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Coevolutionary resolution of the public goods dilemma in interdependent structured populations

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Abstract – We study the coevolution of strategies and network interdependence in the context of a public goods dilemma. Specifically, players occupy the nodes of a network and engage in public goods games, with a twist that those who post a good result in terms of payoff are allowed to form external links with players from another network. These external links may bring additional utilities to players. Moreover, the links between players on different networks become stronger if players keep posting good results, but weaken otherwise. By means of Monte Carlo simulations, we show that, as long as the benchmark for recognition is neither too high nor too low, a "wave of heterogeneity" gives rise to cross-network links with a wide range of different strengths. This spontaneous emergence of heterogeneity seeds strong cooperative clusters that protect cooperators from the invasions of defectors. Ultimately, cooperation prevails, thus revealing a resolution of the public goods dilemma in structured populations.

Introduction. – Because cooperation – *i.e.*, helping others at one's own expense – is seemingly incompatible with the Darwinian evolution, the search for cooperation-promoting mechanisms has captured the interest of scientists across a broad range of disciplines, including physics. Network reciprocity in particular, inspired by the seminal work of Nowak and May [1], has been widely regarded as one of the five basic mechanisms enabling the evolution of cooperation [2,3]. Accordingly, networks as strong promoters of cooperation have received ample attention in the past decade (see [4–6] for reviews), although empirical evidence that networks really do promote cooperation is fairly recent [7,8].

Research on network reciprocity has largely focused on isolated networks [9–19], thus for a long time setting aside the fact that human interaction networks form distinct yet interdependent social layers. Generally, such interdependence has the potential to magnify minor changes in one of the interdependent networks into unexpected and rather catastrophic consequences in the other network [20]. Having recognized the importance of interdependence in a general setting, scientists have therefore turned their attention to the role of interdependent social networks in the context of cooperative behavior.

The concept of interdependent network reciprocity has been shown to maintain healthy public cooperativeness even in the face of adverse conditions [21]. Furthermore,

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it has been shown that there exists an optimal level of interdependence at which cooperation is promoted best [22], while sharing information about strategic choices between players residing in two interdependent networks has a reinforcing effect on the emergence of cooperation [23]. From a physicist's perspective, the spontaneous symmetry breaking in interdependent networked games is perhaps particularly interesting, whereby the frequency of cooperation in one network suddenly diverges from the corresponding frequency in the other network as the large clusters of purely cooperative pairs of players are gradually accompanied by cooperator-defector pairs [24].

Here, we explore the self-organization of the strength of interdependence and its impact on the evolution of cooperation. In this sense, we extend the work of Wang *et al.* [25] and more recently Luo *et al.* [26] by starting from two essentially independent networks in which players from either of the networks have the option to establish, and subsequently strengthen (or weaken and ultimately sever), a unilateral link to successful (unsuccessful) players in the other network. This is to reflect the fact that, say, a business person in one country is likely to pair up with a successful business person doing a similar business in another country in order to eventually benefit from such a pairing. Accordingly, a player whose strategy is well-adjusted to the surroundings will attain a higher payoff, and once this payoff exceeds a certain threshold, a link with the corresponding player in the other network will be strengthened, imparting an additional utility. In this way, a well-adjusted strategy (be it cooperation or defection) will be reinforced. An analogous process will discourage a maladjusted strategy. The key consideration is that the interdependence strength and the player's strategy both coevolve in response to the performance during the game.

Hereafter, we proceed to describe in detail our game setup for the coevolution of interdependence strength and player strategy, followed by the presentation of the main results. We round off the discussion with concluding remarks.

Model. – Evolutionary games were staged on two disjoint square lattices called networks A and B, each of size N and with periodic boundary conditions. Every player had four internal links to immediate neighbors (a von Neumann neighborhood) and the option to form one external link to the corresponding player in the other network. We defined parameter $\alpha_{ij} \in [0,1]$ (i indicates a player in own network; j indicates a player in the other network) as the interdependence strength, *i.e.*, the weight of external links. When $\alpha_{ij} = 0$ the two networks are independent and our model falls back to traditional network reciprocity of a single network. When $\alpha_{ij} = 1$ the two networks are maximally interdependent [21].

