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Noise-induced spatial periodicity in excitable chemical media

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Abstract

We show that spatiotemporal noise acting on the illumination intensity of an excitable photosensitive Belousov–Zhabotinskii medium, modelled by the diffusively coupled two-component Oregonator kinetics, is able to extract a characteristic spatial frequency of the system in a resonant manner. We emphasize that thereby the system is locally initiated from steady state excitable conditions so the observed patterns are exclusively noise induced. Thus, the reported phenomenon is a novel observation of spatial coherence resonance in excitable chemical media.

1. Introduction

Since the early experiments with the Belousov–Zhabotinskii (BZ) medium [1,2], target patterns have become the most distinctive and visually compelling examples of self-organization in chemical systems. Coherent spatial structures have also been observed in catalytic surface reactions [3], cardiology [4] and neurophysiology [5], as well as optical devices [6].

Given the ubiquity of wave-like behaviour in chemical media, several studies have been devoted to the analysis of these fascinating phenomena [7–13]. A particularly interesting aspect of media dynamics that has recently attracted much interest is the ability of noise to induce spatiotemporally ordered behaviour [14]. Spatiotemporal stochastic resonance has been first reported in [15] for excitable systems, while explicit spatial coherence resonance has been introduced in [16] for systems near pattern-forming instabilities. Moreover, there exist studies reporting noise-induced spiral growth and enhancement of spatiotemporal order [17–19], noise sustained coherence of space-time clusters and self-organized criticality [20], noise-induced excitability [21], noise-induced propagation of harmonic signals [22], as well as noise sustained and controlled synchronization [23] in space extended systems. Particularly for chemical systems, there also exist several theoretical and experimental studies reporting a constructive effect of noise on the spatiotemporal dynamics of chemical media [24–28]. Despite vast literature existing on this topic, little attention has been devoted to the explicit analysis of characteristic spatial frequencies of non-linear media. Besides the work of Carrillo et al. [16], there exist no studies reporting resonant enhancement of an inherent spatial frequency in space extended systems.

In the present study, we analyse spatial frequency spectra of excitable chemical media in dependence on different noise intensities. We show that spatiotemporal noise acting on the illumination intensity of an excitable photosensitive Belousov–Zhabotinskii medium [29], locally modelled by the Oregonator kinetics [30], is able to extract an inherent spatial frequency of the system in a resonant manner. By calculating the average spatial structure function, we present first evidences for spatial coherence resonance in excitable media. Note that although coherence resonance phenomena have been extensively studied in arrays of dynamical systems [31,32] as well as systems with spatial degrees of freedom

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[25–28], our work focuses explicitly on the spatial [16] rather than temporal or spatiotemporal system scale.

Given the novelty of reported spatial coherence resonance in excitable media, our work hopefully outlines some possibilities for future experimental work, especially in chemistry, but also in other fields of research such as cardiology and neurophysiology, where excitability and noise in space extended systems appear to be universally present.

2. Mathematical model

The spatial dynamics of the excitable Belousov–Zhabotinskii medium is studied by using the photosensitive version [29] of the reduced Oregonator model [33] given with the equations:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{\varepsilon}\left[u - u^2 - (fv + \phi)\frac{u - q}{u + q}\right] + D\nabla^2 u, \tag{1}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = u - v, \tag{2}$$

