

**PAPER**

Assortativity provides a narrow margin for enhanced cooperation on multilayer networks

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Abstract

Research at the interface of statistical physics, evolutionary game theory, and network science has in the past two decades significantly improved our understanding of cooperation in structured populations. We know that networks with broad-scale degree distributions favor the emergence of robust cooperative clusters, and that temporal networks might preclude defectors to exploit cooperators, provided the later can sever their bad ties soon enough. In recent years, however, research has shifted from single and isolated networks to multilayer and interdependent networks. This has revealed new paths to cooperation, but also opened up new questions that remain to be answered. We here study how assortativity in connections between two different network layers affects public cooperation. The connections between the two layers determine to what extent payoffs in one network influence the payoffs in the other network. We show that assortative linking between the layers—connecting hubs of one network with the hubs in the other—does enhance cooperation under adverse conditions, but does so with a relatively modest margin in comparison to random matching or disassortative matching between the two layers. We also confirm previous results, showing that the bias in the payoffs in terms of contributions from different layers can help public cooperation to prevail, and in fact more so than the assortativity between layers. These results are robust to variations in the network structure and average degree, and they can be explained well by the distribution of strategies across the networks and by the suppression of individual success levels that is due to the payoff interdependence.

1. Introduction

Evolutionary games have a long history of research that is linked to the importance of network structure on which the evolution takes place [1–6], with breakthrough discoveries still being made [7]. The impetus for this fascinating development was the discovery of network reciprocity [8], which stands for the fact that a limited interaction range, as dictated by lattices or other types of networks, facilitates the formation of compact clusters of cooperators that are in this way protected against invading defectors. And this even if the temptation to defect is strong. The interest of the physics community was further spurred on by the discovery that scale-free networks provide a unifying framework for the evolution of cooperation [9], and over the past almost two decades, methods of physics have been applied to many contemporary societal challenges [10]. Examples include traffic [11], crime [12], epidemic processes [13], vaccination [14, 15], cooperation [16], climate inaction [17], as well as antibiotic overuse [18] and moral behavior [19], to name just some examples.

While the bulk of research concerning evolutionary games on networks has been devoted to single and isolated networks, such as small-world [20–26], scale-free [27–41], coevolving [42–50], hierarchical [51, 52] and community [53] networks, the focus has in recent years been shifting towards multilayer and interdependent networks [54–68]. This research has revealed interdependent network reciprocity, probabilistic

interconnectedness, and information transmission across network layers as potent facilitators of cooperation in social dilemmas. Apart from cooperation, many other process have been considered and studied, including cascading failures [69–72], competitive percolation [73–75], diffusion [76], epidemic spreading [77], and abrupt transition in structural formation [78], and multilayer networks have indeed become a strong new paradigm of network science [79–81].

