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PAPER

Doubly effects of information sharing on interdependent network reciprocity

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Abstract

Understanding large-scale cooperation among unrelated individuals is one of the greatest challenges of the 21st century. Since human cooperation evolves on social networks, the theoretical framework of multilayer networks is perfectly suited for studying this fascinating aspect of our biology. To that effect, we here study the cooperation in evolutionary games on interdependent networks, such that players in one network layer play the snowdrift game (SDG), and the prisoner's dilemma game (PDG) in the other layer. Importantly, players are able to share information across two layers, which in turn affects their strategy choices. Monte Carlo simulations reveal that the transfer of information about the strategy of the corresponding player in the other network layer alone is enough to significantly promote the overall level of cooperation. However, while the cooperation is markedly enhanced in the layer where the PDG is played, the opposite is true, albeit to a lesser extent, for the layer where the SDG is played. The net increase in cooperation is thus due to a doubly effect of information sharing. We show further that the more complete the information transfer, the more the overall level of cooperation is promoted, and that this holds as long as the information channels between the player do not vary over time. We discuss potential implications of these findings for future human experiments concerning the cooperation on multilayer networks.

1. Introduction

With the rapid development of new-generation information technology, the means and channels acquiring the information become much more diverse. A huge amount of information will be created at each moment and we have entered into the big data era [1–3]. Meanwhile, it is the rich availability of these information that changes the paradigm of scientific researches and accelerates the data-driven efforts within the studies of many problems, such as behaviorial decision making [4, 5], population mobility [6, 7], information and rumor diffusion [8–10], product marketing strategy [11, 12], epidemic spreading [13–15], virus or malware propagation [16, 17], and even social movements and political campaigns [18, 19], to name some examples. On the one hand, making the information much more available is beneficial to make the rational choices during the everyday life, studies, career and research, even help to implement the scientific discoveries across many disciplines. However, on the other hand, the incomplete or unavailable information will not help to make the right decision, and may even lead towards wrong or unintended results.

According to the theory of games [20–22], any rational individual will not perform the cooperative strategy in order to pursue the maximization of his personal benefit. However, in the real-world cases, the cooperation is still widespread inside the natural, engineering and even social systems including the human society [23]. Thus, exploring the emergence of cooperation within the population has become a long-standing puzzle, which is one of the most challenging 25 scientific problems encountered in this century [24]. At present, five key rules to favor the persistence of cooperation [25], including kin selection, direct and indirect reciprocity, group selection, spatial or network reciprocity, have been proposed to understand how the cooperation evolves under the realistic environments, some fruitful and surprising consequences have been found [26–28].

Recently, the great progresses have been made in the field of network science [29], and the impact of interaction topology on the cooperative behavior has been extensively investigated [30–42]. Beyond the well-mixed and regular topology hypotheses, the scale-free network has been found to provide a unified framework to support the collective cooperation [43–45]. After these seminal contributions, many works are devoted to illustrating the role of complex topology in the evolution of cooperation [26–28], or accounting for the connection between the evolutionary game theory and the agent topologies [46, 47]. In particular, it is often found that different types of systems may interact and/or interrelate with each other, and hence seemingly irrelevant changes in one system may create very much unexpected and even catastrophic consequences in another one. The multilayer or interdependent network becomes a powerful framework to gain deep insights into these emergent phenomena [48–50]. Taking an example, the disease spreading and social contagion on multilayer networks has presented distinct critical properties from those on single-layered networks [51, 52]. Meanwhile the information available within one network may be helpful to make a decision for the players inside another network, which becomes more obvious in the realm of evolutionary games on networks [27, 28] since the players need to maximize their utility through the strategy competition based on their success in previous game rounds. However, most works implement the network interdependence by coupling the individual payoff on different networks and the resilience of cooperation can be largely enhanced by means of the non-trivial organization of players across different networks [53–63].

A distinct example is provided by Szolnoki and Perc [64], who introduce a unique mechanism of information sharing regarding the strategy state between the players on two different networks. In their model, they assume that identical strategies between two players residing on two networks can reinforce themselves by lessening their propensity to change, and they observe the spontaneous emergence of correlated behavior between the two networks, which further deters the defective players. But a strong assumption is adopted here that the information from the other network can be completely and accurately transmitted. In fact, there always exists the errors during the information transmission due to the noise or interference. In addition, many previous works deal with the evolution of cooperation by playing the identical games (say, prisoner's dilemma game (PDG) or snowdrift game (SDG)) on multiple interdependent networks [53–63], and less works consider the situation where the players may perform the distinct game behaviors within different networks or populations. Although Santos *et al* [65] investigated the evolutionary dynamics of two games within distinct layers of interdependent networks, they leverage a biased strategy imitation process among players which allows the individual on one layer to adopt the strategy of players on the other layer, but there are also some scenarios which do not permit the players to pass on the strategy between different networks. Thus, it is an interesting topic to explore the behavior of cooperation on two different networks where a different game type is played within one of two networks. To this end, we integrate a novel mechanism of incomplete information sharing into the individual strategy update process among the same population on one network, and meanwhile we hypothesize the players on different networks to play distinct games so that we can further characterize the evolution of cooperation under some specific backgrounds.

