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Graphical interface as a teaching aid for nonlinear dynamical systems

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Abstract

Chaos theory is an important breakthrough in understanding nonlinear dynamical systems. It is generally accepted that chaotic systems involve transitivity, a high density of periodic points in metric space, and extreme sensitivity to changes in initial conditions. Many systems taught on undergraduate courses satisfy these conditions, such as electronic circuits or computer simulations of nonlinear dynamical systems. However, a good understanding of these concepts is broadly difficult to achieve on such courses, and especially so at the undergraduate level. In view of this, we have developed an interactive standalone graphical user interface, which is platform-independent. The interface didactically simulates six chaotic systems. It allows the user to study a particular system without prior knowledge of computer programming, demonstrating rich dynamical behaviour such as fixed points, periodic cycles and deterministic chaos. At the same time, the interface provides details of the simulation and the parameters used, as well as some of the leading references. We provide the complete code and a detailed installation guide, which should make it easy to use the interface reliably and with little overhead work. We also provide a comparison with other available software for reference, and as a guide to readers who would like to explore this fascinating subject in more depth or with alternative approaches. Lastly, we present the results of a survey of 30 students who used our graphical user interface, revealing that it is an easy-to-use tool in the process of learning about chaotic dynamical systems.

Keywords: dynamical system, chaos, educational software, nonlinear dynamics

(Some figures may appear in colour only in the online journal)

1. Introduction

Chaotic systems have received much attention over the past few decades [1–5]. However, the investigation of such systems started much earlier with the pioneering works of Poincaré, who studied concepts of nonlinearity and chaos in the investigation of the movement of celestial bodies at the beginning of the 1900s [6]. With the advent of computer technology, it was Edward Lorenz who first observed chaotic behaviour in climate phenomena at MIT [7]. Although there is no generally accepted definition of chaos [8], one of the most widely accepted views is based on the theory that a system must exhibit chaotic behaviour if it operates nonlinearly, presents transitivity, has a high density of the periodic points of the function f in metric space and is sensitive to initial conditions [9].

Currently, many works deal with nonlinear dynamics, studying and applying methods to better understand computational simulations [10–13]. Considering that most of the systems found in nature are nonlinear and that we can still see several chaotic features in parts of them, their study is critical, and their teaching should be started as soon as possible on undergraduate courses [14–17]. However, the study of such systems takes great effort, due to the mathematical concepts and proofs involved in this [18]. Thus, alternatives that can reduce learning time are desirable, for example interactive tools and visual effects using electronics and computing [19–21].

The graphical user interface (GUI) is presented as an educational tool to provide the user with interactive visual communication with the tasks required. The GUI can give the student a way to solve problems more efficiently, as shown by related works [22–26]. The GUI environment keeps most of the tedious and repetitive calculations in the background, allowing the user to spend more time analysing the results.

Among the many alternatives available for developing a GUI, Matlab is certainly one of the most reliable computing environments. There are a myriad of works dealing with the elaboration of a GUI in Matlab for educational purposes. For instance, a GUI for the simulation and analysis of power electronic circuits has been developed in [27]. In a similar fashion, a GUI for power electronics converters using Matlab/Simulink and SimPowerSystems is elaborated in [28]. For power systems flow analysis there is a GUI in [29] and an embedded system for online acquisition of biomedical signals can be found in [30]. The reader interested in a broader application of GUIs can refer to [31], where the authors have developed an interactive learning GUI for electrical engineering subjects. However, few studies have focused on developing a GUI for nonlinear dynamical systems, particularly for chaotic systems in a Matlab environment and free of cost. In fact, a detailed literature review has shown that the majority of them are not free of cost. The examined software is also outdated and does not have a great deal of graphic tool options (for further information see section 7). Furthermore, a literature search using Web of Science and IEEE Xplore on 15 May 2018, using the search terms 'Matlab GUI chaos', resulted in only three works. The first work is devoted specifically to one-dimensional chaotic maps [32], while the second is focused on

Table 1. Number of citations for each type of dynamic system related to its principal reference. Data retrieved from Google Scholar on 15 May 2018.

Chaotic system	References	Number of citations
Lorenz system	[7]	19492
Logistic map	[5]	6777
Hénon map	[35]	3086
Chua’s circuit	[36]	619
Rössler attractor	[37]	3489
The double chaotic pendulum	[38]	123

modulation techniques. The third is yet more specific, as the authors elaborate a tool for functional neuroimaging, wherein some ‘chaotic properties’, such as the Lyapunov exponent and entropy, are analysed [33].

In an effort to fill this gap, this paper describes an elaboration of a GUI as a teaching aid for dynamic systems, focusing on chaotic systems. The GUI simulates didactically six chaotic systems, allowing the user to study the methods without prior knowledge of computer programming, and demonstrating aspects of their rich dynamical nature, like fixed points, periodic cycles and chaotic behaviour. Details of the simulation and parameters used are given, as well as some leading references. This work also differs from other works in the procedure used to develop the GUI; it has been elaborated using Matlab Runtime, which enables the execution of compiled code, and therefore, the sharing of the applications royalty-free with users.

The remainder of this paper is organised as follows: section 2 shows the objectives of this project and summarises its scope. Section 3 outlines important concepts for understanding the rest of the text. Sections 4 and 5 describe the development of the interface. A student survey, comparison with similar software and the results and evaluation are shown in sections 6, 7 and 8, respectively. Finally, section 9 presents some final remarks.

2. Objectives and scope

In general, undergraduate courses on electrical engineering cover only linear circuit theory. A exception to this rule can be found at UC Berkeley, where Chua and colleagues have devoted decades of work to demonstrating the importance of teaching nonlinear circuit theory. One of the results is the development of the book *Introduction to Nonlinear Circuits and Networks* [34], which is due to be released in 2019. Nonlinear chaotic systems represent an essential topic in graduate courses, the objectives of which consist of the mathematical analysis, identification and control of these particular systems. However, in undergraduate classes the focus is only on linear models, creating a tremendous gap in the knowledge of students who wish to attain a Masters or Ph.D. degree.