where the dimensionless concentrations of bromous acid $u(x, y, t)$ and the oxidised form of the light-sensitive catalyst $v(x, y, t)$ are considered as two-dimensional scalar fields on a $n \times n$ square lattice with mesh size $\Delta x = 0.3125$. Parameters $\varepsilon = 0.077$ and $q = 0.002$ determine the kinetics of the reaction, $f = 1.4$ is a stoichiometric parameter, whereas ϕ is proportional to the illumination intensity, which is given by $\phi = \phi_0 + \xi$, where ξ is Gaussian noise with zero mean, white in space and time, and variance σ^2 [14]. The Laplacian $D\nabla^2 u$, D being the diffusion coefficient for u, is integrated into the numerical scheme via a five-point finite-difference formula with no-flux boundary conditions. For above fixed parameter values and small ϕ_0 the local deterministic kinetics is oscillatory, while beyond the supercritical Hopf bifurcation at $\phi_0 = 4.4 \times 10^{-3}$ an excitable steady state governs the dynamics. In following calculations, we set $\phi_0 = 0.01$ and initial conditions satisfying $\mathrm{d}u/\mathrm{d}t = \mathrm{d}v/\mathrm{d}t = 0.0$. Thus, under deterministic conditions all lattice sites would remain forever in their initial excitable steady states and the medium strictly unable to exhibit wave-like behaviour. On the other hand, small noisy perturbations of the illumination profile locally evoke non-trivial spike-like behaviour, which can induce regular wave propagation in the spatial domain of the space extended system [26–28]. Note that noise acting additively on ϕ_0 is in fact multiplicatively coupled to the state of the system u, thus introducing a deterministic non-negative contribution to the dynamics at first order in noise intensity [14,26]. However, since in our case the applied σ^2 are small (see below), this effect does not induce a shift in the dynamics governing the system, and thus the observed phenomenon is not a consequence of a noise-induced phase transition as reported in [21], for example, but must be attributed to noise-induced threshold crossing events. In what follows, we will show that there exists an optimal noise intensity for which a particular spatial frequency of the studied system is resonantly enhanced, thus providing first evidences for spatial coherence resonance in excitable chemical media.

3. Spatial coherence resonance

To quantify effects of various noise intensities on the spatial scale of the studied system we calculate the structure function according to the equation

$$P(k_x, k_y) = \langle H^2(k_x, k_y)\rangle / S, \tag{3}$$

where $H(k_x, k_y)$ is the spatial Fourier transform of the u – field at a particular t, S is the area of the system, and $\langle \ldots \rangle$ is the ensemble average over noise realizations. Note that $P(k_x, k_y)$ can also be interpreted as the spatial power spectrum of the system. To study results obtained according to Eq. (3) in more detail, we exploit the spherical symmetry of the spatial power spectra as proposed in [16]. In particular, we calculate the spherical average of the structure function according to the equation

$$p(k) = \int_{\Omega_k} P(\vec{k})\, \mathrm{d}\Omega_k, \tag{4}$$

where $\vec{k} = (k_x, k_y)$, and Ω_k is a spherical shell with radius $k = |\vec{k}|$. Fig. 1a shows results for various σ^2. It can be well observed that there indeed exists a particular spatial frequency, marked with the vertical dashed line at $k_{\max}$, that is resonantly enhanced for some intermediate noise intensity. To quantify the ability of each σ^2 to extract the characteristic spatial periodicity in the system more precisely, we calculate the signal-to-noise ratio (SNR) as the peak height at $k_{\max}$ normalized with the background fluctuations existing in the system. This is the spatial counterpart of the measure frequently used for quantifying constructive effects of noise in the temporal domain of dynamical systems, whereas an identical measure for quantifying effects of noise on the spatial scale of space extended systems was also used in [16]. Fig. 1b shows how the SNR varies with σ^2 for three different diffusion constants D. It is evident that there always exists an optimal noise intensity for which the peak of the spherically averaged structure function is best resolved, thereby indicating the existence of spatial coherence resonance in the studied excitable chemical media.

The existence of a preferred spatial periodicity in the studied excitable media for certain noise intensities can be well corroborated by studying snapshots of typical u – field configurations for optimal σ^2 and various D, as presented in Fig. 2. It is evident that optimal σ^2 at each particular D clearly enhance a particular spatial scale, thus providing visible evidences that corroborate

