UNCERTAINTIES FACILITATE AGGRESSIVE BEHAVIOR IN A SPATIAL HAWK–DOVE GAME

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We study effects of additive spatiotemporal random variations, introduced to the payoffs of a spatial hawk–dove game, on the evolution of the reconciliatory and the aggressive strategy. We show that uncertainties facilitate aggressive behavior for a broad range of resource values. In particular, aggressors thrive best if stochastic influences are of the order of magnitude of deterministic payoff values. We argue that random payoff variations are potent and plausible promoters of aggressive behavior in human as well as animal societies if only the hawk–dove game payoff ranking applies.

Keywords: Evolutionary game theory; hawk–dove game; noise; spatial games.

1. Introduction

The evolutionary game theory [Maynard Smith & Price, 1973] has proven excellent for studying the evolution and success of different behavioral patterns in human as well as animal societies. The two games receiving the most attention are the hawk–dove and the Prisoner's Dilemma game [Axelrod & Hamilton, 1981]. In both games, the cooperative strategy, i.e. dove strategy in the hawk–dove game, warrants the highest collective payoff that is equally shared among the players. Mutual cooperation is, however, challenged by the defecting strategy, i.e. hawk strategy in the hawk–dove game, that promises the defector a higher income at the expense of the neighboring cooperator. The crucial difference that distinguishes both games is the way defectors are punished when facing each other. In the Prisoner's Dilemma game, a defector encountering another defector still earns more than a cooperator facing a defector, whilst in the hawk–dove game the ranking of these two payoffs is switched. Thus, in the hawk–dove game a cooperator facing

a defector earns more than a defector playing with another defector. This seemingly minute difference between both games has a rather profound effect on the success of both strategies. In particular, whilst by the Prisoner's Dilemma spatial structure often facilitates cooperation this is often not the case in the hawk–dove game [Hauert & Doebeli, 2004].

Recently, we studied the impact of noise, added to the payoffs of a spatial Prisoner's Dilemma game, on the evolution of the cooperative and the defective strategy [Perc, 2006a]. We found that temporally and spatially uncorrelated additive Gaussian disturbances are able to revert the extinction of cooperators, occurring by a large enough temptation to defect, in a resonant manner depending on the intensity of noise, thus marking a coherence-resonance-like phenomenon in the spatial Prisoner's Dilemma game. There exist also several other studies acknowledging the importance of external disturbances in evolutionary dynamics, affecting both the overall population gain [Traulsen *et al.*, 2004] and equilibrium selection [Szabó *et al.*, 2005], or even the nature

of phase transitions from one equilibrium towards the other [Szabó & Töke, 1998].

In view of interesting differences between the evolution of cooperation in the spatial versions of the two discussed games pointed out by Hauert and Doebeli [2004], we presently study the impact of temporally and spatially uncorrelated Gaussian noise on the cooperative and the defective strategy also for a spatial hawk–dove game. In the hawk–dove game, also known as the snowdrift or chicken game, the dove strategy is considered reconciliatory or cooperative, as often observed by doves, whilst the hawk strategy is the aggressive or defecting one, describing the natural behavior of hawks. However, it is important to note that the two behavioral patterns are not reserved only for doves and hawks, but may very much apply also to other species. Unlike by the Prisoner's Dilemma game [Perc, 2006a–2006c; Perc & Marhl 2006], we show that uncertainties actually facilitate the aggressive behavior instead of the dove-like cooperative strategy. Thus, similarly as in [Hauert & Doebeli, 2004], we emphasize that the reverse payoff ranking of punishment and suckers payoff in the hawk–dove and the Prisoner's Dilemma game has a profound effect on the evolution of both strategies, not just in the deterministic case, but also in the noisy case. Moreover, we emphasize that by the introduction of additive spatiotemporal noise to the payoffs, we are able to exactly determine and control the intensity of noisy perturbations, and thus are able to systematically analyze their effects on the evolution of both strategies. Thus, unlike by nondeterministic strategy adoption rules, such as for example the proportional adoption rule, which also introduces stochastic effects to the game, our approach warrants an easily controllable investigation of differently pronounced stochastic effects on the evolution of different strategies of the game. Since uncertainties are a part of everyday life, we argue that explicit random payoff variations present a viable mechanism that affects the outcome of evolutionary games in human and animal societies or economic cycles.