The tool developed in this work includes some dynamic and widely cited systems (see table 1), which represent typical systems studied in engineering and related courses. Moreover, the GUI has been developed using one of the most reliable numerical computing platforms currently available, Matlab[5]. We have used the Matlab Runtime option, which enables the execution of compiled Matlab applications on any computer. This feature is essential for students who want to carry out experiments on their personal computers.

[5] For more information visit https://mathworks.com.

Finally, our main goals are summarised as follows.

- Show that systems and circuits, seen broadly as linear in undergraduate courses, can present chaotic behaviours (e.g. the double pendulum and Chua's circuit).
- Teach concepts such as periodicity, bifurcation and chaos in state space and the time domain with the help of animations.
- Present a tool to save the data of simulations to assist in the elaboration of educational examples.
- Stimulate teachers and students to join the research on nonlinear chaotic systems.

3. Dynamical systems background

3.1. Lorenz system

In 1963, Lorenz investigated the dynamic system proposed by Saltzman [7, 39], a paradigm suggested for the study of the convection amplitude that realises simplifications that apply to a reduced atmospheric model, commonly used for meteorological prediction. The simplified model described by a set of three differential equations, shown in equation (1), exhibits atypical and sensitive behaviour to the initial conditions. The set of equations is as follows:

$$\begin{cases} \dot{x} = -\sigma x + \sigma y, \\ \dot{y} = rx - y - xz, \\ \dot{z} = xy - bz, \end{cases} \tag{1}$$

where σ and r are called Prandtl and Rayleigh numbers, respectively. The state x represents the intensity of the convection movement, y is proportional to the temperature variation between upstream and downstream currents, and z is equivalent to the distortion of the vertical temperature profile, starting from linearity, where a positive value indicates stronger gradients near the limits.

3.2. Logistic map

Nonlinear maps present a rich and complex dynamic, the logistic map being one of the most studied [5] as represented by equation (2):

$$f(x_n) = x_{n+1} = rx_n(1 - x_n), \tag{2}$$

where r is a control parameter that belongs to the interval $1 \leqslant r \leqslant 4$ and x_n belongs to the interval $0 \leqslant x_n \leqslant 1$. An important concept for understanding this map is the fixed point, such as the point of $f(x_n)$, where

$$x_n^* = f(x_n^*), \tag{3}$$

denotes that if the orbit of the map reaches the fixed point in a given iteration, the orbit therein will remain in the following iterations. By equation (3) the fixed points of the logistic map are given by $x_1^* = 0$ and by $x_2^* = 1 - (1/r)$. The fixed point x^* is said to be stable if after a small perturbation, the map iterations remain close to x^*.

3.3. Hénon map

The Hénon map [35] was proposed to describe the Poincaré section of the Lorenz system. Hénon observed that a discrete system of only two recursive equations could have rich dynamics, presenting chaotic behaviour.

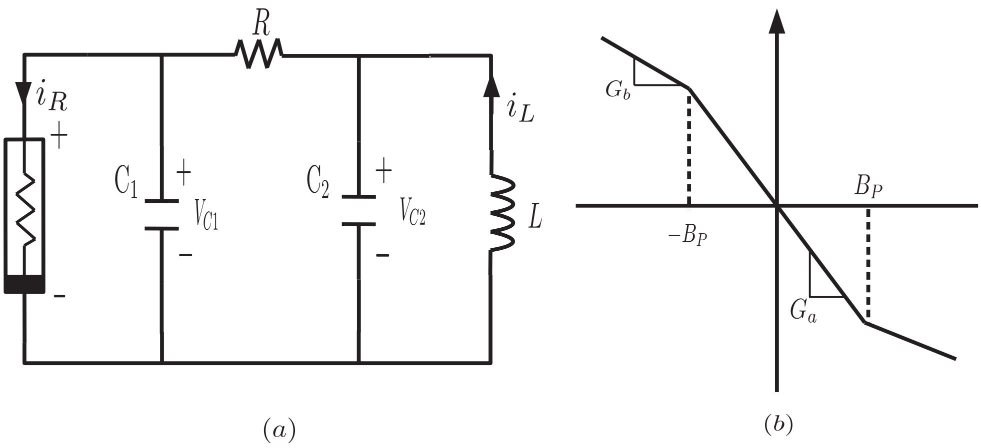

Figure 1. (a) Chua's circuit. C_1 and C_2 are capacitors, v_{C_1} and v_{C_2} are the corresponding voltages, L is an inductor and R a resistor. The i_R and v_R, respectively, represent current and voltage in the nonlinear resistor, i.e. Chua's diode. (b) Chua's diode curve. m_0, m_1 e B_p are parameters of the nonlinear element.

Equation (4) represents the Hénon map, from which is possible to observe the reason why it is highly applicable in the studies of dynamic systems: the low dimension, and therefore the computational time for the simulation, is smaller than in other *N*-dimensional systems:

$$\begin{cases} x_{n+1} = y_n + 1 - ax_n^2, \\ y_{n+1} = bx_n \end{cases} \tag{4}$$

In equation (4), x and y are dynamic variables, a is the nonlinearity factor and b the dissipation factor. The constructive parameters of the equation are commonly used as 1.4 and 0.3, respectively. From the Hénon map, it is possible to obtain chaotic saddles and periodic points as seen in [40].

3.4. Chua's circuit

The circuit developed by Leon O Chua [36] exhibits nonlinear behaviour, and it was elaborated with the purpose of simulating the chaotic behaviour proposed by Lorenz. This circuit had already been investigated by Matsumoto in an earlier work [41]. Chua and Matsumoto have proved the existence of chaos [42] for this circuit. Since then, the circuit has been extensively studied, implemented and simulated computationally.