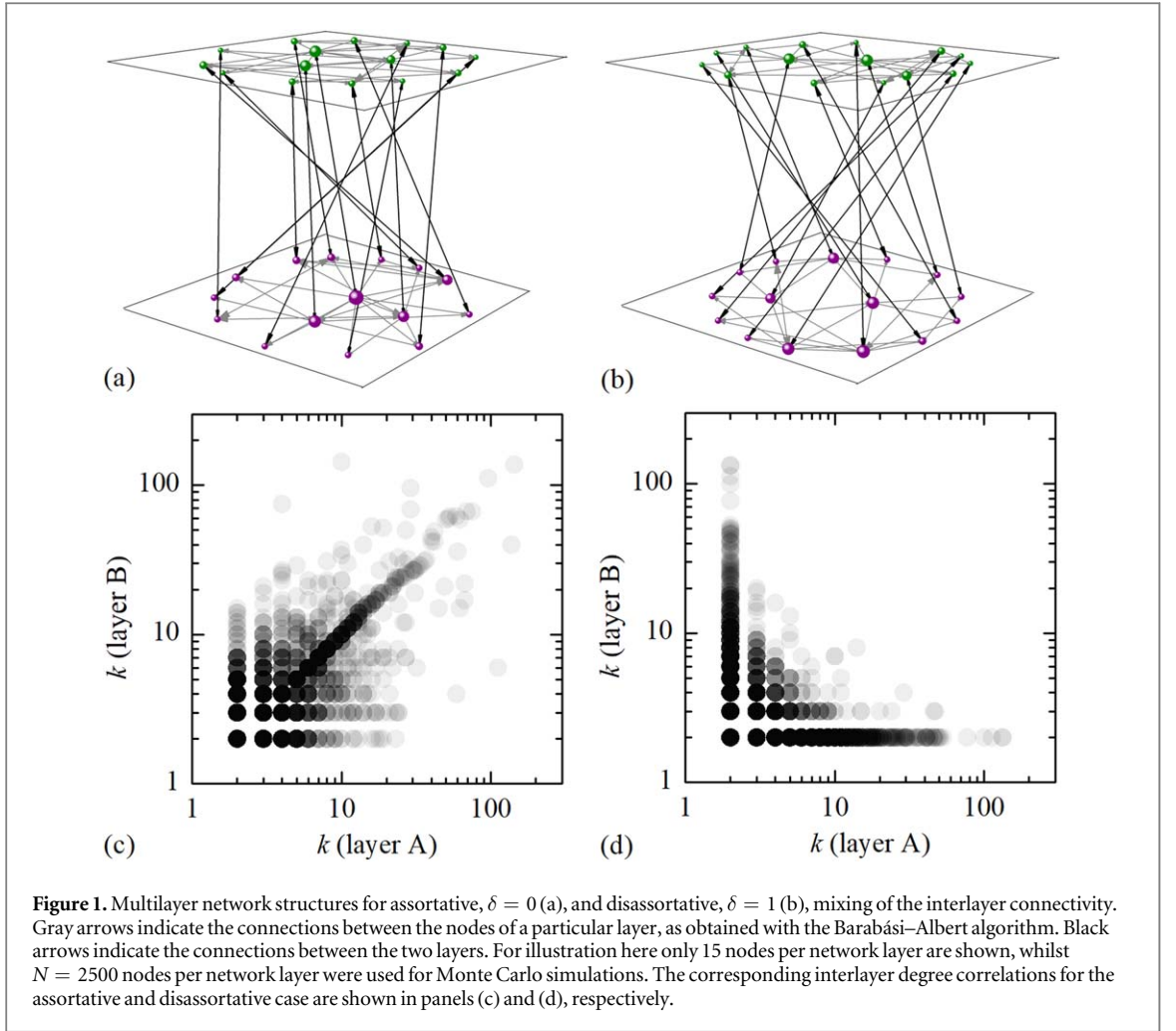
Here we address an open problem in the evolution of cooperation in multilayer networks, namely the role of assortative and disassortative matching between the layers. Focusing on human societies for a moment, it is for example easy to appreciate that leaders from one network will want to link with the leaders of the other network, hence giving rise to assortative matching. Imagining reasons for disassortative matching is a bit more challenging, but it may apply to less intelligent biological systems, where the cognition to consciously choosing partners from the other network is simply not there. Either way, it is of interest to determine how the two different types of matching between two network layers affect public cooperation. To study this we use the well-known public goods game [16, 82], where cooperators contribute to the common pool, defectors do not, and lastly all enjoy equal benefits regardless of their contribution. This is evidently a social dilemma, where individual best interests are at odds with the best interests of the group and the population as a whole. Knowing what works best for public cooperation to be maintained is therefore crucial for avoiding a tragedy of the commons [83]. As we will show, assortative matching between the two layers does enhance cooperation, but only with a relatively modest margin in comparison to random or disassortative matching. This is true regardless of the network degree and structural properties in the layers, and it can be explained well by looking at the distribution of strategies and at the individual success levels of different groups of players in dependence on the level of payoff interdependence.

In what follows, we first present the public goods game and the procedure for the construction of assortatively and disassortatively linked networks. We then continue with the main results, and finally we discuss their implications and outline open problems in this line of research.

2. Mathematical model

We study the public goods game on two interconnected network layers, A and B, where each node within a layer is occupied by one player. In most of our calculations we use the Barabási–Albert scale-free network model [84] with an average degree 4 (or in some calculations 8) for each individual layer, so the degree distribution is a power law $p(k) \sim k^{-3}$. Alternatively, we also use the growing and adjacent random (non-preferential) attachment model to study the public goods game on less heterogeneous network layers, where the degree distribution is exponential $P(k) \sim e^{-k/\langle k \rangle}$ [85]. However, players that occupy nodes in different network layers interact not only with their neighbors within the same layer, but also with players in the other network layer. In particular, the interdependence between both layers is introduced by means of an interdependent utility function, as explained in more detail in what follows.

We are specifically interested in how the nature of connections between network layers affects the evolution of public cooperation. To that effect, we propose a new model for connections between the layers that enables a systematic realization of different degree mixing patterns that interpolate smoothly between assortative and disassortative matching between players on different network layers. Firstly, nodes in both layers are ranked in accordance with their degrees. Then, each node within the first layer A is connected with an interlayer link to a node in the other layer B with the same rank, thereby leading to a completely assortative structure of the links between both networks. To obtain a disassortative interlayer mixing by degree, we introduce a rewiring probability δ that determines the likelihood that the i th node in layer A with rank degree $RD_i(A)$ will get disconnected from the i th node in layer B with the same rank $RD_i(B)$, and instead will become connected with the j th node in layer B with rank $RD_j(B) = N - RD_i(B)$. And simultaneously, that the j th node in layer A with rank $RD_j(A) = N - RD_i(A)$ will get disconnected from the j th node in layer B with rank $RD_j(B)$, and will then become connected with the i th node in layer B. Accordingly, if $\delta = 1$, each interlayer connection is rewired so that the nodes in layer A with the highest rank will be connected with the nodes in layer B with the lowest rank, which represents a maximally disassortative interlayer mixing pattern. By varying the parameter δ between 0 and 1, various degrees of assortative/disassortative mixing in the interlayer connections are thus attained, with $\delta = 0.5$ corresponding to a random, i.e. non-correlated, interlayer connectivity. In figures 1(a) and (b) we present typical two-layer networks with an assortative and disassortative mixing pattern between the two layers. The latter are quantified in figures 1(c) and (d), where the corresponding node degrees in layer A are plotted against their interlayer neighbor degrees in layer B. Evidently, for $\delta = 0$ a clear tendency of hubs connecting with other hubs and less connected nodes connecting with less connected nodes between the two layers can be observed. For $\delta = 1$, on the other hand, an opposite correlation can be observed, such that connections between the two layers exist primarily among high degree and low degree nodes.