The social dilemma faced by players in our model was the public goods dilemma. Each player together with their von Neumann neighborhood was considered a group

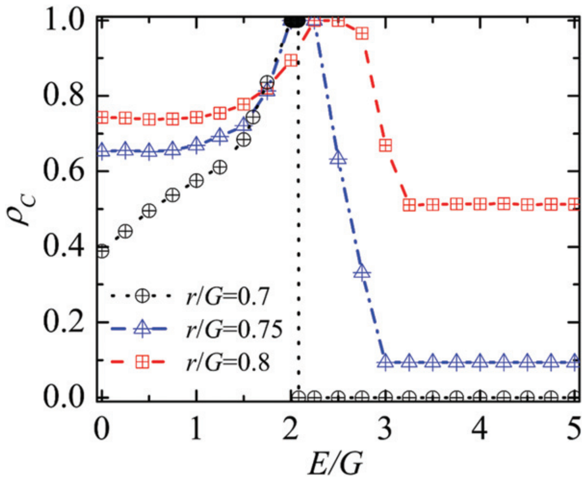

Fig. 1: (Color online) Synergy between two interconnected networks resolves the spatial public goods dilemma. Shown is the fraction of cooperators in both networks as a function of the normalized recognition threshold, E/G, for different values of the normalized enhancement factor, r/G. High E/G values prevent interconnectedness and the system practically functions as two independent networks. Only when E/G becomes sufficiently small, synergistic effects can manifest themselves. Interestingly, the synergy is strongest for intermediate values of E/G.

playing a variant of the Public Goods Game. This meant that every single player belongs to $g = k + 1$ overlapping groups of size $G = g$, where $k = 4$ is the internal node degree in lattices with the von Neumann neighborhood structure. Each player started as either a cooperator or a defector with equal probability. Payoffs P_i and P'_j in networks A and B, respectively, were calculated following the same procedure. Cooperators contributed 1 unit to the common pool of each group, while defectors contributed nothing. The total contribution was subsequently multiplied by an enhancement factor r and then equally shared by G group members irrespective of their individual strategies. Payoffs P_i and P'_j represent the grand totals obtained from all g groups to which a player belongs.

We assumed that the two networks are not in direct physical contact. Instead, the benefit from interconnectedness arises as an intangible consequence of establishing a (*e.g.*, business) relationship. This was reflected in utility functions [25–27] defined via

$$\begin{cases} F_i = P_i + \alpha_{ij} P'_j, \\ F'_j = P'_j + \alpha_{ji} P_i, \end{cases} \tag{1}$$

where α_{ij} is the directed interdependence strength between the two networks. Initially, we assigned $\alpha_{ij} = \alpha_{ji} = 0$. We further assumed that players can form external links only when their current payoff exceeds recognition threshold E. Thus, when $P_i > E$, the interdependence strength α_{ij} was strengthened by $\delta > 0$ as a sort of reward. Otherwise, the interdependence strength α_{ij} was weakened by the same value δ as a sort of punishment.