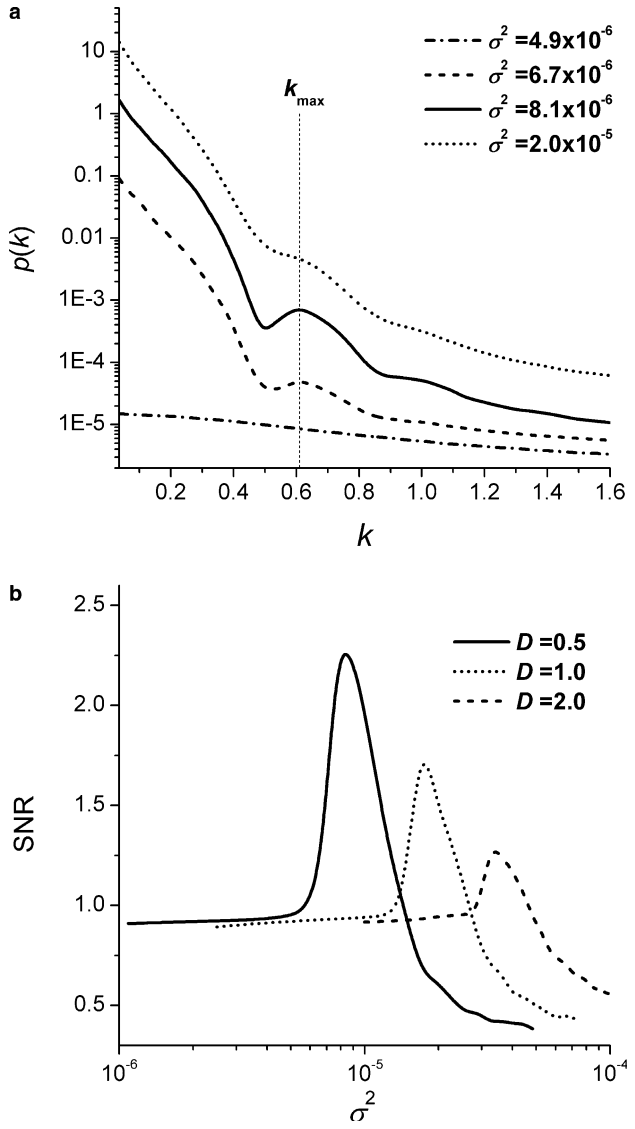

Fig. 1. Spatial coherence resonance in the studied excitable media: (a) spherical average of the structure function for various σ^2 at $D = 0.5$; (b) SNR in dependence σ^2 for various D.

results presented in Fig. 1. Interestingly, note how the width of the wave pattern representing the characteristic scale of the system, i.e., the inverse of k_{max}, increases with increasing D. As we will argue next, this property is crucial for explaining the observed spatial coherence resonance in the studied chemical media.

To explain the above-reported phenomenon, we first briefly summarize findings regarding the temporal coherence resonance in excitable systems [34]. It is known that excitable systems have a characteristic firing time t_e, termed excursion time, which is well preserved under variable noisy perturbations. Contrary, the average time between consecutive firings t_a, termed activation time, depends heavily on the noise intensity, i.e., decreases with increasing σ^2. The time coherence of the system is best pronounced when σ^2 is large enough so that $t_a \ll t_e$, but still small enough so that fluctuations of t_e remain moderate and thus the outline of excursion phase smooth [34].

These different noise dependencies of t_e and t_a, together with the rate of diffusive spread that is proportional to $\sqrt{D}$, hold also the key to understanding the spatial coherence resonance in excitable media. We argue that during t_e each particular lattice site acts like a circular (after local initialisation all directions for spreading are equally probable) front initiator. After initialisation the front starts to spread through the media with a rate proportional to $\sqrt{D}$. When embarking on neighbouring sites the front can, depending on σ^2, cause new excitation or eventually die out. In particular, if σ^2 is large enough, i.e, t_a short enough, neighbouring sites have a large probability to also become excited, which eventually nucleates a wave that propagates through the media. Analogous to the time domain, for this to happen the noise level also has to be sufficiently small so that the outline of the excursion phase remains smooth, which constitutes a nearly deterministic nucleus formation in the spatial domain and guarantees that locally initiated excitations can merge into spatially coherent structures. Since larger D constitute faster diffusive spread, it is understandable that the characteristic spatial scale of coherent structures induced by increasing D increases (see Fig. 2). However, since for larger D local excitations tend to die out more quickly, and larger coherent structures also require a higher rate of local excitations to propagate through the media, it is evident that shorter t_a (larger σ^2) are required to produce sustained waves. This also explains the increasing σ^2 that is required for the optimal response at ever larger D, as shown in Fig. 1b. Furthermore, since larger σ^2 blur local excursion phases (t_e) as well, the maximal spatial coherence that can be achieved by noise decreases with increasing D (see Fig. 1b).

