different periods, periodic window, and crises. The biological model (ADD) contains period-doubling with different periods, pitchfork bifurcation, periodic window, and crises. Applying the modified indicators on these two models shows their generality in different applications. The proposed method showed proper performance in predicting different tipping points. The main advantage of the proposed indicators is their ability to anticipate different TPs in a period-doubling route to chaos. Previous indicators were unable to predict these TPs, and they can only predict TPs of period-one. Noise and external perturbations can cause faults for anticipating TPs in the proposed methods just like with previous indicators. To investigate the influence of noise on the performance of the proposed methods, white Gaussian noise n_k with zero mean and variance σ_n is added to the state of the system x_k as follows:

$$\begin{aligned} x_{k+1} &= f(x_k), \\ z_k &= x_k + n_k, \end{aligned} \tag{4}$$

z_k is the noisy observation of the system. Figure 10 shows the sample time interval of prediction of the modified autocorrelation (a), variance (b), kurtosis (c), and skewness (d) method in the Ricker-type model, for seven landmarks, and for ten values of $\sigma_n \in [0.0001, 0.001]$ which are depicted by colors as shown in the color bar. It can be seen that increasing the power of the noise decreases the sample time interval of prediction of each landmark in the modified autocorrelation method. Also, the variance method has the most robust result by increasing the power of the noise. Kurtosis and skewness are less robust to noise.

In the real applications, one can window the signal with specified overlap and calculate indicators in each window. Then, evolution of those indicators can tell us how close the system is to a TP.

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