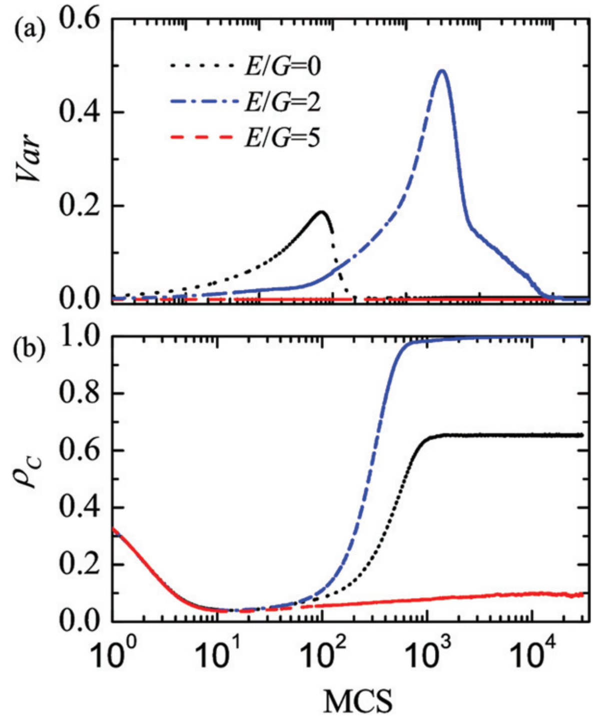

Fig. 2: (Color online) Cooperativeness emerges in the wake of heterogeneity "waves". Shown is (a) the variance of the interdependence strength as a function of time for the different values of the normalized recognition threshold, E/G, and (b) the corresponding fraction of cooperators in both networks taken together. Higher heterogeneity "waves" establish and maintain a larger fraction of cooperators. Here, the normalized enhancement factor is $r/G = 0.75$. MCS stands for Monte Carlo Steps.

The resulting equation is

$$\begin{cases} \alpha_{ij} = \min\{\alpha_{ij} + \delta, 1\}, & \text{if } P_i \geq E, \\ \alpha_{ij} = \max\{\alpha_{ij} - \delta, 0\}, & \text{if } P_i < E, \end{cases} \tag{2}$$

where we fixed $\delta = 0.05$ throughout the study. Once utilities were calculated, player i from network A with utility F_i adopted the strategy of neighbor k (from the same network A) with utility F_k in a probabilistic manner. Precisely, the probability of strategy adoption was

$$W = \frac{1}{1 + \exp\left(\frac{F_i - F_k}{K}\right)}. \tag{3}$$

Strategy adoption in network B was determined analogously. We fixed the strength of the selection parameter to $K = 0.5$ [28]. We carried out Monte Carlo simulations in a square lattice of size 200×200 and 1000×1000 to avoid finite-size effects. The key quantity, *i.e.*, the fraction of cooperators ρ_c, was determined in the last 10^3 time steps, while the total number of time steps was $5 \cdot 10^4$, such that the stationary state has been reached before we have begun recording results. In particular, we have verified that the average of the strategy frequency over a sliding time window became time independent. We have also calculated the ensemble average over up to 50 independent runs for each set of parameter values, although oftentimes 10 realizations have proven sufficiently many for the statistical fluctuations to become negligible given that we have used relatively large lattices to begin with.

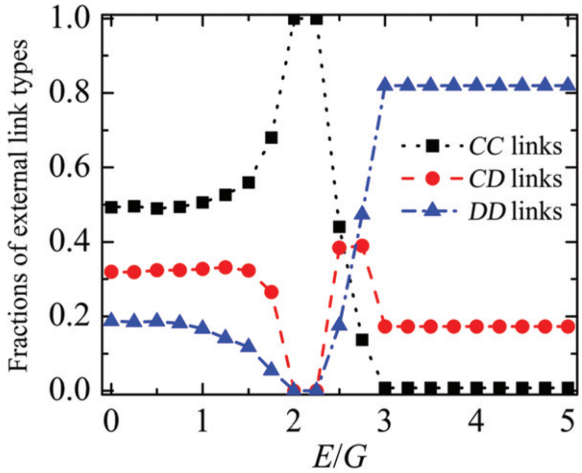

Fig. 3: (Color online) Cross-network links conducive to cooperation form when the recognition threshold is optimal. Shown is the fraction of external link types (CC, CD, or DD) as a function of the normalized recognition threshold, E/G. For large E/G values, predominantly defectors form external links. For small E/G values, cooperators can also form external links, but not exclusively. Finally, for intermediate E/G values, cooperators exclusively form external links, which helps establish cooperative clusters to protect the emerging cooperativeness. Here, the normalized enhancement factor is $r/G = 0.75$.

Results. – Examining how cooperation fares under our coevolutionary setup (fig. 1), we find that synergy between two interconnected networks can resolve the spatial public goods dilemma. Expectedly, for large values of the normalized recognition threshold, E/G, there are no synergistic effects because the two networks function independently [28]. Only when the recognition threshold becomes sufficiently small, synergy manifests itself. Perhaps surprising is that the synergy is strongest for the intermediate E/G values [22,29] rather than the small ones. The strictness of the dilemma, as quantified by the inverse of the normalized enhancement factor, r/G (*i.e.*, the lower the value of r/G, the stricter the dilemma), leads to qualitatively similar results with one notable exception. Namely, stricter dilemmas (*i.e.*, low r/G values) cause an abrupt transition from a non-cooperative to a cooperative state, whereas weaker dilemmas (*i.e.*, high r/G values) cause a non-linear, but continuous transition. In what follows, we seek to establish a better understanding of these results.