The circuit is composed of linear passive elements: two capacitors, an inductor and a resistor, which are connected to an active, nonlinear element called Chua's diode, as shown in figure 1. In Kennedy [43], the diode was replaced by a network of operational amplifiers. Other simplifications can be seen in [44], where the authors have replaced the inductor by an operational amplifier realisation.

Using Kirchhoff's law, it becomes possible to obtain the differential equations governing the dynamics of the circuit, as shown in equation (5):

$$C_1\frac{dv_{c_1}}{dt} = \frac{v_{c_2} - v_{c_1}}{R} - i_R(v_{c_1}),$$
$$C_2\frac{dv_{c_2}}{dt} = \frac{v_{c_1} - v_{c_2}}{R} - i_R(v_{c_1}),$$
$$L\frac{di_L}{dt} = -v_{c_2}. \tag{5}$$

The resistive effect of the inductor is considered as negligible. The current through the nonlinear element $i_R(v_{C_1})$ is given by equation (6):

$$i_R(v_{C_1}) = \begin{cases} G_b v_{C_1} + B_p(G_b - G_a) & v_{C_1} < -B_p, \\ G_a v_{C_1} & |v_{C_1}| \leqslant B_p, \\ G_b v_{C_1} + B_p(G_a - G_b) & v_{C_1}) > B_p. \end{cases} \tag{6}$$

3.5. Rössler attractor

In 1976, Rössler obtained a system with three differential equations with the minimum parameters, sufficient to generate a continuous chaotic system [37]. The system proposed by Rössler is shown by equation (7) and, according to the author, many natural and artificial systems are governed by this type of equation:

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay, \\ \dot{z} = bx - cz + xz. \end{cases} \tag{7}$$

In equation (7), the parameters x, y and z are the system variables and a, b and c the parameters of it. Although Rössler has studied the equation by the parameter values $a = 0.2$, $b = 0.2$ and $c = 5.7$, the system is very rich and it is possible to obtain the most diverse behaviours varying these values, from fixed points and periodic orbits up to chaotic attractors.

3.6. The double chaotic pendulum

The literature reports several types of mechanical systems that exhibit chaotic behaviour. An example that synthesises the main characteristics of chaotic systems is the double pendulum based models [38, 45]. The most used formulation to demonstrate this system is the Newtonian, based on the balance of forces. Figure 2 shows the configuration composed by a double pendulum subjected to the action of gravity, modelled by the following systems of equations:

$$\ddot{\theta}_1 = \frac{g(\kappa_2\beta_1 - M\kappa_1) - (l_2\dot{\theta}_2{}^2 + l_1\dot{\theta}_1{}^2\beta_1)\beta_2}{l_1(M - \beta_1{}^2)}, \tag{8}$$

$$\ddot{\theta}_2 = \frac{gM(\kappa_1\beta_1 - \kappa_2) - (Ml_1\dot{\theta}_1{}^2 + l_2\dot{\theta}_2{}^2\beta_1)\beta_2}{l_2(M - \beta_1{}^2)}, \tag{9}$$

where $\alpha = \theta_1 - \theta_2$, $\beta_1 = \cos\alpha$, $\beta_2 = \sin\alpha$, $\kappa_1 = \sin(\theta_1)$, $\kappa_2 = \sin(\theta_2)$, $M = 1 + (m_1/m_2)$, m_1 and m_2 are the masses of the rods, l_1 and l_2 are the length of the rods, and θ_1 and θ_2 are the angles between the normal line and the inner and outer rods, respectively. The double pendulum represents the disorder and irregularity of deterministic systems, since although governed by specific and simple laws, it is highly sensitive to the initial conditions.

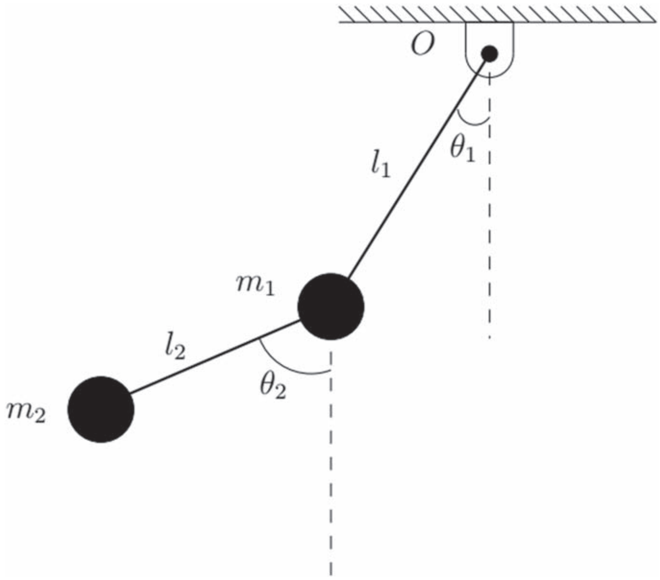

Figure 2. The double chaotic pendulum. The parameters are described in section 3.6. A full description of this system can be found in [38].

4. The GUI

The interface presented in this work was created using a built-in Matlab tool called GUIDE, which permits developers to design GUIs for many applications [46]. The GUIDE is a Matlab application that allows the development of parallel scripts with tools that create graphic designs, process control and simulations in real time. It enables scientists to simplify high-level codes for manipulation by novice users.

The demand for easy-to-use tools is an essential factor in the development of devices based on the exploration of interactivity as a pedagogical basis for teaching and learning activities. Interaction is a process of communication between people and interactive systems. In this procedure, the user and system exchange shifts where one speaks, and another listens, interprets and performs an action [47]. Figure 3 illustrates the interaction process of the interface and user.

5. Development and tool description

In a teaching session using the tool presented for dynamic systems, the user needs to specify the studied system and the desired parameters to start the simulation. Then, after the processing is finished, the results are displayed graphically or stored in text files. In this way, the algorithms and confusing technical details are hidden from the user, showing only what is needed: the graphical and data results.