In the rest of this paper, we firstly illustrate the game model with incomplete state information sharing on two interdependent networks in detail in section 2, and then the extensive numerical simulation results are provided in section 3. At last, some concluding remarks are summarized so as to highlight the characteristics of cooperation and potential applications under the current model.

2. Mathematical model

In our model, the system is made up of two-layered lattices where each lattice contains $N = L \times L$ agents and every focal agent can only interact with 4 nearest neighbors (i.e., von Neumann neighborhood). On either of lattices, a distinct game is played and illustrated in figure 1. Without loss of generality, we consider the typical SDG on the upper layer and the PDG on the bottom network. Before the evolution, each player x in network UP (upper) and corresponding one x' in network DOWN (bottom) is stochastically designated as a cooperator (C) or a defector (D) with the equal probability. Then, in network UP, player x accumulates the payoff P_x according to the traditional SDG model in which the payoff matrix can be summarized as follows

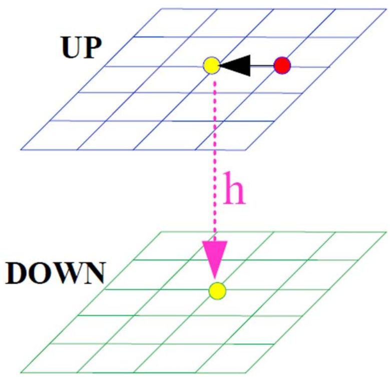

Figure 1. Role of incomplete information sharing in the evolution of cooperation between two interdependent networks. The snowdrift game will be played on the UP network, but the prisoner's dilemma game is performed in the DOWN network. Each focal player on one network will refer to the state of his corresponding partner on the other one with the probability h during the strategy update.

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix} \end{array}, \tag{1}$$

where r denotes the ratio of benefit to cost and the SDG condition $T > R > S > P$ can be strictly obeyed provided that we take the setup $T = 1 + r, R = 1, S = 1 - r, P = 0$ and $0 < r < 1$ into account.

Meanwhile, in the network DOWN, the PDG is played and each agent obtained his payoff through the following matrix

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & -r \\ 1+r & 0 \end{pmatrix} \end{array}, \tag{2}$$

where r has the same meaning as in equation (1) and PDG condition $T > R > P > S$ can be ensured if we consider the parameter setup $T = 1 + r, R = 1, S = 1 - r, P = 0$ and $0 < r < 1$.

After determining the payoff for a focal player, where his current utility can be fully represented by this payoff, the strategy imitation can take place between this player and one of his nearest neighbors on the same given lattice, but never allowed to spread the strategy between players locating on different networks. Meanwhile, the strategy adoption of a player on one network may refer to the state information of the corresponding partner on the other one. However, different from [64], the focal one may utilize this kind of information on account of limited perception ability, and thus incomplete information sharing is applied here. That is to say, any player will perform the strategy spread by virtue of information sharing between two networks with the probability h, or update the current strategy without any information sharing with the complementary probability $(1 - h)$. Taking the top network as an example, with the probability $(1 - h)$, player x in network UP can adopt the strategy s_y of one of his nearest neighbors (say, y) chosen at random with the Fermi probability [66] as follows

$$W(s_x \leftarrow s_y) = \frac{1}{1 + e^{\frac{P_x - P_y}{K}}}, \tag{3}$$

where K denotes the amplitude of irrationality during the strategy adoption. When $K \to 0$, the strategy adoption will be deterministic and player x will definitely take the strategy s_y if $P_x < P_y$ (i.e., with the probability 1); while for $K \to \infty$, the imitation procedure will be at random and player x will stochastically consider the strategy s_y (i.e., with the probability 0.5). Without lacking the generality, we set $K = 0.1$ unless directly stated.

Alternatively, with the probability h, player x try to imitate the strategy s_y of player y with the following Fermi-like probability

$$W(s_x \leftarrow s_y) = \omega_x \frac{1}{1 + e^{\frac{P_x - P_y}{K}}}, \tag{4}$$

where the pre-factor ω_x of player x depends on the strategy of corresponding player from the other network, and the information sharing between the players among two networks can be implemented here. For the simplicity, we assume that ω_x of player x in network UP is only correlated with the corresponding one x' in network DOWN, and vice versa. Here, ω_x will be minimal ($\omega_{\min}$) if $s_x = s_{x'}$ and maximal ($\omega_{\max}$) if s_x is unequal to $s_{x'}$. To avoid frozen states, $\omega_{\min}$ will be set to 0.1 as the minimal scaling factor, while ω_x takes the maximal value $\omega_{\max} = 1$.

