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The Impact of Static Disorder on Vibrational Resonance in a Ferroelectric Liquid Crystal

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Vibrational resonance manifests itself in certain nonlinear systems that are subject to the action of a biharmonic force with very different frequencies. A proper frequency and/or amplitude of the high-frequency harmonic signal can lead to a resonantly enhanced response of the system with respect to the low frequency signal. In the present study we extent the concept of vibrational resonance to a thermotropic bistable surface-stabilized ferroelectric liquid crystal cell. Furthermore, we systematically investigate the impact of different origins of static disorder and reveal that different kinds of static disorder affect the resonant response in a qualitative different way.

Keywords vibrational resonance; liquid crystal; static disorder

1. Introduction

Weak signal detection by nonlinear systems can be considerably affected by external influences. Perhaps the most famous example of this verity is stochastic resonance, where a proper intensity of random fluctuations resonantly enhances the response of a nonlinear system to a weak deterministic signal [1]. The very general phenomenon of stochastic resonance appears in the processing of harmonic [2] and aperiodic signals [3] by nonlinear systems and has thus been recognized as important in several scientific disciplines ranging from physics [4–6] to biology [7, 8] and chemistry [9]. Interestingly, even the popular nanoscale systems have become associated with stochastic resonance. Badzey et al. [10] have shown that nanomechanical oscillators in a dynamic bistable state exhibit a more controllable switching in the presence of noise. Furthermore, experiments on a single-walled carbon nanotube transistor have confirmed that a proper value of noise can help a threshold-like nanotube transistor to detect subthreshold electric signals [11].

Different kinds of external influences have been perceived to improve the signal processing. Besides the most commonly employed white noise also colored noise [12, 13] and chaotic signal [14] were found to enhance the response of a nonlinear system to weak forcing. Remarkably, Landa and McClintock [15] have revealed that a resonant behavior in a bistable potential with respect to a low-frequency force can be provoked by a high-frequency periodic force. This phenomenon, called vibrational resonance (VR), closely resembles stochastic resonance, except that in this case the role of noise is replaced by a

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high-frequency harmonic signal. Here, the dependence of the system response versus the amplitude of the high-frequency action also exhibits a bell-shaped resonant form. Besides bistable systems [15–17] the phenomenon of VR has been studied in excitable [18, 19], multistable [20] and spatially extended systems [21, 22]. Notably, special attention has also been devoted to the role of additive noise on VR as well [23–25]. First experimental evidences of VR have been provided by Chizhevsky et al. [17] in a bistable vertical cavity laser system and by Baltanás et al. [16] in analog electronic circuits. The most recent experimental and theoretical investigations of VR are devoted to weak signal detection [26], signal transmission [22] and its applications in biosciences [27, 28].

In the present paper we aim to extend the scope of VR to a polymer stabilized liquid crystal (LC) confined within a plan-parallel cell. We have recently shown that under proper conditions the dynamics of this LC cell can be mapped to an ensemble of coupled bistable oscillators and thus represents a suitable candidate for the manifestation of stochastic resonance [29]. Inspired by our previous findings we explore the VR phenomenon in the setup as well, giving special emphasis to the role of static disorder. As mentioned above, a polymer network is usually introduced to the LC cell, which on one side significantly increases the mechanical strength, but on the other side it introduces a certain degree of static randomness in the system [30, 31]. Notably, Tessone et al. [32] have shown that a proper degree of static disorder can, following the model of stochastic resonance, enhance the response of the system to weak external signals by means of diversity-induced resonance. Subsequent studies have revealed that in various systems also the interplay between dynamical noise and static disorder leads to a nontrivial resonant behavior [29, 33, 34]. However, to the best of our knowledge, the influence of static disorder on the VR phenomenon has not yet received any attention. We address this issue in the present study by systematically analyzing the impact of different types of static disorder. Our results reveal that resonant responses provoked by the high-frequency signal are indeed affected by static disorder as well as that different kinds of disorder change the behavior in a qualitatively different way. Moreover, it turns out that any type of static randomness has a completely different impact on VR as additive dynamic noise.