When the student's need is established, the interface design process is initiated. During this process, the most diverse scenarios are evaluated to build the interaction with the user [49]. The advantage of making recurring evaluations during development is that interaction problems are identified and repaired before your design is completed. The sooner a problem is discovered and fixed, the lower the cost of the code changes required, leading to resulting

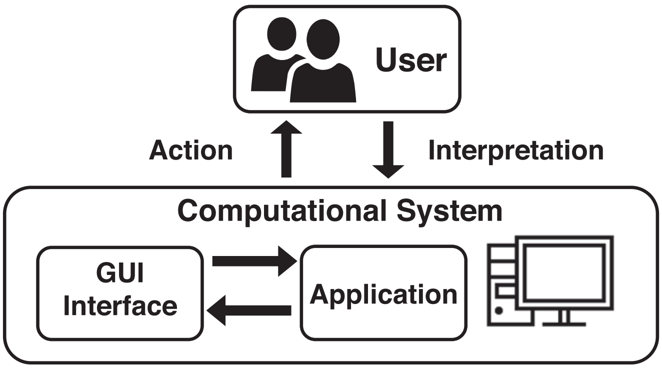

Figure 3. The process of human–computer interaction. A detailed description of the importance of human–computer interaction in many aspects of society has been presented in [47].

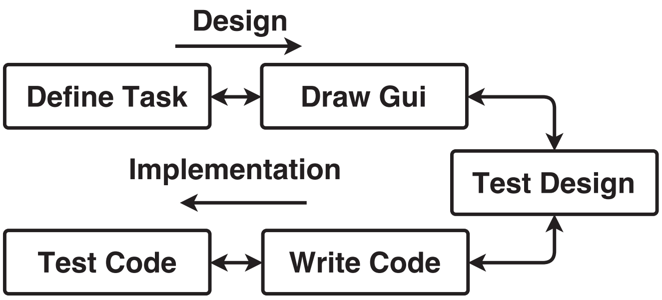

Figure 4. The phases of constructing the GUI. The reader can refer to [48] for a detailed description of these steps.

software of better quality [50]. Once the interface design is established, the code is written, and subroutines are organised that represent the mathematical processes to perform the simulations. Figure 4 shows the steps of development of the GUI [48].

After the design and implementation cycles, the GUI in its final version has the following characteristics.

- All parameters display default values, set appropriately, which allow the user to accept the settings suggested by the computer and prepare the way for automatic program execution and for the system.
- The interface comprises information for each system, including a brief introduction and main references about the models. It is obtained by activating the 'Help' button.
- The possibility of saving the data from the simulations in a text file format for the elaboration of educational examples.
- The tool allows the user to manipulate the graph window, such as moving, changing the axis limits, rotating and identifying point-to-point values.

Different dynamic behaviours of the systems can be observed by animations, which becomes didactic in the study of such chaotic systems. Figure 5 shows the simulation of the Lorenz system after the end of the animation by the standard parameters, and the 'Help'

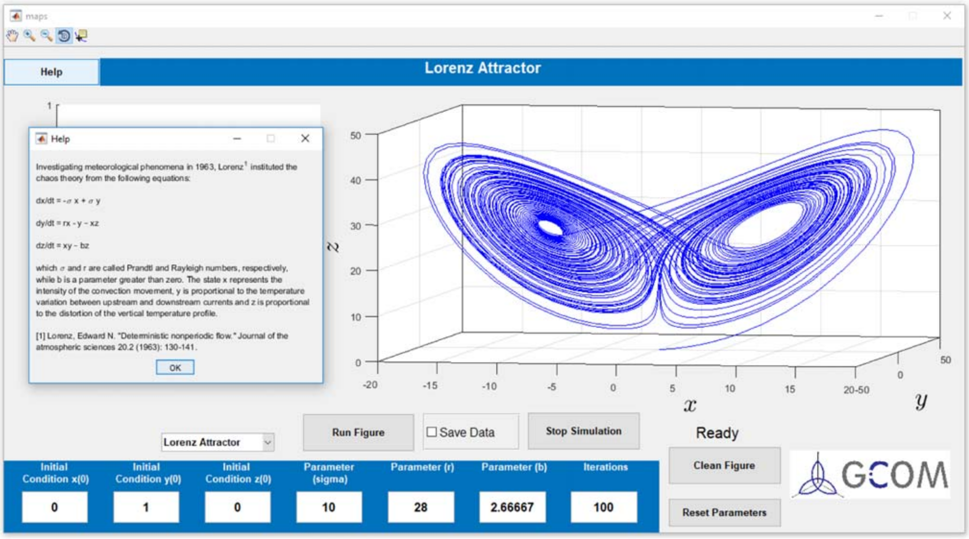

Figure 5. GUI available, indicating a chaotic behaviour for the Lorenz system and the help menu. An important feature of the GUI has been addressed for each classic chaotic system, that is, the user has easy access to the parameters used in classic papers, as well the indication of this source. For each chaotic system, a similar 'Help' function has been built, showing a brief description of the system, important parameters, and the reference. By default, the parameters initially set are those used by the authors of the cited reference.

information about the system sampled. Likewise, figure 6 demonstrates the animation of the double pendulum and exemplifies the saving of the data generated in the simulation, by checking the option 'Save Data' on the GUI.

Although the platform is developed from Matlab, a non-free software, it is not necessary to have the license to run this GUI. Mathworks provides Matlab Runtime, which is a standalone set of shared libraries that enables the execution of compiled Matlab applications or components on any computer. The interface package, compiler, executable and installation instructions are available online.[6]

6. Classroom experience

The device was utilised on the electrical engineering course at the Federal University of São João del-Rei, Brazil. The authors of this study were the instructors, and 30 students, with or without prior knowledge of dynamic systems, were chosen to evaluate and participate in the use of the platform.

Altogether, four experiments, with different approaches, were performed. Each experiment contained a step-by-step guide and was followed by a brief explanation and discussion of the results. These activities were centred on showing the most diverse qualitative behaviours of such systems. A succinct description of the exercises and their objectives, as well some results, follows.