The variance, defined as the square of the standard deviation, and as such the second central moment of a distribution, characterizes the degree of heterogeneity among individuals of a given group. Thus, the larger the variance, the more heterogeneous the group. Examining how group heterogeneity in terms of the interdependence strength variance evolves over time for different values of

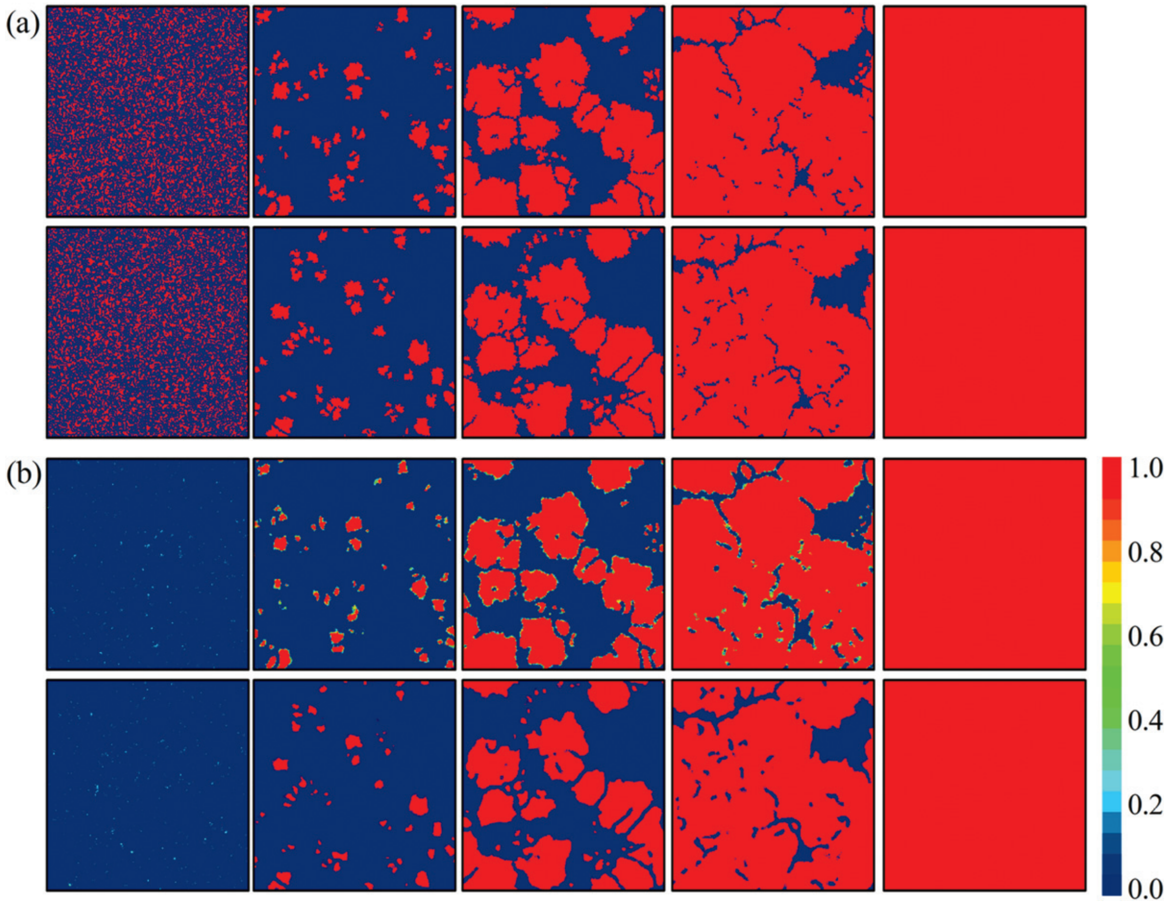

Fig. 4: (Color online) Cross-network links conducive to cooperation act as seeds of cooperative clusters. Shown are (a) the evolutionary snapshots of the distribution of cooperators (red) and defectors (blue) in network A (upper panels) and network B (lower panels), as well as (b) interdependence strength α_{ij} (color coded) of all external links originating from network A (upper panels) and network B (lower panels). From left to right, panels display Monte Carlo Steps 0, 100, 300, 500, and 9999, respectively. The lattice size is $L = 200$, while the parameter values are $r/G = 0.75$ and $E/G = 2$.