2. Vibrational Resonance in a Polymer Sliquid Crystal Cell

We consider a ferroelectric LC confined to a plan-parallel cell of thickness d. For illustration purposes we consider rod-like LC molecules. In the helicoidal smectic C phase and thin enough cells the LC forms a layered structure which is schematically depicted in Fig. 1 together with the coordinate system. The translational order is characterized by smectic layers that are stacked along the z-axis, and the normals of the cell walls are parallel with the x-axis of the system. The cell confinement can suppress the helicoidal structure if the thickness d of the cell is less or comparable to the pitch of the helix p [35]. The resulting unwound SmC structure is on average homogeneously tilted along a single direction. In the case of isotropic planar anchoring (i.e. molecules tend to lie in the confining plane within which all directions are equivalent), the molecules tilt either along the positive or the negative y-axis and the polarization is aligned either along the positive or the negative x-axis, respectively. In other words, the equilibrium configuration corresponds to spatially homogeneously tilted molecules either for an angle θ or $-\theta$, whereby the states are separated by en energetic barrier. One can switch between these configurations by an external field electric field E applied along the x-axis. Standard SSFLC cells are extremely susceptible to imperfections of the confining walls. One can substantially improve their mechanical

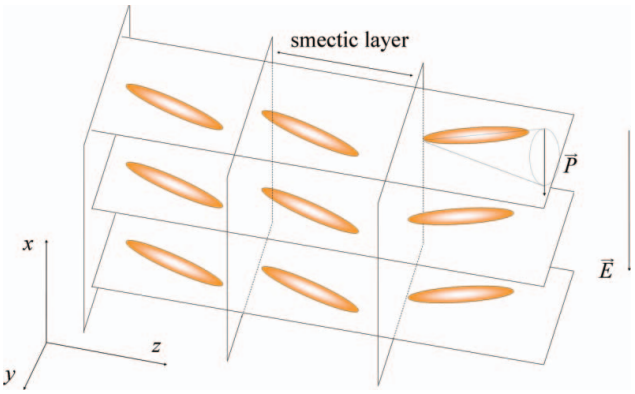

Figure 1. The SSFLC cell – LC molecules are collected in smectic layers that are stacked along the z-axis. For $d < p$ the molecules are either tilted along the positive or the negative y-axis. Consequently $\vec{P}$ is aligned along the negative or the positive x-axis, respectively. An external electric field along the x-axis favors one of the two possible configurations.

stability by introducing a proper concentration of polymer network into the LC [36]. The resulting system is referred to as the PSFLC. In this manner the mechanical stability of the LC component is gradually improved, but on the other side the network introduces a kind of a bulk field.

We proceed by describing the bistable equilibrium structure in the unwound SmC phase quantitatively in terms of the tilt angle θ and the polarization $\vec{P}$. Since the molecules can only be tilted for an angle θ or $-\theta$ the director field $\vec{n}$ is defined as follows: $\vec{n} = (0, |\sin\theta|, \cos\theta)$ and the polarization is defined as $\vec{P} = (\pm P, 0, 0)$. A simple form of the appropriate free energy density f expression in terms of θ and P, taking into account all the key features of the system, can be expressed as [37]

$$f = f_0 + \frac{\alpha_0(T - T_c)\theta^2}{2} + \frac{\beta\theta^4}{4} - CP\theta + \frac{2\theta P^2}{\chi\varepsilon_0} - EP + \frac{J}{2}|\nabla\theta|^2 - \frac{w\theta^2}{2} \qquad (1)$$