Finally, a full Monte Carlo simulation (MCS) step will be completed if the above-mentioned sub-steps have been finished. The system will arrive at the stationary state after some temporary steps are discarded. In our

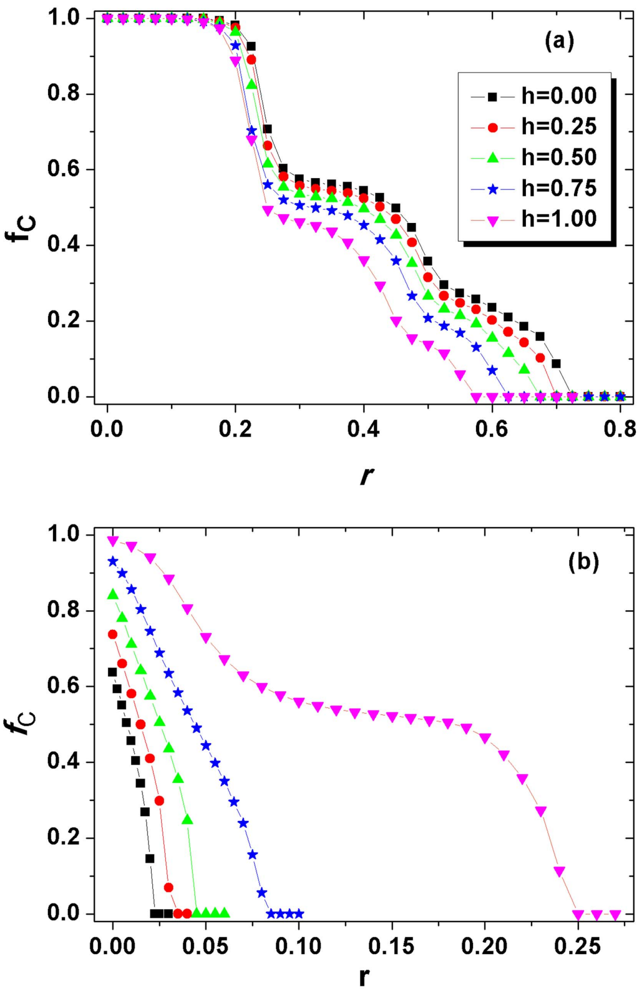

Figure 2. Fraction of cooperators at the stationary state among the population as a function of ratio of cost-to-benefit (r). The top panel (a) denotes the results of SDG, while the bottom panel (b) represents the cooperator's fraction of PDG. Two lattices have the same size $L \times L = 200 \times 200$, MCS = 50 000, and the amplitude of irrationality of strategy adoption $K = 0.1$.

current simulation setup, the total MCS steps are up to 50 000 (if unstated clearly) and the stationary state is obtained over the last 5000 time steps.

In terms of the motivation for the proposed model, it is worth noting that a particular social dilemma can be perceived differently by different players, and this is properly taken into account by considering an environment with different evolutionary games. At the individual level, a simple example to illustrate the point entails two drivers meeting in a narrow street and needing to avoid collision. While the first driver drives a cheap old car, the second driver drives a brand new expensive car. Obviously, the second driver will be more keen on averting a collision. This example can be easily extended to populations (or network layers in our case), where when we face a conflict, we are likely to perceive differently what we might lose in case a player in the other layer choose to defect. The key question then is how the presence of different payoff matrices, motivated by the different perceptions of a dilemma situation, will influence the cooperation level. However, knowing what the other player chooses, i.e., having information about its strategy, can also crucially affect our strategic choice. Accordingly, we have introduced the parameter h as the probability of this information transfer between the two network layers as a minimal model that captures the essence of such a situation. More precisely, with probability h this information is provided and affects the strategy transfer probability (the probability of strategy change is higher/lower if the strategy of the player in the other layer is same/different), while with probability $1 - h$ there is no information transfer. By varying h between 0 and 1, we thus capture all possible aspects of information sharing between the two layers.

3. Results

Firstly, we will depict the fraction of cooperators (f_C) at the stationary state as a function of the model parameter r in figure 2. Since two distinct game dynamics are carried out over two-layered networks, we will count the