the normalized recognition threshold (fig. 2(a)), we find that high E/G values make all agents virtually the same. This is because the requirement for recognition is so high that interdependence never really takes off, and networks keep functioning independently. For the chosen strength of the dilemma ($r/G = 0.75$), this generates very low cooperativeness (fig. 2(b)). For low values of E/G, player heterogeneity increases from low values, peaks, and then declines (fig. 2(a)), but this "wave" of heterogeneity establishes a certain degree of interconnectedness, and a much larger fraction of cooperators is maintained than before (fig. 2(b)). Finally, for intermediate values of E/G, player heterogeneity evolves in qualitatively the same way as for low E/G values, but its peak is much higher. The fraction of cooperators maintained in the wake of such a high "wave" of heterogeneity is even larger than before (fig. 2(b)). These results are thus reminiscent of previous studies on heterogeneity in the context of human cooperation [30,31].

The type of external (*i.e.*, cross-network) links has been shown to play a crucial role in spreading cooperativeness in interdependent networks [32]. In this context, we recognize three link types: CC, CD, and DD.

Examining these link types in our coevolutionary setup, we find that for large values of the normalized recognition threshold, E/G, some external links are formed, but they are mostly of the DD type that is conducive to defection (fig. 3). For small E/G values, the fraction of CC external links improves considerably, but still the system retains both CD and DD external links, thus limiting the extent of cooperativeness in the networks. Finally, for intermediate E/G values, the latter external link types almost disappear from the system, leaving only CC external links that are conducive to cooperation. The exact mechanism of how CC external links promote cooperation is that they help build compact clusters wherein cooperators are protected from defectors. Such a mechanism has, in fact, recently been observed in social-dilemma experiments with human participants [7].

To illustrate the mentioned clustering mechanism at a microscopic level, we examine snapshots of the time evolution of the two interdependent networks. Initially, an equal number of cooperators and defectors is randomly distributed throughout the system (fig. 4(a)). As the system evolves, cooperators get exploited by defectors and cooperativeness diminishes. This process continues until only small and scattered cooperative clusters remain. At this point, mutual defection prevails through the system which, on the one hand, greatly reduces the payoff of defectors and, on the other hand, makes cooperative clusters advantageous if these clusters provide sufficient protection for cooperators at the edges (*i.e.*, those who are exposed to defection). Interestingly, cooperative clusters form almost exclusively around cooperators with external links (fig. 4(b)). It is in this sense that external links act as seeds of cooperation. The formation and the subsequent growth of cooperative clusters in the two interdependent networks are closely synchronized. One final observation of interest is that cooperators enjoy the highest interdependence strength, which is unattainable for defectors.

Conclusion. – We have studied the coevolution of cooperation and network interdependence in a context of the public goods dilemma. Here, this dilemma can be seen as an endeavor, involving several potential partners, the success (or profitability) of which depends on how many partners decide to cooperate. Players who choose a more successful strategy in terms of payoff get recognized and subsequently form external links with players from another network. Such external links bring additional utility to linked players based on the strength of interdependence. The results show that cooperation best evolves for intermediate recognition thresholds due to i) heterogeneity "waves" that modulate the interdependence strength and ii) cooperative clusters that flourish around agents with external links.

We conclude that the interconnectedness of structured populations may help resolve the public goods dilemma, along with many other mechanisms that have recently been studied [33]. Some have also been studied experimentally in recent years, revealing that indeed some circumstances promote cooperation rather expectedly [34] and others somewhat surprisingly [35]. We find herein that cooperation evolves because external (*i.e.*, cross-network) links seed cooperative clusters which provide protection against defectors. That clusters provide protection against defectors is consistent with empirical observations [7]. For a fuller verification of the theory, further experiments are of course needed. We believe that out of this need, experimental evolutionary game theory will emerge as a major driver of advances in the context of human cooperation.

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