Here P stands for the in-plane spontaneous polarization of LC molecules, f_0 is the free energy density contribution of the undistorted smectic A phase, α_0, β, C and χ are material constants, ε_0 is the dielectric constant, J is the representative elastic constant of the smectic C phase, E is an external electric field applied along the x-axis, and $w = w(\vec{r})$ describes the coupling strength between LC molecules and the polymer strands. We proceed with minimization of f with respect to P and introduce dimensionless scaled quantities $\Delta\tilde{f} = (f - f_0)\frac{\beta}{\alpha_0^2(T_c-T)^2}$ $\tilde{\theta} = \theta\sqrt{\frac{\beta}{\alpha_0(T_c-T)}}$, $\tilde{E} = EC\varepsilon_0\chi(\frac{\beta}{\alpha_0(T_c-T)})^{3/4}$, $\tilde{K} = K\frac{\beta}{d^2\alpha_0^2(T_c-T)^2}$, $\tilde{w} = w\frac{\beta}{\alpha_0^2(T_c-T)^2}$, $\tilde{\nabla} = d\nabla$. Discarding the tildes it follows

$$\Delta f = -\frac{\theta^2}{2} + \frac{\theta^4}{4} + \frac{J}{2}|\nabla\theta|^2 - \theta E - \frac{w\theta^2}{2} \qquad (2)$$

The first two terms enforce values $\theta = \theta_0 \equiv \pm 1$, the 3rd term penalizes deviations from a spatially homogeneous ordering, the 4th term introduces the source of the coherent input signal, and the last term introduces coupling between LC molecules and the polymer strands. Taking into account a standard form of the dissipation free energy term [29, 38] and

after discretization $\theta_i = \theta\vec{r}_i$), we obtain the following system of dimensionless dynamical equations:

$$\frac{\partial\theta_i}{\partial t} = \theta_i - \theta_i^3 + \sum_j J_{ij}(\theta_j - \theta_i) + E + \theta_i w_i \tag{3}$$

Here θ_i represents the scaled tilt angle value at the i-th site of the lattice which follows from the discretization, w_i stands for the local LC-polymer interaction and J_{ij} measures the coupling strength between the i-th and j-th LC molecule, the index j runs over the nearest neighboring sites of the i-th site, and t stands for the dimensionless time. Note that J_{ij} values could be site dependent due to the spatially random structure of the perturbing polymer network.

In Eq. (3) the properties of the coupling matrix J_{ij} and the static random field w_i uniquely determine qualitatively different origins of static disorder. In particular, a binary distribution of J_{ij} being either 0 or J_0 results in the random dilution (RD) universality class, whereby p stands for the fraction of diluted links. Randomly distributed elements of J_{ij} inside $[J_0 - \Delta J, J_0]$ yield the random bond (RB) universality class, whereby the limiting case $\Delta J = 2J_0$ represent the spinglass (SG) universality class. Moreover, we consider the random field (RF) universality class, where $J_{ij} = J_0$ and values of w_i are distributed randomly in the interval $[-\Delta w, \Delta w]$.

In order to explore the vibrational resonance phenomenon we consider the external electric field being constituted by a low- and a high-frequency periodic signal:

$$E = A\cos(\omega_1 t) + B\cos(\omega_2 t), \tag{4}$$

where $\omega_2 >> \omega_1$ and A and B are the amplitudes of the low- and high-frequency signals, respectively. Note that the amplitude $A = 0.05$ is too low to provoke switching between the two possible configurations of the liquid crystal molecules without a proper high-frequency electric periodic field.

In order to quantify the collective response of the system to the low-frequency periodic field, we calculate the Fourier coefficients Q for the mean field

$$\Theta = \frac{1}{N}\sum_i \theta_i, \tag{5}$$

where the coefficients are defined as

$$Q' = \frac{\omega_1}{\pi n}\int_0^{2\pi n/\omega_1} \Theta(t)e^{i\omega_1 t}dt, \quad Q = \left|Q'\right|. \tag{6}$$

In Eq. (6) $n = 100$ signifies the number of oscillation periods used for the calculation, after 20 initial periods of the temporal traces were discarded as transients.