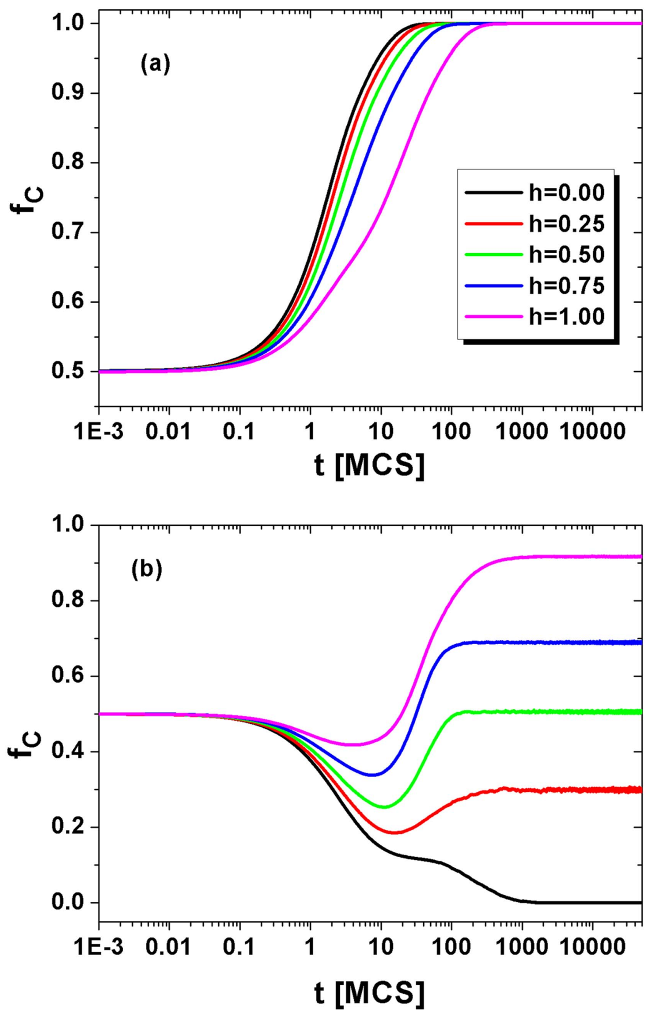

Figure 3. Fraction of cooperators as a function of time step (MCS). Likewise, the top panel (a) depicts the simulation of SDG, while the bottom panel (b) represents the cooperator's fraction at each time step under the PDG. Here, the ratio of cost-to-benefit is set to be $r = 0.025$. Two lattices have the same size $L \times L = 200 \times 200$, MCS = 50 000, and the amplitude of irrationality of strategy adoption $K = 0.1$.

fraction of cooperators on each network at the stationary state, respectively. That is, in figure 2, the upper panel gives out the fraction of cooperators for the SDG, while the lower panel describes f_C as a function of r in the PDG. As the parameter r increases, the temptation to defect becomes stronger, and it renders to be more difficult to favor the collective cooperation. However, the information sharing between two networks changes the cooperating behavior as compared to the spatial SDG or PDG model. On the one hand, for the PDG in the bottom layer, the cooperation can be greatly promoted as the information sharing probability h augments when r is not too large (less than 0.25). As an example, $h = 0.5$, the extinct threshold of cooperators for the ratio of cost-to-benefit is $r_{\mathrm{th}} \approx 0.048$ while r_{th} approaches about 0.02 for the original PDG model. In addition, for the same ratio $r = 0.01$, the fraction of cooperators at the stationary state is around 0.4567 without any information sharing, but this fraction will grow up to about 0.7119 where each agent can hold the probability of 50% to refer to the strategy state of corresponding partner on the other network. On the other hand, if r is larger than 0.25, the total defection exhibits within the whole population for any information sharing probability h since the temptation to defect is too large and all individuals make a ration choice during the decision making.

Meanwhile, the cooperation behavior for the SDG on the upper layer becomes a little more complex as r changes. As r is smaller (e.g., less than 0.2), it can be observed that the cooperation can be almost not influenced by the information sharing. However, when r becomes larger ($r > 0.2$), we can find that the cooperative behavior for the SDG is actually impaired by this kind of state or information sharing between two networks. Also, the larger the information sharing probability h, the more the level of cooperation reduction in the SDG, but the reduction extent is much less than the promotion of cooperation in the PDG. Thus, different from previous work [64], the cooperative behavior will exhibit the distinct properties for different game dynamics and the doubly effects exist here. Henceforth, the current results will enhance our understanding of evolution of cooperation within real-world environments.