In terms of the governing public goods game, initially each player in layers A and B is assigned either as a cooperator or defector with equal probability. The accumulation of payoffs P_x and P'_x on both networks follows the same procedure, according to the rules of the public goods game. Namely, each individual x participates in $g = k_x + 1$ groups, where k_x is the number of its direct neighbors in the same network layer (the degree of individual player). In each group the player will contribute 1 to each instance of the game if it adopts the cooperation strategy ($s_x = 1$), while defectors contribute nothing ($s_x = 0$). Subsequently, the sum of contributions is multiplied by the factor R that is greater than 1.0 and reflects synergistic effects of cooperation. The resulting amount is then divided equally among all players in the group. Thus, the payoff of a player x in every group g obtained on the network layer A is

$$P_x^g = \frac{RN_C^g}{k_x + 1} - s_x, \quad (1)$$

where N_C^g is the number of cooperators in the group g . Likewise, the payoff of a player x' in every group g' obtained on the network layer B is $P_{x'}^{g'}$. Accordingly, the total payoff received in all the groups can be calculated as $P_x = \sum_g P_x^g$ and $P_{x'} = \sum_{g'} P_{x'}^{g'}$. Importantly, different individuals have a different number of direct neighbors. Therefore, to ensure a relevant comparison of the results, the multiplication factor R is normalized with the size of the corresponding groups [35].

As we have noted above, there exist connections between both layers. Each player x in the network layer A has exactly one external partner x' in the network layer B, and vice versa. The interdependence between the network layers A and B is introduced via the utility function

$$U_x = \alpha P_x + (1 - \alpha) P_{x'}, \quad U_{x'} = (1 - \alpha) P_{x'} + \alpha P_x, \quad (2)$$

where α determines the bias in the consideration of payoffs collected by the players x in the network layer A and x' in the network layer B. Accordingly, at low α values player x is guided dominantly by the payoff of its external partner x' , while the player x' is only slightly influenced by the payoffs of the player x . For $\alpha = 0.5$ both P_x and $P_{x'}$ are taken into consideration equally strongly by both players x and x' , and the evolution on both network layers is in this case virtually identical (lest the differences that emerge due to the differences in the initial distributions

of strategies and in the realization of network structure for each layer). For $\alpha > 0.5$ the roles are exchanged and the treatment is fully symmetric with respect to $\alpha < 0.5$. Evidently, at $\alpha = 1$ ($\alpha = 0$) the game on network A (B) behaves equally as if played on a single and isolated network, while the game on network B (A) is governed exclusively by the payoffs of players in network A (B) [54].

We use the established Monte Carlo method to simulate the evolutionary processes on the multilayer network. The first step involves randomly selecting one player x , one of its neighbors y on the layer A, and the corresponding external partners x' and y' on layer B. Following the accumulation of payoffs P_x and P_y on layer A and payoffs $P_{x'}$ and $P_{y'}$ on layer B as described above, the corresponding utility functions as per equation (2) can be calculated. Finally, player y compares its payoff to that of player x , and adopts the strategy of player x with a probability determined by the Fermi function

$$W(s_x \rightarrow s_y) = \frac{1}{1 + \exp[(U_y - U_x)/K]}, \quad (3)$$

where K denotes the uncertainty by strategy adoptions. As is standard practice, we use $K = 0.5$ without loss of generality [16, 86]. Importantly, strategy invasions are possible from nearest neighbors on a given network layer only. Accordingly, the player x' adopts the strategy from player y' with a probability determined likewise, only that utilities U'_x and U'_y are used in equation (3).