[6] http://ufsj.edu.br/gcom/program.php

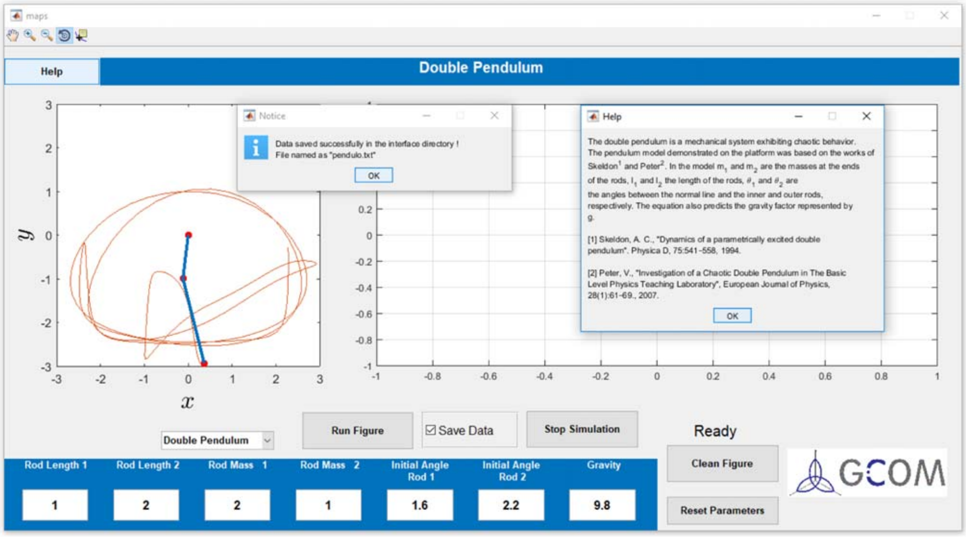

Figure 6. Dynamic presented by the double pendulum and the notice that the data was saved successfully in text format.

6.1. Activity 1: logistic map

An example of a system that can generate complex patterns despite having a simple representation is the logistic map, considered as an exemplary dynamic system that can model population growth.

It has a didactic nature because it represents a one-dimensional and deterministic system. Thus, this one-dimensional map is perfect to introduce the topics of nonlinearity, feedback (as the map is recursive), periods and chaos.

For students to become familiar with the behaviour of the map, three instructions with the respective parameters were given: initial condition $x_0 = 0.294\,11$, *iterations* $= 100$ and the following values of *r:*

- $\underline{r = 1.5}$: in this regime, regardless of the initial population value, it stabilises at a determined value, representing a fixed point;
- $\underline{r = 3.4}$: the behaviour corresponds to a cycle in which the orbit takes two different values at alternating instants. A sudden change in the stability of a fixed point or orbit is called bifurcation;
- $\underline{r = 4}$: orbit no longer tends to a known period and evolves irregularly, characterising a chaotic system.

By varying only the parameter r of the equation, the map presents three distinct behaviours: period 1, from period 1 to period 2, indicating a bifurcation as shown in figure 7, and chaos as shown in figure 8.

6.2. Activity 2: Rössler system

The student must identify period and chaos from phase space graphs. From the continuous system of Rössler [37], students identified periods 1, 2, 3 and 4 as well as a chaotic attractor. The parameters to determine such behaviours are described as follows: parameters

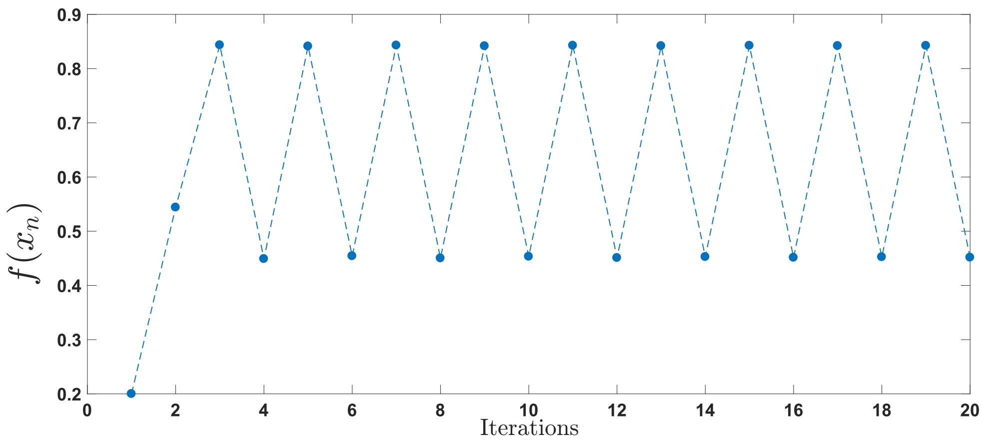

Figure 7. Time series of the logistic map for $r = 3.4$. For this simulation, the logistic map presents a fixed point of period 2.

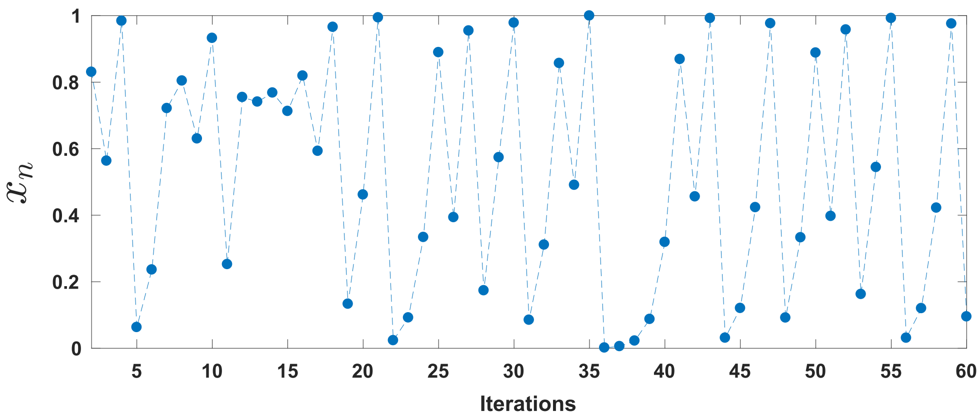

Figure 8. Time series of logistic map showing chaos. In this case, the parameter $r = 4$.