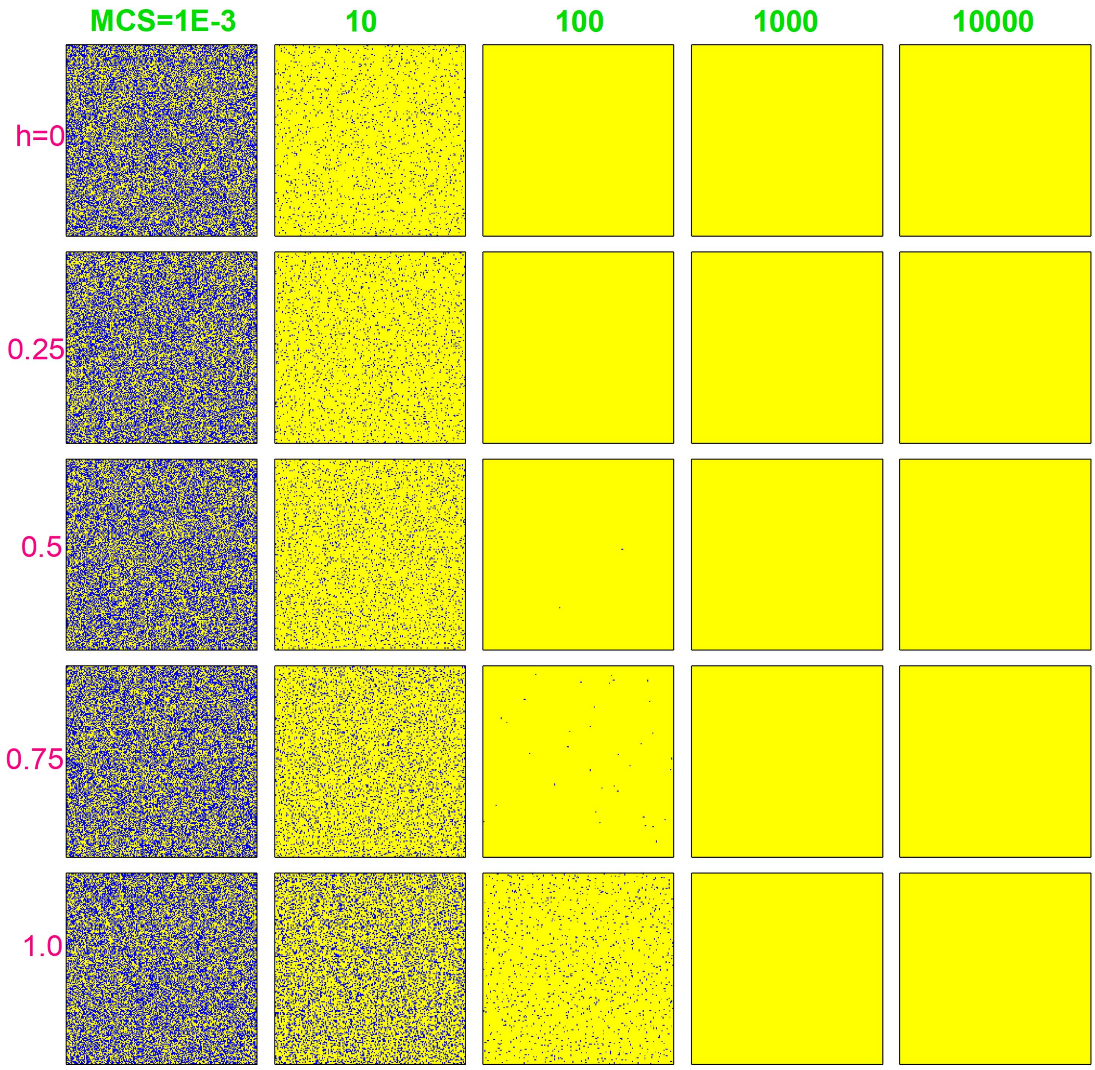

Figure 4. Characteristic patterns regarding the distribution of cooperators and defectors on the upper panels for different information sharing probabilities, where the snowdrift game is played by all individuals. In the simulations, two lattices have the same size $L \times L = 200 \times 200$ and the amplitude of irrationality of strategy adoption $K = 0.1$. From top to bottom, the information sharing probability between corresponding players is set to be $h = 0, 0.25, 0.5, 0.75$ and 1.0, respectively. Meanwhile, at each row of panels, we depict the distribution of cooperators and defectors at different time step $\mathrm{MCS} = 1E - 3$, 10, 100, 100, 1000 and 10 000. For all panels, the yellow (gray) dots represent the cooperators and blue (dark) ones denote the defectors.

Secondly, in figure 3 we record the fraction of cooperators at each time step [$f_C(t)$] at $r = 0.025$ to illustrate the origin of different cooperation behaviors. Likewise, the upper sub-figure (panel (a)) depicts the behavior of SDG game evolving, and the bottom panel (panel (b)) gives out the dynamical process of evolution of cooperation for the PDG game. In order to better scrutinize this process, here we divide each time step into 1000 mini-time steps so that the dynamical process can be clearly observed. Since $r = 0.025$ is smaller, and the final fraction of cooperators in the SDG (upper panel) arrives at the same total cooperation, but the dynamical process to attain the complete cooperation becomes slower when the information sharing probability h becomes larger. As an example, for $h = 0$ it takes about 30 MCS steps to reach the stationary and total cooperation state, but this steady state can only be arrived at after around 300 time steps when $h = 1.0$. Nevertheless, for the PDG game, it is found that the different stationary states can be obtained for different h, and the higher the information sharing probability h, the larger the stationary fraction of cooperators f_C. Being worthy of noting that, in the first several time steps (MCS $\leqslant$ 10), players cannot resist the temptation of being defected and the fraction of cooperators continuously shrinks. After that, without any information sharing, it can be noticed that the cooperation cannot persist and all individuals will choose to defect in the end, which means that the spatial reciprocity is not enough to support the evolution of cooperation. As the information sharing is introduced here, the player in the bottom layer can refer to the state of corresponding partner in the upper layer where more than half of agents are cooperators, and thus it facilitates the cooperator to hold the current strategy and the defector to switch to the opposite strategy. On the contrary, the player on the upper layer may face the opposite scenario, but the final state cannot be affected since r is too small and not enough to change the eventual fate for the SDG.