2. Spatial Hawk–Dove Game

We consider an evolutionary hawk–dove game with players located on vertices of a two-dimensional square lattice of size $n \times n$ with periodic boundary conditions. Moreover, we assume that each individual interacts, i.e. plays the game, only with its four nearest neighbors located to the north, south, east

and west, whereby self-interactions are excluded. The game starts with doves (D) and hawks (H) that are uniformly distributed on the square lattice. Each player P_i changes its strategy after each full iteration cycle of the game so that it adopts the strategy of its nearest neighbor P_j with the largest cumulative payoff. Thus, we apply the deterministic best-takes-over strategy adoption rule to focus solely on the stochastic disturbances delivered by the Gaussian payoff variations. The cumulative payoffs acquired during each iteration cycle of the game are calculated in accordance with the payoff matrix [Nowak & May, 1993]:

$$
\begin{array}{c|cc}
\dfrac{P_i}{P_j} & D & H \\
\hline
D & \begin{array}{c} \dfrac{G}{2} + \xi_i \\ \dfrac{G}{2} + \xi_j \end{array} & \begin{array}{c} G + \xi_i \\ 0 + \xi_j \end{array} \\
H & \begin{array}{c} 0 + \xi_i \\ G + \xi_j \end{array} & \begin{array}{c} \dfrac{G-C}{2} + \xi_i \\ \dfrac{G-C}{2} + \xi_j \end{array}
\end{array}
\tag{1}
$$

The payoff matrix is subjected to temporally and spatially uncorrelated additive Gaussian noise, satisfying the correlation function $\langle \xi_i(k)\xi_j(l) \rangle = \sigma^2 \delta_{ij} \delta_{kl}$, whereby indexes (i, j) mark any of the two neighboring players, whilst k and l index two consecutive pair interactions. Moreover, $0 \leq G < 1$ is the resource, $C = 1$ is the fitness loss endured by any two hawks that are involved in a fight, whereas σ^2 is the variance of payoff variations that exactly determines the level of stochasticity in the game.

The studied spatial hawk–dove game is iterated forward in time using a synchronous update scheme [Huberman & Glance 1994; Nowak *et al.*, 1994; Hauert, 2002]. After every full iteration cycle of the game all players simultaneously update their strategy according to the best-takes-over strategy adoption rule and reset their cumulative payoffs to zero. For a large enough number of game iterations ($\geq 10^4$) and large system sizes ($n \geq 200$), the average frequencies of doves F_D and hawks F_H approach an equilibrium value irrespective of the initial conditions, provided long enough discard times are taken into account. However, in the purely deterministic case ($\sigma = 0$) the best-takes-over strategy adoption rule prohibits an appropriate equilibration process and thus F_D and F_H might vary slightly in

dependence on the initial uniform distribution of players. We eliminated these fluctuations by averaging F_D and F_H over 30 game realizations with different initial distributions of players.

3. Results

We start the study by visually inspecting three characteristic spatial distributions of doves and hawks obtained by various values of σ for $G = 0.68$. Figure 1(a) proves that by $\sigma = 0$ neither of the two strategies is particularly favored since both doves and hawks are rather equally spread on the spatial grid. However, as stochasticity is introduced to the game via additive Gaussian distributed payoff variations the situation changes substantially since hawks start to dominate the field. Already by $\sigma = 0.08$ [Fig. 1(b)] the reconciliatory strategy gives way to the aggressive hawk strategy so that doves cover only 1/4 of the spatial grid. The facilitating effect of noise on the hawk strategy is even more pronounced by $\sigma = 0.2$ in Fig. 1(c), where doves struggle against extinction by mere 5% chance for survival. By increasing the level of stochasticity even further ($\sigma \geq 0.21$) hawks start to dominate the field completely.

To quantify the ability of each particular σ to facilitate the aggressive hawk strategy more precisely, we calculate F_H in dependence on various σ and G. Results presented in Fig. 2 clearly show that the range of resource values G where hawks completely dominate the game increases steadily as σ is enlarged. In particular, by $\sigma = 0$ the total dominance of aggressors ends already by the resource

value $G = 0.98$, whilst by $\sigma = 0.25$ it lasts up to $G = 0.67$. Importantly, however, the facilitative effect appears to be of saturating nature with respect to larger σ since by $\sigma = 0.4$ the dominance of hawks still ends by $G = 0.672$, which is only a marginal improvement with respect to $\sigma = 0.25$. Thus, there appears to exist an optimal amount of noise that is in the order of magnitude of deterministic payoffs by which the aggressive strategy thrives best, whilst still allowing a fairly fast equilibration process of F_D and F_H. Note that larger σ prolong the equilibration process substantially, especially near the extinction thresholds, and thus unnecessarily burden the expensive numerical procedure. However, due to the added stochasticity, the transitions marking complete dominance of either of the two strategies become increasingly smooth, as can be inferred from Fig. 2.