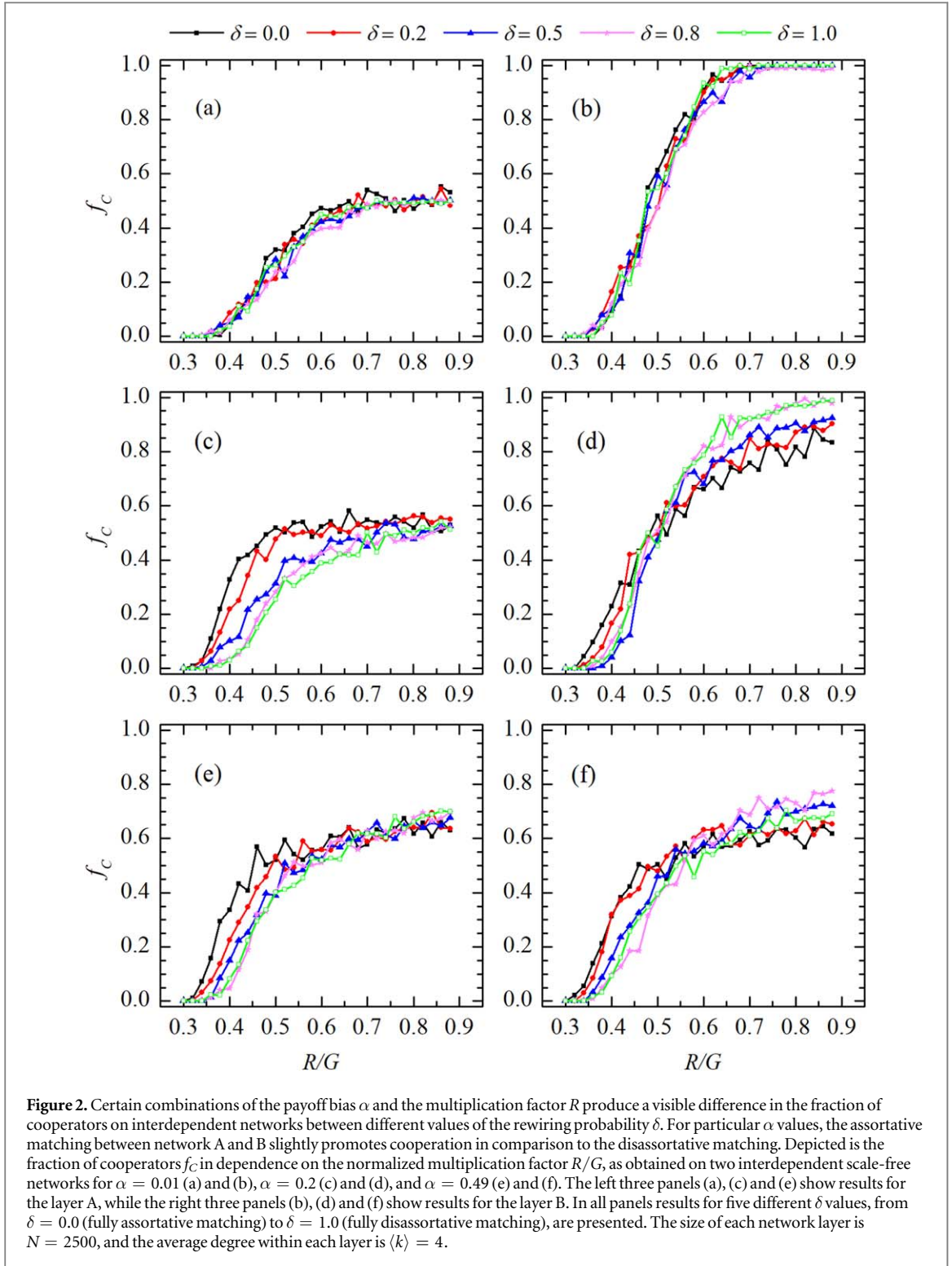
The simulation results presented below have been obtained on two network layers, each with $N = 2500$ nodes. During one full Monte Carlo step we repeat the described elementary steps $2N$ -times, such that every player in the two network layers has a chance to change its strategy once on average. The fraction of cooperators (f_C) has been determined as the average within the last 10^4 out of the total of 10^5 Monte Carlo steps, when the system reaches a steady state. For each set of parameter values this procedure was repeated with 10 different initial conditions. Since the degree distributions of players within each layer are scale-free and thus strongly heterogeneous, this might introduce additional uncertainties. Therefore, for the final results, we have additionally averaged the outcome over 20 different network realizations.

3. Results

We begin by showing the fraction of cooperators in dependence on the group size normalized value of the multiplication factor R , as obtained for different combinations of the payoff bias α and the assortativity parameter δ . Panels in figure 2 show the results for layers A and B on the left and right, respectively. It can be observed that different δ values hardly evoke a visible difference in the fraction of cooperators. Only on the A layer for $\alpha = 0.2$, where thus the evolution is by 80% determined by the payoffs of players on layer B and only by the remaining 20% by the actual payoffs obtained on layer A, is a difference quite clearly visible. There, at best a 30% margin in the fraction of cooperators between the fully assortative ($\delta = 0$) and the fully disassortative matching ($\delta = 1$) can be observed around $R \approx 0.4$. Notably, exactly the same results can be observed on the B layer for $\alpha = 0.8$, given the symmetry in the payoff functions between the two network layers (see equation (2)). As the value of R increases, the margin for enhanced cooperation gradually vanishes, and by $R \approx 0.8$ it is again hardly visible. No other combinations of α and δ produce a statistically more significant difference, and only for $\alpha = 0.49$, in the bottom row, the assortative matching between the two layers seems to also confer a very slight evolutionary advantage to cooperators in comparison to the disassortative matching.