Finally, it is of interest to explain the existence of a particular spatial periodicity. We argue that the characteristic noise robust excursion time t_e, combined with the diffusive spread rate proportional to $\sqrt{D}$, marks a characteristic spatial scale of the system that is indicated by the resonantly enhanced spatial wave number k_{max}. Since the characteristic spatial scale is determined by the inverse of the resonantly enhanced spatial wave number k_{max}, our reasoning thus predicts the dependence $k_{max} = 1/\sqrt{\tau D}$, whereby $\tau \propto t_e \approx$ constant. Fig. 3 shows numerically obtained k_{max} for different D. It is evident that obtained values are in excellent agreement with the inverse square root function, thereby validating our above explanation. Nevertheless, an open question remains how the constant τ is explicitly linked to t_e, which is left as a problem to be solved in future studies. The main point is that the inverse square root function fits to the numerically obtained values with a *constant* τ, which reflects a noise robust t_e that is characteristic

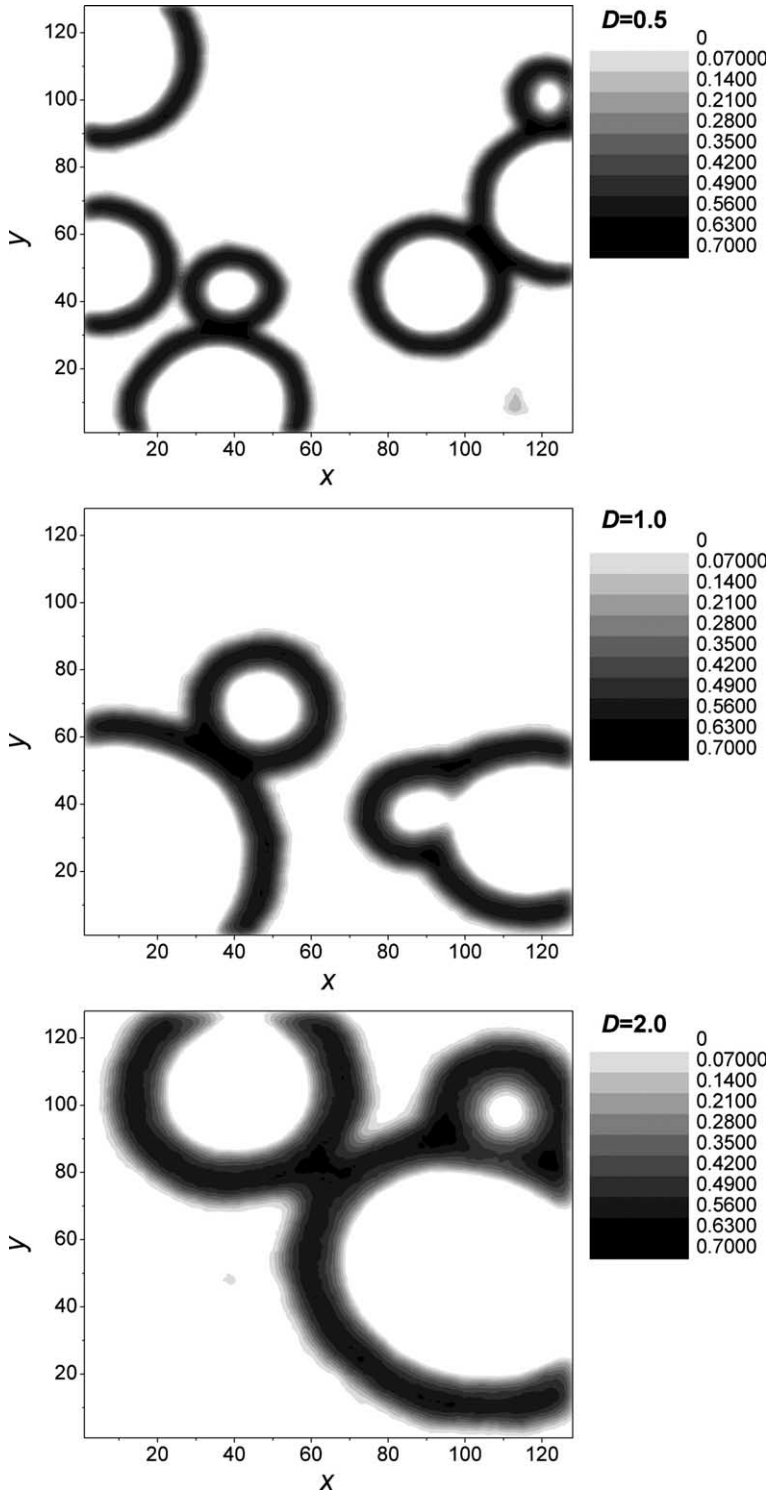

Fig. 2. Characteristic snapshots of the spatial profile of u for various D at optimal σ^2. Note that all figures are depicted on 128×128 square grid.

for excitable systems [34]. Together with a given D, this property of excitable systems constitutes an inherent spatial scale that can be resonantly enhanced by noise, thus explaining the existence of spatial coherence resonance in the studied excitable chemical media.

4. Discussion

We show that spatiotemporal noise is able to extract a characteristic spatial scale of the excitable chemical media in a resonant manner. In particular, there exist

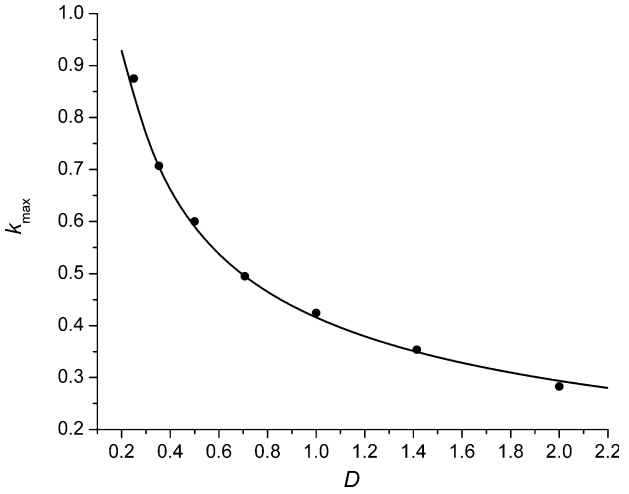

Fig. 3. Resonantly enhanced spatial wave number k_{max} (see Fig. 1a) in dependence on different values of D. Dots indicate numerically obtained values, whereas the solid line indicates the predicted $k_{max} = 1/\sqrt{\tau D}$ dependence for $\tau = 5.8$.