$a = b = 0.1$, initial conditions $x_0 = 2$, $y_0 = -6$, $z_0 = 0$, *iterations* = 200 and the following values of c:

- Period 1: $\underline{c = 4}$;
- Period 2: $\underline{c = 6}$;
- Period 3: $\underline{c = 12}$;
- Period 4: $\underline{c = 8.5}$;
- Chaotic: $\underline{c = 18}$.

In this experiment, the students are encouraged to investigate the properties of chaotic systems, where a chaotic system is characterised by a strange attractor, whose orbits never repeat the same path, although they are confined (attracted) to a limited region of the phase space. Figures 9 and 10 show the identification of period 2 and chaos, respectively.

The next example is to demonstrate the concept of sensitivity to initial conditions. It was proposed to the students to simulate the chaotic attractor twice, but once the initial condition

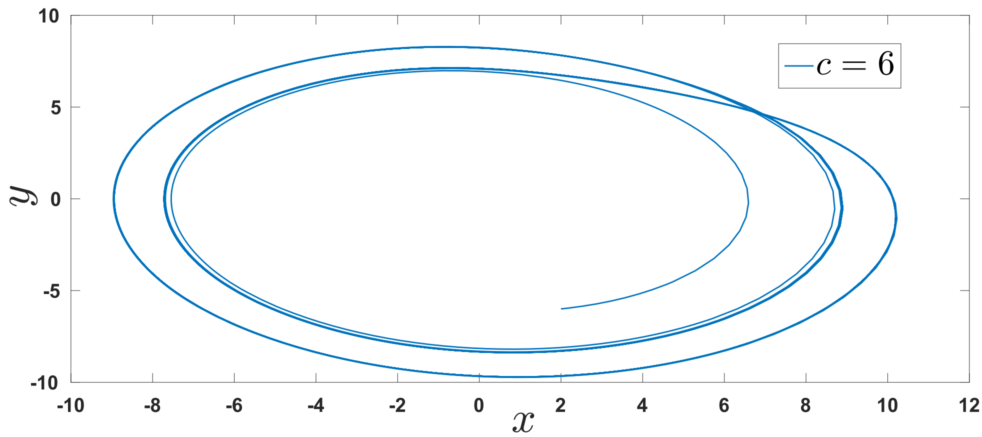

Figure 9. Simulation of the Rössler system resulting in period 2 in the plane $x - y$. The parameters are $a = b = 0.1$, $c = 6$ and initial conditions given by $x_0 = 2$, $y_0 = -6$ and $z_0 = 0$.

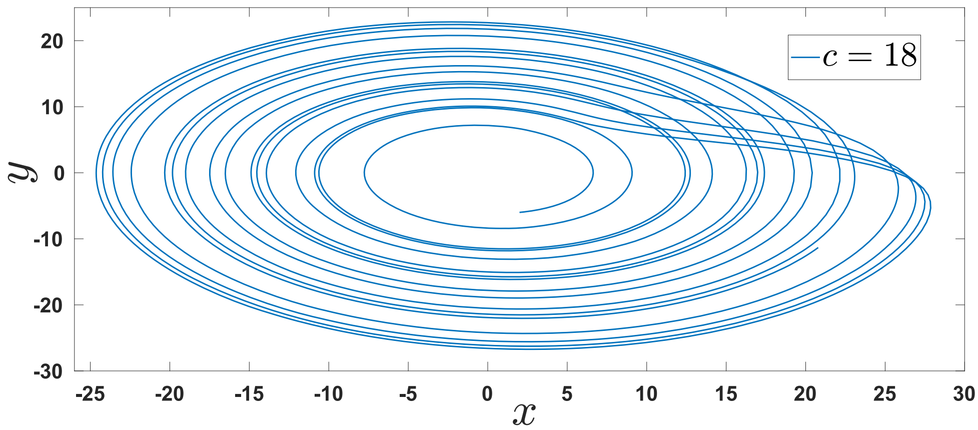

Figure 10. Simulation of the Rössler system resulting in chaos. The parameters are $a = b = 0.1$, $c = 18$ and initial conditions given by $x_0 = 2$, $y_0 = -6$ and $z_0 = 0$.

would be $x_0 = 2$ and the other time it would be $x_0 = 2.0001$. With the help of the 'Save Data' tool, the undergraduates could plot the orbits and observe the differences between the data along the iterations (see figure 11). The students learned that in the chaotic regime, any small variation is widened over the iterations until they assume enormous proportions.

6.3. Activity 3: Chua's circuit

The study of chaotic behaviour can be performed reasonably simply in circuits constructed from basic discrete components, such as resistors, capacitors, inductors and diodes, designed to reproduce differential equation systems. An example is Chua's circuit, wherein can be observed several strange attractors, bifurcations and routes to chaos by simply adjusting the parameters of its electronic components.

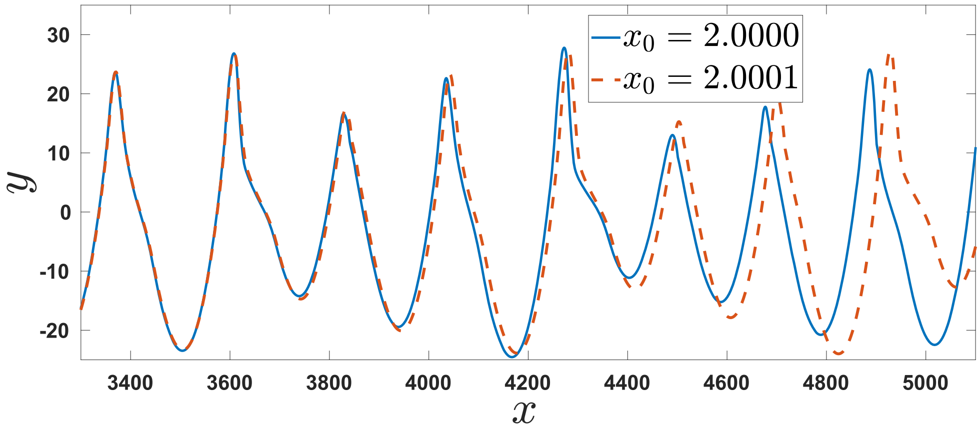

Figure 11. Simulation of the divergence of two orbits for different initial conditions for the Rössler system. This system is in the chaotic regime. The parameters are the same as described in figure 10. The range of iterations from 3400 to 5000 is shown, where the divergence becomes visible.