Looking at these results from the perspective of the payoff bias α , as presented in figure 3, it can be observed that the bias itself can enhance cooperation more markedly, and this regardless of the level of assortativity between the two network layers. Looking at the fraction of cooperators on the right side for layer B, there is a consistent gap of almost 50% between $\alpha = 0.01$ and $\alpha = 0.8$ for sufficiently large R values (above $R \approx 0.6$). This is true as much for $\delta = 0$, where the matching is assortative such that hubs of one network are linked with the hubs in the other network, as it is for $\delta = 1$, where the matching is disassortative such that the hubs of one network are linked with the low-degree hubs in the other networks. As in figure 2, the same results would be observed for layer A provided we would start with $\alpha = 0.99$ and then decrease this value. Indeed, the reverse trend can be clearly observed in panels on the left side of figure 3 that show the fraction of cooperators on layer A. It is also worth pointing out that the comparatively marginal benefits of assortativity for the evolution of cooperation on layer A can still be observed, but one has to focus on the $\alpha = 0.2$ line (red with circles), and then compare the $\delta = 0$ and the $\delta = 1$ case at $R \approx 0.4$.

To corroborate the robustness of these results, we show in figure 4 the fraction of cooperators f_C in dependence on the normalized multiplication factor R/G , as obtained for different values of the rewiring probability δ at a fixed value of the payoff bias $\alpha = 0.2$. We remind that at the latter value of α the difference between assortative and disassortative linking between two scale-free network layers in promoting cooperation was found to be the largest (see figure 2). For comparison, we show in figure 4 again the results obtained on two interdependent scale-free network layers with an average degree $\langle k \rangle = 4$ in panels (a) and (b), while in panels (c)



and (d) we show the same results obtained on two scale-free network layers with an average degree $\langle k \rangle = 8$, and in panels (e) and (f) we show these results obtained on two networks with an exponential degree distribution. It can be observed that, regardless of the variations in the network structure and average degree, the promotion of cooperation that is due to the nature of the linking between the two layers is rather marginal (compare $\delta = 0$ and $\delta = 1$ as the two extremes), albeit visible especially on the layer A (left three panels). Notably, due to the symmetry of the payoff interdependence, the same results would be obtained on the B layer at $\alpha = 0.8$.

Given the robustness of the presented results, it remains of interest to explain why assortativity provides only a narrow margin for enhanced cooperation on multilayer networks. To that effect we show in figure 5 the distribution of the two competing strategies across the network nodes. We focus on the layer A at the payoff bias $\alpha = 0.2$, which would be the same as network layer B at the payoff bias $\alpha = 0.8$. Comparing first the