an optimal level of additive noise acting on the illumination intensity of an excitable photosensitive Belousov–Zhabotinskii medium for which the spatial periodicity of the system is best pronounced. Thereby, no additional deterministic inputs were introduced to the system and the latter was initiated from steady state excitable conditions. Thus, the presented results offer convincing evidence for the existence of spatial coherence resonance in the studied excitable chemical media.

Noteworthy, the phenomenon of spatial coherence resonance has been first reported by Carrillo et al. [16] for the chlorine dioxide–iodine–malonic acid chemical reaction system. In dependence on the illumination intensity, and for large differences between the diffusion coefficients of the activator and inhibitor, the latter system exhibits a pattern-forming supercritical Turing bifurcation, which gives rise to a preferred noise-induced spatial periodicity. In the present case, the situation is different since the resonant spatial frequency does not emerge as a noisy precursor of a pattern-forming bifurcation, but is a consequence of a noise robust excursion time that is characteristic for excitable systems whereby the diffusion constant, representing the rate of diffusive spread, determines the actual resonant spatial frequency, which decreases with increasing D.

Our results thus extend the theoretical knowledge as well as possibilities for future experimental work regarding the analysis of spatial dynamics from systems near pattern-forming instabilities, present for example in chemical reaction systems, electro-convection, Rayleigh-Bénard convection and granular media [16], to excitable media, which are also ubiquitous in all areas of science ranging from chemistry, neurophysiology, cardiology and physics of optical devices [35].

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