In order to shed light on the observed phenomenon, we first briefly summarize the intuitive explanation as to why uncertainties promote the cooperative strategy in the Prisoner's Dilemma game, instead of the defective strategy as reported here. In [Perc, 2006b] we argued that clustered cooperators (note that clustering of cooperators is the key phenomenon facilitating cooperation in the two-strategy Prisoner's Dilemma game on the spatial grid) have an edge with respect to defectors since in times of an unlucky draw of the unpredictable disturbances they help each other out by mutually sharing the reward. Although this income is often smaller as would be obtained by the defecting strategy, it is, on the other hand, reliable and this turns out to be the key enabling survival in

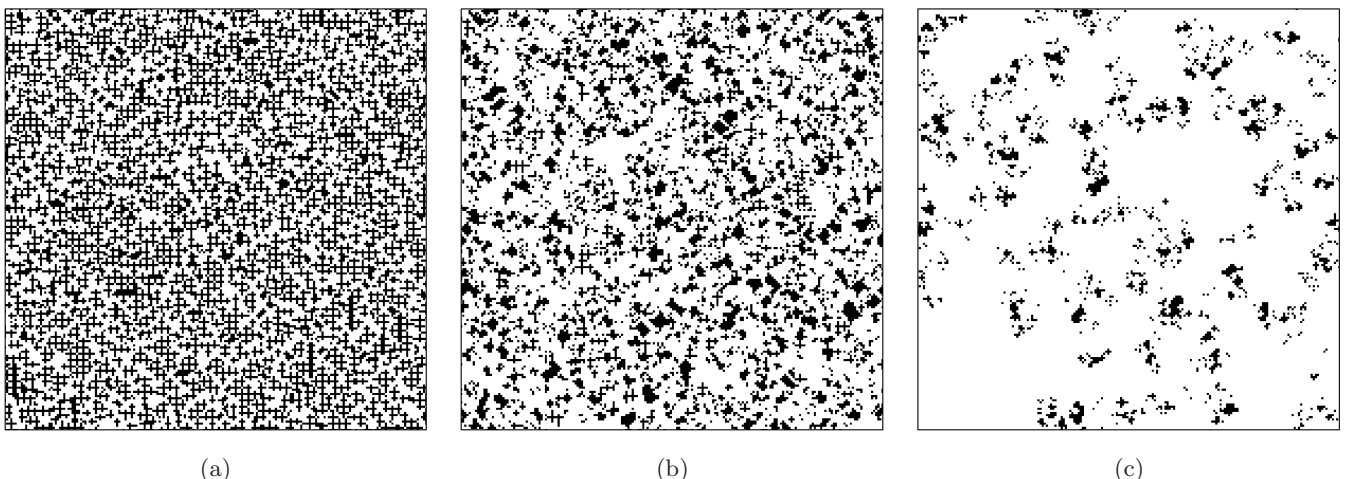(a)   (b)   (c)

Fig. 1. Characteristic equilibrium spatial distributions of doves (black) and hawks (white) obtained by $\sigma = 0.0$ [$F_H = 0.62$, (a)], $\sigma = 0.08$ [$F_H = 0.75$, (b)], and $\sigma = 0.2$ [$F_H = 0.95$, (c)] for the resource value $G = 0.68$. All panels are depicted on a 200×200 spatial grid.
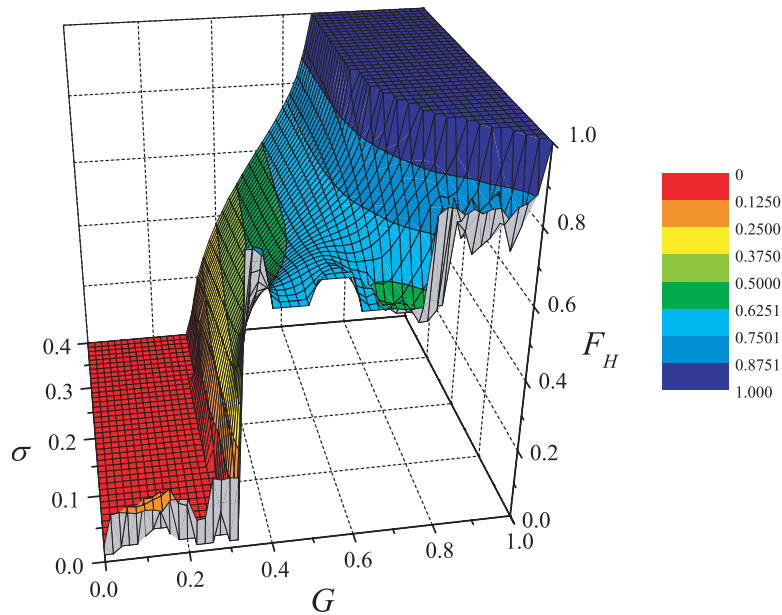
Fig. 2.   Enhancement of aggressive behavior via additive Gaussian payoff variations. Notice how hawks dominate the game up to $G = 0.67$ by $\sigma = 0.25$ whilst by $\sigma = 0$ total dominance of aggressors ends already by $G = 0.98$. Notice also how the discontinuous jumps of F_H vanish as the deterministic best-takes-over strategy adoption rule is increasingly blurred by noise. The vertical axis (depicting F_H) has a color-coded surface to enable a better visualization of results.