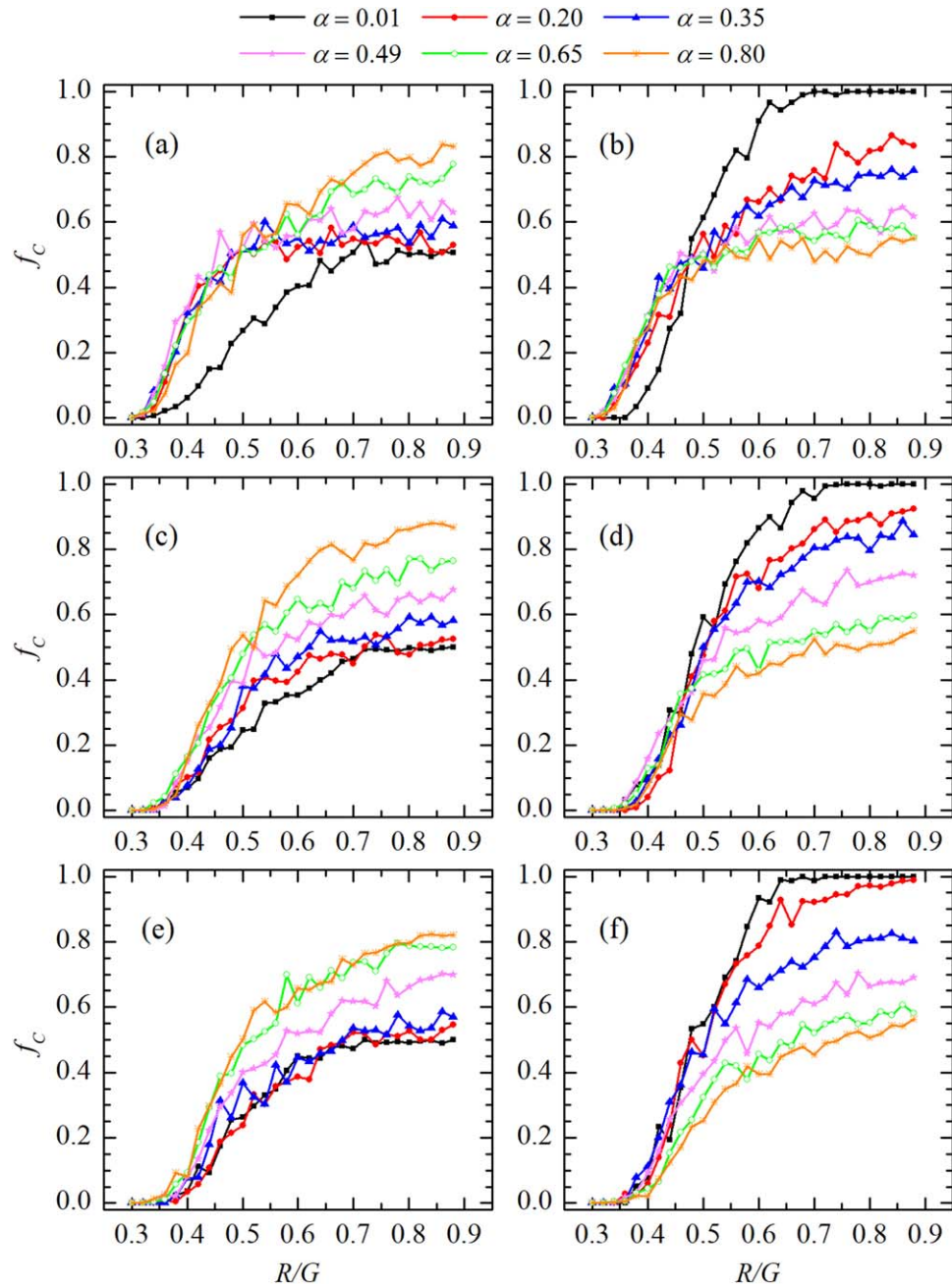


Figure 3. The evolution of public cooperation on the two network layers depends significantly on the value of the bias α . The stronger the bias, the higher the level of cooperation regardless of the nature of connections between the layers. All six panels present the fraction of cooperators f_c in dependence on the normalized multiplication factor R/G , as obtained on two interdependent scale-free networks for three different rewiring probabilities: $\delta = 0.0$ (a) and (b), $\delta = 0.5$ (c) and (d), and $\delta = 1.0$ (e) and (f). The left three panels (a), (c) and (e) show results for the layer A, while the right three panels (b), (d) and (f) show results for the layer B. In all panels results for six different α values, from $\alpha = 0.01$ to $\alpha = 0.8$, are presented. The size of each network layer is $N = 2500$, and the average degree within each layer is $\langle k \rangle = 4$.

distributions obtained at $R/G = 0.42$, it can be observed that for $\delta = 0$ (a), where the matching is assortative such that hubs of one network are linked with the hubs in the other network, the hubs are only somewhat more commonly populated with cooperators (red), than for $\delta = 1$ (b), where the matching is disassortative such that the hubs of one network are linked with the low-degree hubs in the other networks. This is indeed the parameter region, where the difference is the largest. Going to $R/G = 0.8$ in panels (c) and (d), the difference is basically not there, which is also reflected in the results above. Thus, it can be argued that assortative matching between the layers somewhat strengthens the positions of the hubs to be occupied by cooperators, but indeed only marginally so.

We further support these observations quantitatively by determining the strategy change probabilities of different groups of players on the two network layers. As mentioned above, players in both layers are ranked in accordance with their degrees from $r = 0$ (the node with the highest degree) to $r = 2499$ (the node with the