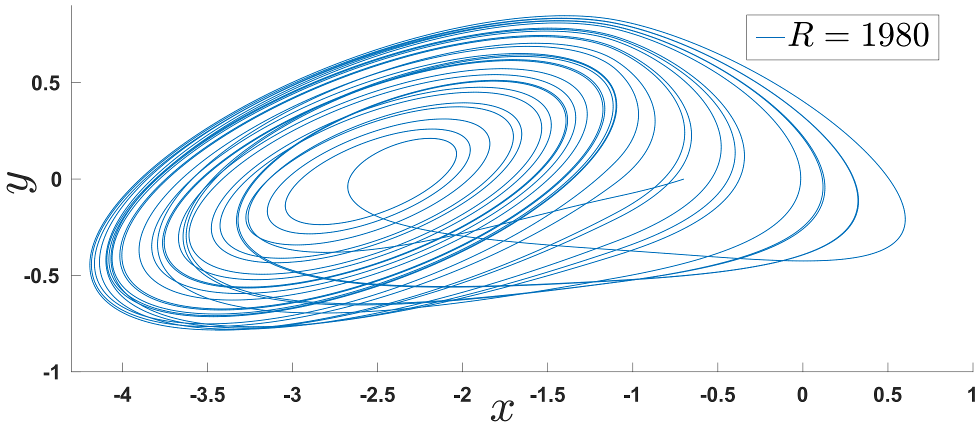

Figure 12. Spiral attractor in Chua's circuit. See section 6.3 to check the parameters and initial conditions used in this example.

Students were motivated to study the relationship between relevant concepts in electrical engineering and dynamic systems. They were asked to simulate the spiral attractor and double-scroll attractor behaviours by the following parameters.

- Spiral attractor: $C1 = 10nF$, $C2 = 100nF$, $L = 19mH$ e $\underline{R = 1980\Omega}$ (figure 12).
- Double-scroll attractor: $C1 = 10nF$, $C2 = 100nF$, $L = 19mH$ e $\underline{R = 1800\Omega}$ (figure 13).

It was possible to verify the appearance of two chaotic behaviours for the circuit just by varying the resistance value. Moreover, it gave the students encouragement to research other circuits, such as [51–53].

6.4. Activity 4: the double pendulum

The theory of dynamic systems is also related to a branch of applied physics. The model studied in this activity consists of two physical pendulums that can rotate freely around their

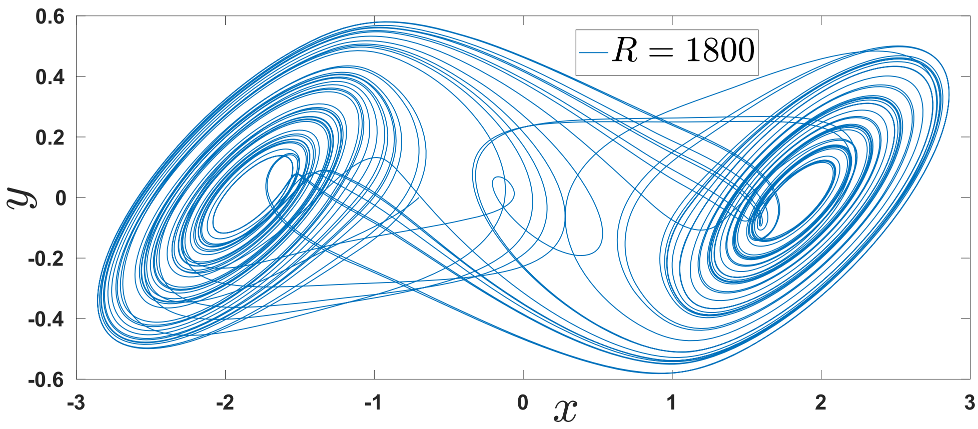

Figure 13. Double-scroll attractor in Chua's circuit. See section 6.3 to check the parameters and initial conditions used in this example.

respective points of fixation. This process aims to establish a physical example in the form of animations, allowing a different perception of the concepts previously discussed.

With the help of the animation in the movement of the two bars, the student is instigated to observe the chaotic evolution of the system for the established standard parameters in the platform. Then, the discovery of new behaviours and observations of how the animation is changed according to the modification in the parameter of gravity, for example, is cited as a task to the user.

7. Comparison of different software tools

In this section, a comparison with software found in the literature dedicated to the study of nonlinear dynamic systems is presented (see table 2). But first, a brief description of each tool is given.

An important motivation to develop tools such as that proposed in this work is to assist novice users in the teaching of nonlinear dynamic systems, enabling the user to understand dynamic phenomena in a simple and visual way through numerical simulations, as explained by [54]. Software such as Dynamics 2 [55] aims at showing how teaching-oriented computing tools should be, providing complete documentation and interactive capabilities such as plotting orbits, changing system parameters and image-oriented tools. However, Dynamics 2 is not a free tool; it is obtained by purchasing the book cited in [55]. Due to the fact that it was last updated in the year 2000, its interface may not be friendly to students and may be incompatible with current computers and operating systems.