an unpredictable environment. The conclusion of [Perc, 2006b] was that in a chaotic or noisy world it appears better to receive smaller but reliable incomes than larger but infrequent ones.

In the presently studied hawk–dove game, however, the above-described reasoning does not hold since clustering of the reconciliatory (cooperative) dove strategy is no longer present. By observing the panels of Fig. 1 it becomes instantly obvious that, unlike by the spatial Prisoner's Dilemma game (see e.g. [Perc, 2006a]), doves do not form large compact clusters but only small fragmented clusters. In [Hauert & Doebeli, 2004] it was argued that this is due to the processes at micro scales where the payoff structure of the hawk–dove game makes it advantageous to adopt strategies that are opposite to the neighboring strategies. Thus, since doves do not form large clusters, they cannot benefit from reliable incomes received by mutual cooperation. In particular, hawks encounter cooperators nearly just as likely as doves. Therefore, the temptation to outperform the opponent by acting aggressively is not opposed to by the promise of receiving slightly smaller but reliable incomes if acting reconciliatory. From this point of view, it is just as risky to act aggressively as it is to act reconciliatory, and thus in a noisy environment the aggressive hawk strategy is more likely to win.

4.   Summary and Discussion

In sum, we show that spatially and temporally uncorrelated additive Gaussian noise introduced to the payoff matrix of an evolutionary spatial hawk–dove game facilitates the aggressive hawk strategy in dependence on the level of random variations. Thereby, noise levels that are of the same order of magnitude as the deterministic payoffs appear to offer the strongest support for hawks, whilst still maintaining a fairly quick equilibration process of the game near the extinction thresholds. The noisy support for the aggressive strategy is attributed to the inability of doves to form large compact clusters in the spatial domain, as this do cooperators in the spatial Prisoner's Dilemma game [Hauert & Doebeli, 2004]. Importantly, we note that noisy additive payoff variations are excellent for modeling stochastic influences on spatial games developed in the framework of evolutionary game theory, thereby warranting a relevant and accurate analysis of the problem.

We emphasize that the subtle differences between the payoff rankings of the Prisoner's Dilemma and the hawk–dove game lead to substantially different responses of both strategies, not just with respect to the facilitation of the cooperative strategy by spatial structure [Hauert & Doebeli,

2004], but also when noisy influences are taken into account, as reported presently. In real-life studies it is therefore extremely important to correctly determine the appropriate payoff ranking of the system under study (see e.g. [Turner & Chao, 1999]), since even minute differences with respect to reality might lead to wrong and meaningless interpretations of observed behavior.

Finally, we argue that random payoff variations are common in real life, and thus represent a viable modeling approach for the introduction of stochasticity to games on grids [Nowak & May 1992]. It is straightforward to imagine that a successful spreading of a behavioral pattern is affected by numerous unpredictable factors, whereby it seems reasonable that the interaction phase between two individuals, each trying to make the best out of the encounter, is the most likely part of the evolutionary process for uncertainties to take effect. Thereby, disturbances can arise either from the players themselves [Dall *et al.*, 2004; McNamara *et al.*, 2004], for example by not adhering to the rules of the game in trying to make an illegal profit, or from the environment which can either favor or hinder the success of each player. Although the presently applied methodology assumes uncorrelated Gaussian distributed disturbances and regular nearest-neighbor interactions of players, future studies will be necessary to clarify the importance of temporal and spatial correlations of such random influences [García-Ojalvo & Sancho, 1999] in conjunction with different topological realizations of the spatial grid [Tomassini *et al.*, 2006].

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