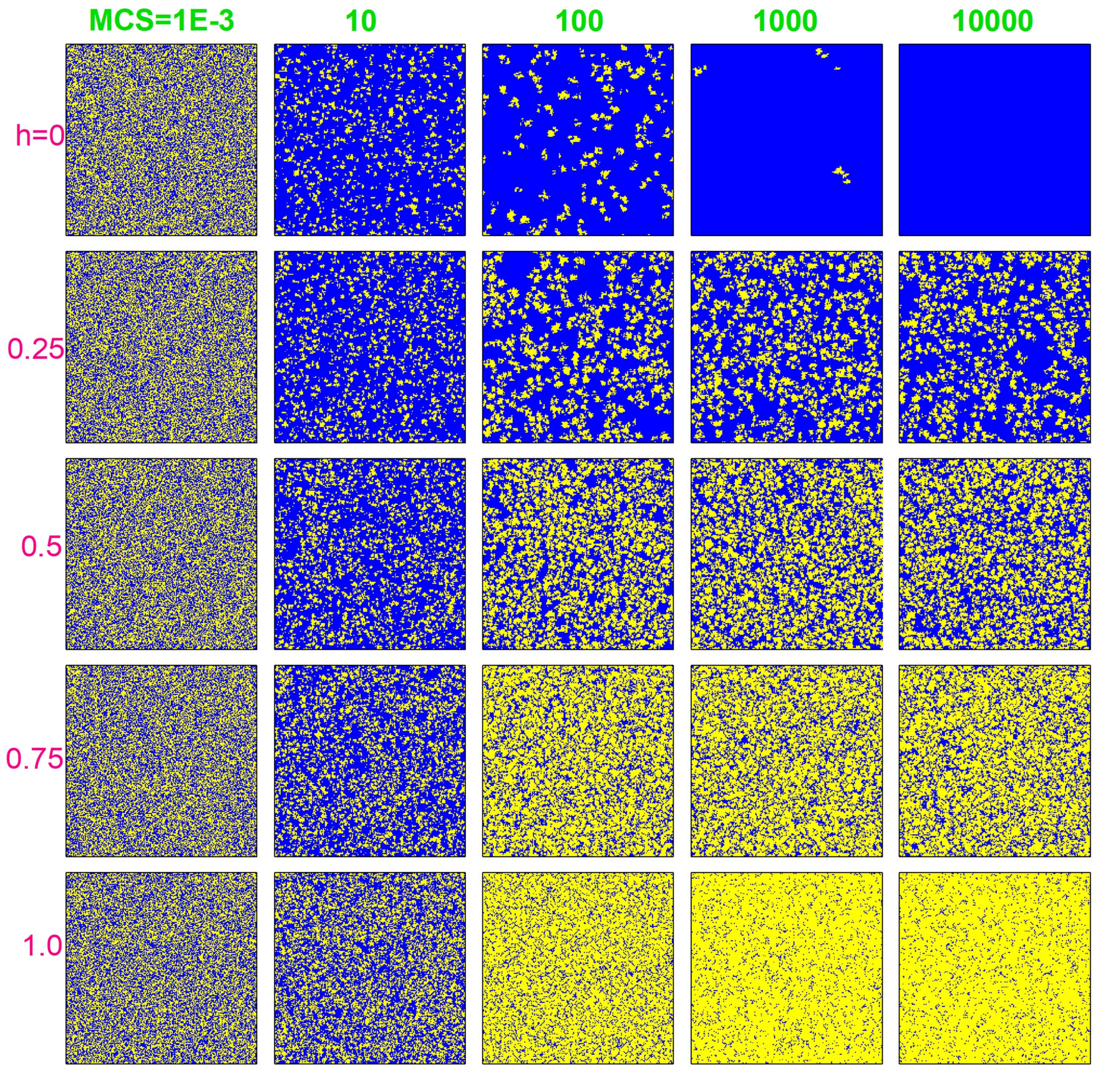

Figure 5. Characteristic patterns regarding the distribution of cooperators and defectors on the lower panels for different information sharing probabilities, where the prisoner's dilemma game is played by all individuals. In the simulations, two lattices have the same size $L \times L = 200 \times 200$ and the amplitude of irrationality of strategy adoption $K = 0.1$. From top to bottom, the information sharing probability between corresponding players is set to be $h = 0, 0.25, 0.5, 0.75$ and 1.0, respectively. Meanwhile, at each row of panels, we depict the distribution of cooperators and defectors at different time step MCS $= 1E - 3$, 10, 100, 100, 1000 and 10 000. For all panels, the yellow (gray) dots represent the cooperators and blue (dark) ones denote the defectors.