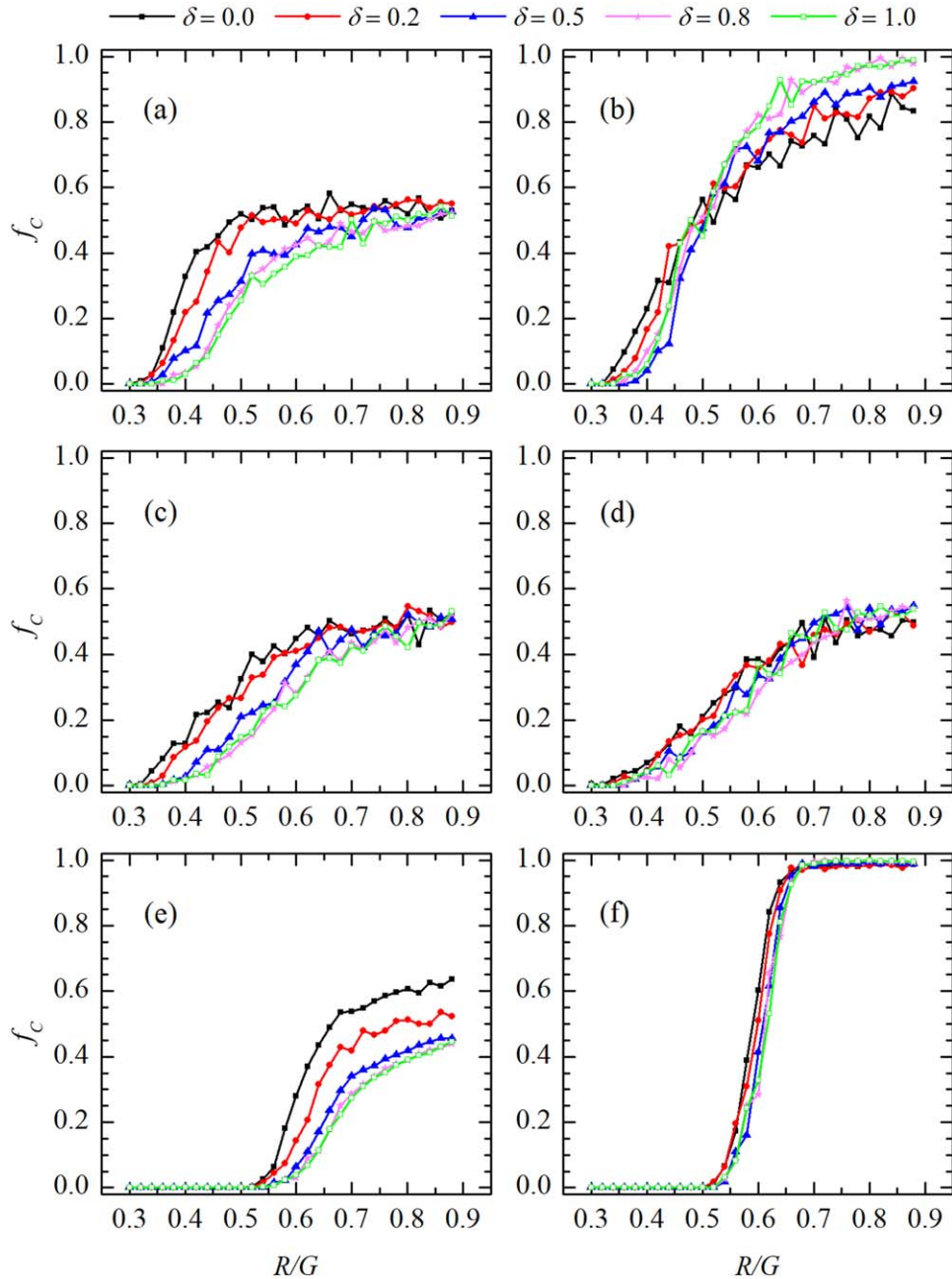
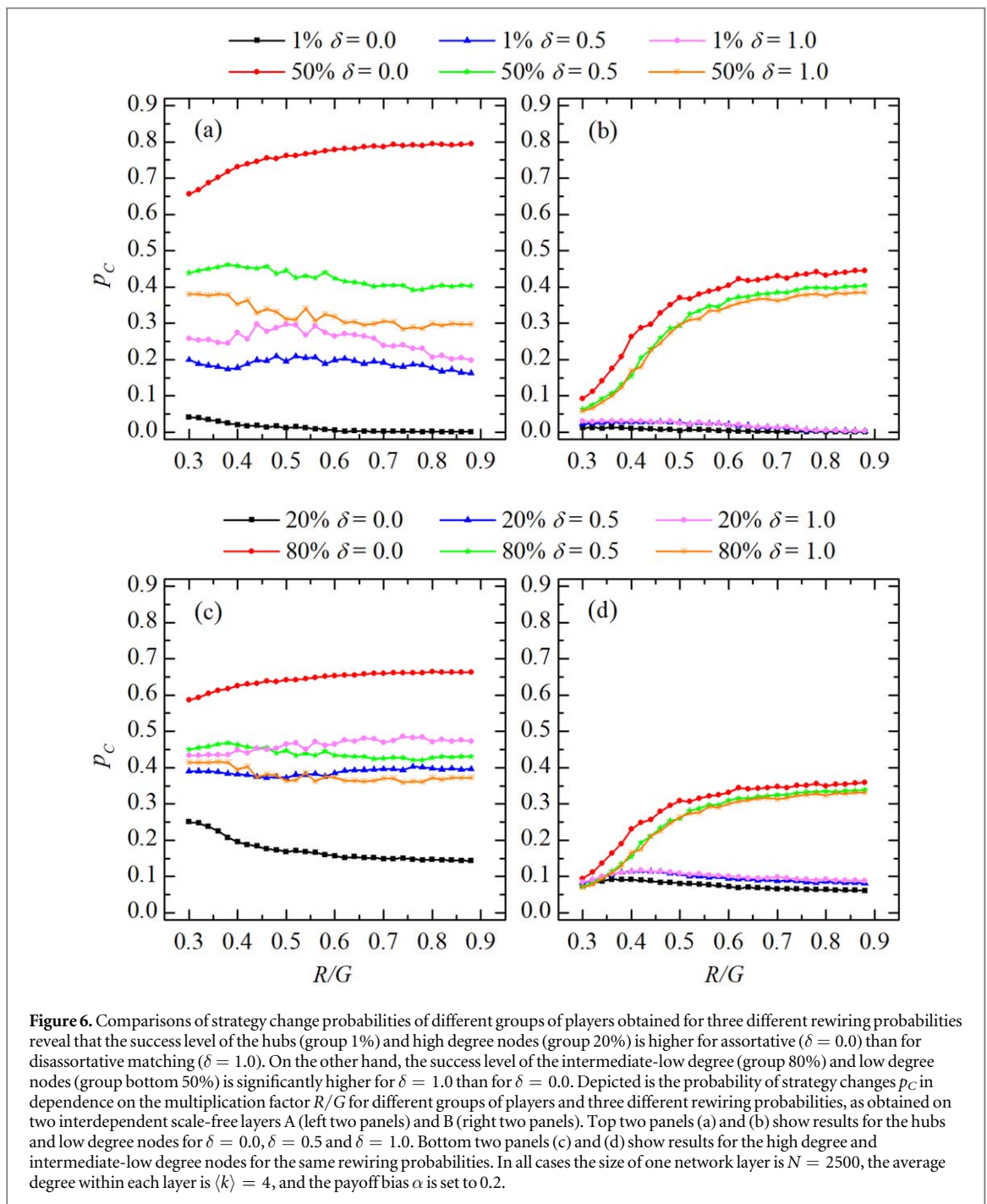
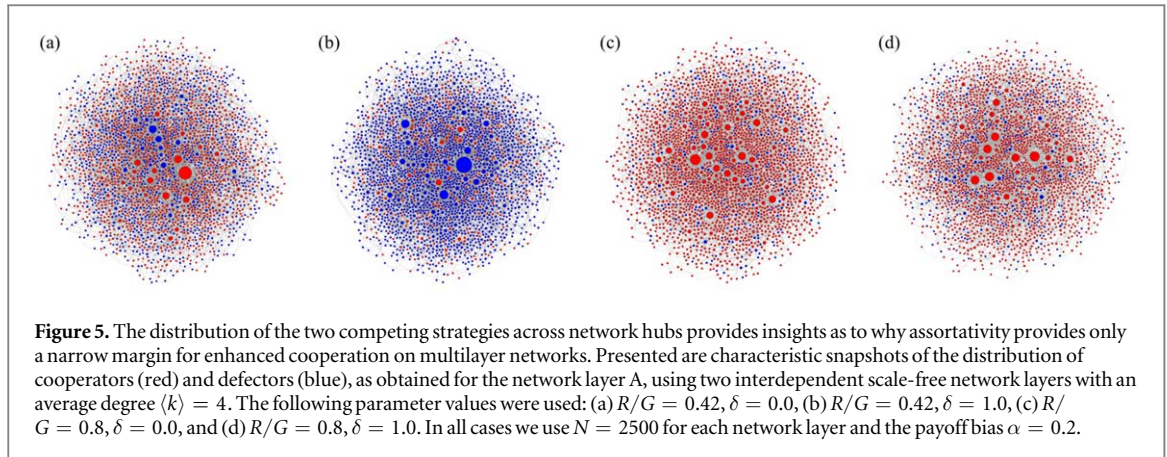