A more recent tool is Chaos for Java [56, 57], which has been written in the Java language and is entirely free and independent. It is software that provides support for all current operating systems, enabling the simulation of classic nonlinear systems. However, Chaos for Java does not provide systems like Chua's circuit, a relevant model in engineering, or Newtonian systems such as the double chaotic pendulum, an interesting system for exemplifying chaos for beginners. The application has several computational tools, accompanied by a friendly and simple interface, saving the graphics in different formats and qualities, but does not display the option to save the numeric data from the simulations.

Table 2. A review of the software under analysis: I—Dynamics 2 [55], II—Chaos for Java [56, 57], III—Interactive Differential Equations [58, 59], IV—cspX: Tools for Dynamics [60], V—Tisean [61, 62].

Features	Software					
	Proposed	I [55]	II [56, 57]	III [58, 59]	IV [60]	V [61, 62]
Free of cost	✓	—	✓	—	—	✓
Organised documentation	✓	✓	✓	✓	✓	✓
Friendly interface	✓	—	✓	✓	—	—
Use of animation	✓	✓	—	—	—	—
Save numeric data	✓	✓	—	—	✓	✓
Save graphics	✓	✓	✓	—	✓	—
Change parameters	✓	✓	✓	✓	✓	✓
Updated software	✓	—	✓	✓	—	✓
Plot tools (e.g. zooming)	✓	✓	✓	✓	✓	—
Matlab support	✓	—	—	—	—	—
Exercises and examples	✓	✓	✓	✓	✓	✓

Another complete tool is Interactive Differential Equations (IDE) [58, 59], which is based on a collection of illustrations and interactive tools to explore various mathematical models. Including dynamic systems, the software consists of a packet of exercises and laboratory experiments for engineering, physics, chemistry and biology. Nonetheless, the program is not free and exhibits limitations in saving the numerical data and images generated by the simulations.

Finally, other software was found that was designed for the same purpose, such as cspX: Tools for Dynamics [60], software that displays the basic tools but is not upgraded to current platforms; and Tisean [61, 62], which shows some features for more advanced users (e.g. entropy and Lyapunov exponents), but does not provide interactive tools and was last updated in 2007.

According to table 2, the proposed GUI is a valuable supplement to the literature, since it has more features. Moreover, the GUI may be more practical, for users and non-users of Matlab, due to the fact that it is updated and free of cost.

8. Student assessment and evaluation

As a way of evaluating student satisfaction with the use of the interface, two analyses were made. The first was quantitative, in which the students were asked to answer a brief survey, in which they rated their experience using a Likert-scale questionnaire. This instrument used a standardised scoring system that included scores that ranged from 1 to 10, where 1 represents substantial disapproval and 10 full agreement with the question. The second analysis was qualitative, in which the students made brief comments, positive or negative, about their experiences with the tool.

The questionnaire and the average and standard deviation scores are given in table 3. From the scores, the interface proved to be a source that impressed users who specialised in the area, such as the study of the Lyapunov exponent, entropy, the Kaplan–Yorke dimension and intermittent dynamics. The students found the tool very accessible, with an average score

Table 3. Survey applied to students who participated in the study in the classroom. The questionnaire was given to 30 students. The average and standard deviation for each question is shown.

	Question	Average Score	Standard Deviation
Q1	Is the interface accessible to students?	9.52	1.03
Q2	Does the interface complement and make it easier to learn the theory of nonlinear systems?	9.06	1.56
Q3	Has the interface motivated the study of concepts in engineering?	8.29	1.59
Q4	How important is the aid of the animation in the execution of the graphs for the understanding of the behaviours presented by the system?	9.33	1.65
Q5	Is observing graphical data through the interface more beneficial than texts for the understanding of initial concepts?	8.76	1.61

of 9.52. Another important point, rated an average of 9.06, was the fact that students found the interface a complement to learning nonlinear systems. Furthermore, the usability of the tool is good for novice users, showing that its use is interactive and that the adoption of animations is beneficial for learning. One point that can be improved on is related to the motivation to study the concepts of engineering, which received the lowest score. A possible explanation for this is the fact that only one of the examples is related to electrical engineering, Chua's circuit. This is a topic that can be covered in a continuation of this work, exploring more examples related to electronic circuits, such as those developed by Sprott [2, 3]. In addition to the scores, the students made several constructive comments about the tool. Many students have concluded that the interface is simple and clear and that the organisation of chaotic systems is welcome.

In general, the tools presented (e.g. zooming and rotating figures) were well received. However, one user found the rotating graphics option confusing and unnecessary, and another suggested translation into other languages, such as Portuguese and Spanish. There was also a student who suggested an option to change the colours of figures. Finally, all the features that the authors of this project aimed at when the interface was created were well accepted by the students. The objectives that were mentioned above in section 2 were achieved, indicating the success of the work.

9. Conclusion

This paper has described the development of a GUI used for teaching dynamical systems. Specifically, six chaotic systems with a rich nature were chosen to demonstrate the tool to students, with the help of animations and periodic and chaotic behaviours. The platform has been developed from Matlab Runtime, for which users do not need to have the license to run the GUI.

The tool has been tested in the classroom with the purpose of evaluating the performance of the GUI. It also allowed students to explore the effects of the variation of the parameters and instantly observe their influence on the systems presented. This type of interaction proved to be useful as it encourages students, and it was discovered that the interactive tool provides a stronger motivation to learn than the traditional way. A questionnaire given to 30 students resulted in high scores, showing that on a scale of 1 to 10, the students found the tool to be accessible with an average score of 9.52. The lowest score was given to the question related to motivation to study concepts in engineering. We believe that this topic reveals the need to include more examples, particularly of electronic circuits.

In the future, we hope to add more improvements, such as increasing the number of experiments, or including the computation of the maximal Lyapunov exponent [63–65]. Both have been pointed out by student assessment as being desirable features. Furthermore, we aim to produce works that deal with nonlinear dynamics, with a focus on studying and applying methods to understand computer simulations better. We also intend to translate the platform to other languages, as was suggested by one of the students.

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