Meanwhile, figures 4 and 5 depict the characteristic snapshots at different time steps for several typical information sharing probabilities, which are used to further characterize the evolutionary process regarding the distribution of cooperators and defectors on two-layered lattices. It can be observed in figure 4 that the evolution of strategy distribution on the upper network is almost identical when $h \leqslant 0.5$, and the cooperative clusters can only be invaded before 1000 time steps only if $h > 0.5$, where the SDG model is adopted and the parameter r is still set to be 0.025. After that, cooperators can finally organize into an effective giant cluster to defend the defectors and then achieve a full cooperation scenario, that is, the stationary state of collective cooperation on the upper network is not changed for this setup. However, the evolutionary behavior can be greatly modified for the individuals on the lower network, which can can be seen from figure 5. A particular note is that the PDG is played here. Initially, the cooperators cannot resist the invasion of defectors and thus the cooperative clusters shrink little by little; if there is no information sharing, the defecting individuals will eventually dominate the population and even the full defection will be arrived at; But this situation will be surprisingly altered provided that the information sharing or state reference exists between two networks, and the cooperative clusters will gradually survive over the seabed of defectors at the steady state. It is worthy of being noted that the cooperators even hold the advantage even when the information sharing probability h is less than 0.5. Again, the information sharing will enhance the cooperation behavior between the interdependent networked population, especially for the population playing the PDG; while the population playing the SDG may suffer from this type of information sharing if $h > 0.5$. That is, the doubly effects of information sharing may exist between two interdependent populations playing different games.

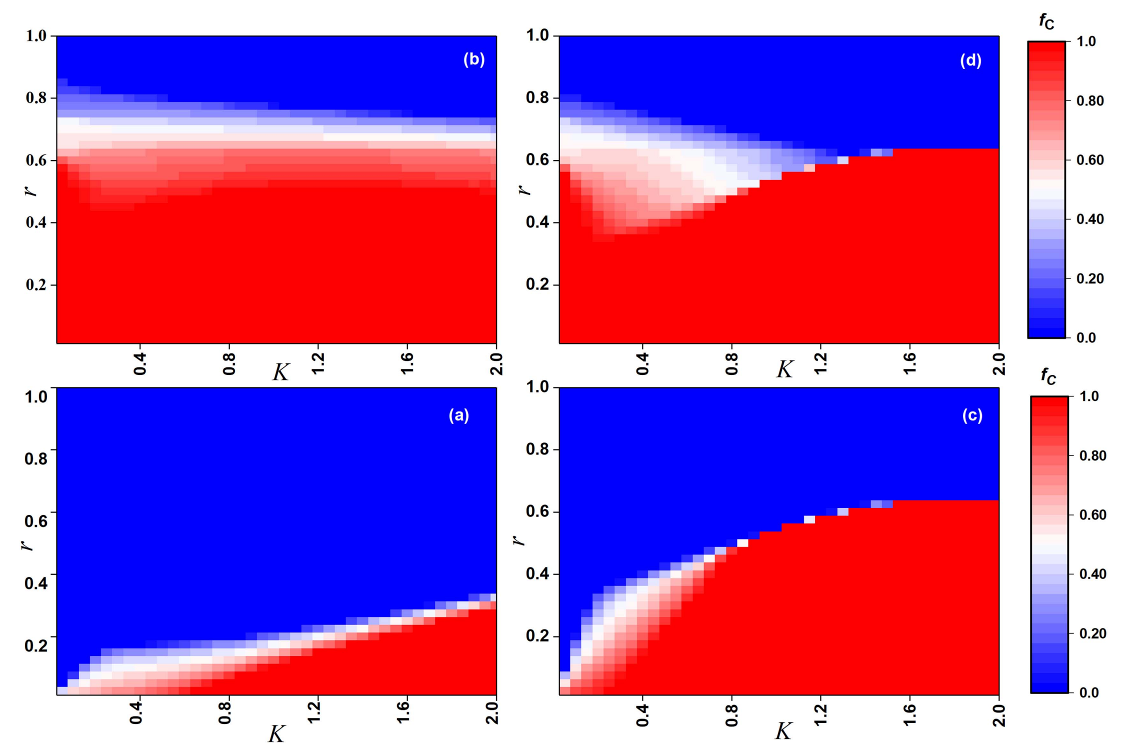

Figure 6. Fraction of cooperators as a function of ratio of cost-to-benefit (r) and individual irrationality (K) on interdependent lattices at the stationary state. The information sharing probability between two corresponding players is set to be $h = 0.25$ in the left two panels and 0.75 in the right two panels, and the top/bottom panels depicts the fraction of cooperators playing SDG/PDG on the UP/DOWN networks. Meanwhile, two lattices have the same size $L \times L = 200 \times 200$ and MCS = 50 000.

Thirdly, in order to more completely check the impact of individual irrationality decision on the cooperation, we plot the fraction of cooperators (f_C) at the stationary state as a function of r and K in figure 6, where f_C is characterized with the color value inside each panel. In the left two panels (panel (a) and (b)), the information sharing probability h is set to be 0.25, while the right two panels (panel (c) and (d)) depict the cases with $h = 0.75$; meanwhile, the top panels plot the fraction of cooperators for the SDG on the upper networks, and the bottom panels give out the cooperator's fraction of PDG on the lower networks. On the one hand, the results can clearly indicate that the cooperation behavior regarding the PDG can be promoted at the cost of being slightly reduced cooperation for the SDG; it is particularly worth mentioning that, at a smaller specific ratio of cost-to-benefit ($r \leqslant 0.3$), the increasing irrationality K further fosters the cooperation within the population playing the PDG on the DOWN network, but the level of cooperation can be maintained unchangeably on the UP one; on the other hand, the increasing information sharing will promote the fraction of cooperators of PDG-played population for the DOWN network at the stationary state into a higher level without influencing the cooperation behavior SDG-played population on the UP one, especially for the larger irrationality (e.g., $K \leqslant 1.6$). Taking together, the cooperation regions for the PDG have been greatly extended under the current framework, while the cooperative areas have not been varied for the SDG. Additionally, two different games have been played on the two-layered lattices and the information sharing is allowed between the corresponding players on these two sub-networks.