Now let us investigate the response of the examined model [Eqs. (3) and (4)] with respect to the varying amplitude and frequency of the high-frequency signal. Results shown in Fig. 2 reveal that for various frequencies ω_2 the response Q first increases with increasing amplitude B and then passes through a maximum and decreases again. This confirms that there is indeed an optimal amplitude of the high-frequency signal at which the switching of the molecules orientation is at best correlated with the weak low-frequency signal. Moreover, we notice that higher frequencies of the high-frequency signal require larger amplitudes of the high-frequency vibration B, whereas the amplitude of the resonant response Q remains more or less intact providing that the difference between the frequencies

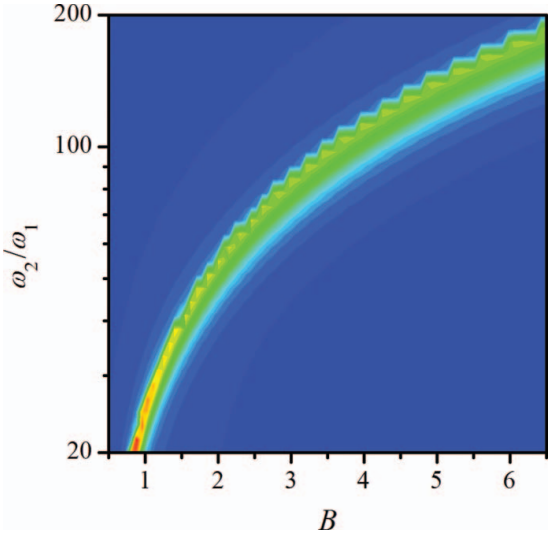

Figure 2. Vibrational resonance in the examined system – the resonant response quantified via Q in dependence on the amplitude of the high-frequency signal B and the ratio between the frequencies of the low- and high-frequency signals. The color coding is linear, blue depicting 0.0 and red 0.4 values of Q. The number of bistable oscillators was $N = 100$ and other parameters used by the calculation are: $J_{ij} = J_0 = 1.0$, $\omega_1 = 0.05$.

is large enough ($\omega_2/\omega_1 > 50$). Results presented in Fig. 2 thus confirm that a ferroelectric LC confined to a plan-parallel cell is a suitable candidate for the manifestation of VR. However, in the next section it remains of great interest to explore the impact of static disorder on the phenomenon, especially because in the examined system it arises naturally due to the random character of the polymer network that is introduced to the LC.

3. Effects of Static Disorder on the Vibrational Resonance Phenomenon

As announced, we examine the effect of different origins of static randomness described in the previous section on the resonant behavior in the examined system. For this purpose we fix the frequency of the high-frequency signal to $\omega_2 = 20$ and focus on the response of the mean field quantified via Q in dependence on the amplitude B and strength and type of static disorder. Results for random dilution, random bond and random field types of static disorder are shown in the left, middle and right panel of Fig. 3, respectively. Evidently, the random dilution disorder weakly affects the resonant response, since the quality of the maximal response is roughly the same for all fractions of diluted links p. Only a slight shift of the optimal amplitude of the high-frequency signal B towards higher values is inferred as p is increased. For the random bond type disorder the dependence of the resonant response on the disorder strength is almost identical as in the case of random dilution as long as $\Delta J < J_0$. However, for higher levels of random bonds the quality of the resonantly enhanced response decreases rapidly, so that in the limiting case where $\Delta J = 2J_0$, signaling the spin-glass universality class, the resonant behavior is more or less vanished. In contrast to the previous cases, the random field type disorder results in a progressive decrease of the resonant response already for small amplitudes of random fields. Furthermore, there is

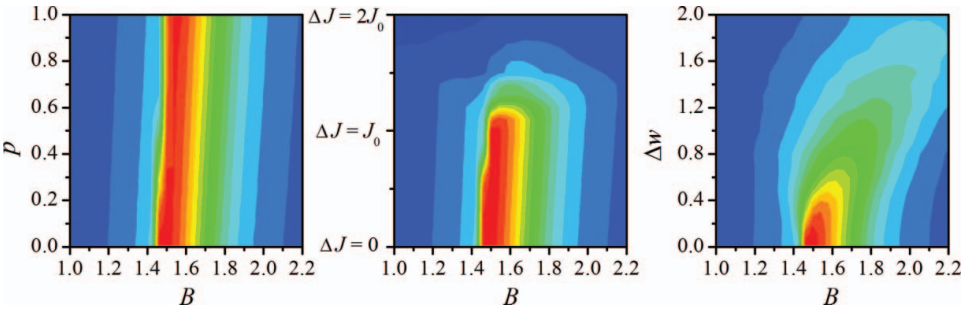

Figure 3. Resonant responses Q as a function of B and strengths of different types of static disorder: random dilution (right panel), random bond (middle panel) and random field (right panel). Parameters used by the calculation are: $\omega_1 = 0.05$, $\omega_2 = 20$, $J_0 = 1.0$, $N = 100$. The color coding is linear, blue depicting 0.0 and red 0.3 values of Q.