Finally, in previous simulations each player on the upper network is strictly paired in order with the corresponding one on the lower network. In figure 7, we randomly match two individuals between two lattices at the initial time step and then keep this matching relationship unchanged during the following steps, and other evolutionary procedures are qualitatively identical with those in our original model. It can be observed once again that this kind of information sharing can still promote the cooperation behavior of prisoner' dilemma game in the lower lattices, and at the same time the cooperative behaviors in the upper one cannot be destroyed greatly. The results indicate that the coupling effect between two types of games will be greatly conducive to the evolution of cooperation of one kind of game with marginally sacrificing the cooperator's ratio for the other class of game, which demonstrates the doubly effects of information sharing during the evolution of cooperation between two different games within the interdependent networked populations. The current results may also provide some hints to boost the collective cooperation with a lower cost in practice.

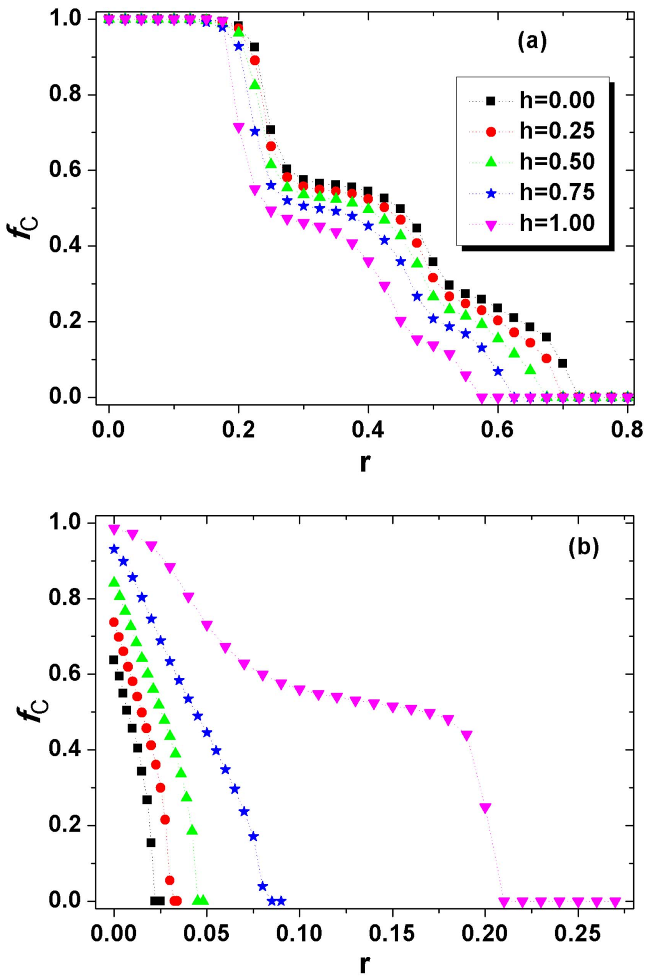

Figure 7. Fraction of cooperators at the stationary state among the population as a function of ratio of cost-to-benefit (r). Here, players between two networks are paired randomly at first and then kept unchanged. Two lattices have the same size $L \times L = 200 \times 200$, MCS $= 50\,000$, and the amplitude of irrationality of strategy adoption $K = 0.1$.

4. Conclusions

In summary, we propose an evolutionary game model on two-layered interdependent networks to explore the evolution of cooperation, in which the interdependency can be implemented through the strategy state sharing. Different from [64], the information sharing is incomplete during the strategy state reference since the individual perception ability may be limited. It is of particular concerns that two different games are played on interdependent lattices, respectively; that is, players perform the traditional SDG on one network (UP), while the PDG is played among the population on the other network (DOWN). After collecting the payoffs through the interactions with its nearest neighbors on the same network, each player will update its current strategy, which will partially refer to the strategy state of his corresponding partner on the opposite network, according to the Fermi-like probability. The numerical simulation results demonstrate that the cooperation of PDG on DOWN network will be promoted due to the incomplete state sharing, but the cooperative behaviors may keep unchanged when the ratio of cost-to-benefit r is smaller ($r \leqslant 0.2$), or be impaired a little for a larger r ($r > 0.2$); and these simulations indicate that doubly effects have been exhibited here. Additionally, this kind of doubly effects can be observed for two different mapping relationships (including one-to-one mapping and random pairing) between players on these two interdependent layers at the initial setup. The current results are of high importance for us to devise some effective mechanisms to enhance the level of collective cooperation within some real-world networking systems.

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