a significant increase in the required amplitudes of the high-frequency signal which ensure optimal synchrony between the low-frequency signal and the tilting of the LC molecules.

Finally it remains of great interest to compare the impact of different types of static disorder on the VR phenomenon with the influence of additive dynamical noise. For this purpose we add the stochastic term $D\xi_i(t)$ to the model described by Eq. (3), where D signifies the intensity of additive Gaussian white noise ξ_i with zero mean and unit variance. Results showing the quality of the resonant response Q as a function of B and D are shown in Fig. 4. It can be observed that the optimal resonant response is reduced as the noise intensity is increased. In addition, the peak is shifted toward lower amplitudes of the high-frequency signal. This effect is somehow expected since the noise associates with the high-frequency driving and advances the resonance. Interestingly, the VR phenomenon disappears for

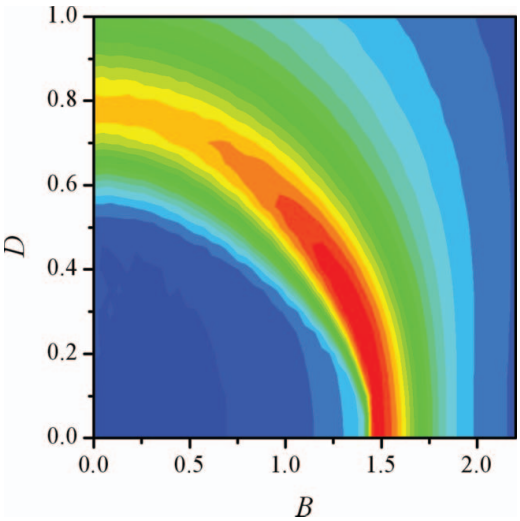

Figure 4. Response of the system Q as a function of the amplitude of the high-frequency signal B and dynamical noise. The color-profile and the parameter values are the same as quoted in the caption of Fig. 3.

higher noise amplitudes and for $B \to 0$ the system exhibits classical stochastic resonance. This observation is in agreement with previous studies [23, 24], where the effect of additive noise on VR in a bistable system has been tackled theoretically. The main reason why we present this more or less known result is to emphasize the radical difference between the impact of static and dynamic disorder on the VR phenomenon. It is rather surprising that the additive noise evokes the opposite effect of all kinds of static disorder.

4. Summary

We have studied the phenomenon of vibrational resonance in a polymer-stabilized liquid crystal that was confined within a plan-parallel cell. The introduction of a polymer network may significantly enhance mechanical strength, but in addition, it also introduces a certain degree of static disorder to the system [30, 31]. Given that the dynamics of this system can be described successfully by an ensemble of coupled bistable oscillators [29], the essential ingredients are thus at hand that warrant the observation of interesting noise induced phenomena. As mentioned, we have here focused on vibrational resonance and the role of static disorder. It is worth pointing out that a properly adjusted level of static disorder may enhance the response of a system to weak periodic driving [29, 32–34], although this has not yet been studied in conjunction with vibrational resonance. We remind the reader that vibrational resonance can be observed upon the action of a biharmonic force with very different frequencies. A proper frequency and/or amplitude of the high-frequency harmonic signal can lead to a resonantly enhanced response of the system with respect to the low frequency signal. Our results reveal that resonant responses provoked by the high-frequency signal are crucially affected by static disorder, and moreover, that different kinds of disorder change the behavior in a qualitatively different way. Indeed, it turns out that any type of static disorder has a completely different impact on vibrational resonance as additive dynamic noise.

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