Figure 4. Assortative matching between network layers A and B slightly enhances cooperation under adverse conditions for different variations in the network structure and average degree $\langle k \rangle$. All six panels present the fraction of cooperators f_C in dependence on the normalized multiplication factor R/G , as obtained for different values of the rewiring probability δ . Panels (a) and (b) show results obtained on scale-free networks with an average degree $\langle k \rangle = 4$, panels (c) and (d) shows results obtained on scale-free networks with an average degree $\langle k \rangle = 8$, and panels (e) and (f) shows results obtained on networks with an exponential degree distribution. The left three panels (a), (c) and (e) show results for the layer A, while the right three panels (b), (d) and (f) for layer B. In all cases we use $N = 2500$ and the payoff bias $\alpha = 0.2$.

lowest degree). To quantify the suppression of individual success levels due to the payoff interdependence, the probability of strategy changes p_C was calculated considering separately four groups: the most highly connected nodes or hubs (1% of all N nodes where $0 \leq r < 25$), the high degree nodes (20% of all N nodes where $0 \leq r < 500$), intermediate-low degree nodes (80% of all N nodes where $500 \leq r < 2500$) and low degree nodes (50% of all N nodes where $1250 \leq r < 2500$).

The results presented in figure 6 show the comparison between the results obtained for three different rewiring probabilities and the payoff bias $\alpha = 0.2$. The top two panels, depict the probability of strategy changes of the hubs (group 1%) and low degree nodes (group bottom 50%) in dependence on the multiplication factor R for layer A (left panel) and layer B (right panel). Comparing first the probability of strategy changes of the hubs and low degree nodes on the layer A, it can be observed that the success level of the hubs is evidently higher for $\delta = 0.0$ where the matching is assortative, than for $\delta = 0.5$ and $\delta = 1.0$, where the matching is random and



disassortative, respectively. For $\delta = 0.0$, where the hubs of one network are linked with the hubs in the other network, the probability of strategy changes is nearly zero. An opposite correlation can be observed by comparing the probabilities of strategy changes of low degree nodes (group bottom 50%) where the success levels are significantly higher for the disassortative than for assortative matching. While these differences are clearly visible for all values of the multiplication factor R on layer A, the differences on layer B are growing as the value of R increases. Nevertheless, a relatively modest margin between different rewiring probabilities for each group of nodes can be observed in comparison to layer A, which is also reflected in the above-presented results concerning cooperation promotion.

The bottom two panels of figure 6 depict the probability of strategy changes of the high degree nodes (group 20%) and intermediate-low degree nodes (group 80%) in dependence on the multiplication factor R for layer A (left panel) and layer B (right panel). In the group of high degree nodes, in addition to the hubs also nodes with not the highest degrees are present, and subsequently the success levels in this group are smaller in comparison to the success levels in just the hubs depicted in figures 6(a) and (b). On the other hand, since the intermediate-low degree group (80%) consist of nodes with low and intermediate degree nodes, the success levels in this group are higher in comparison to the success level of low degree nodes (group bottom 50%). Comparing different rewiring probabilities for each group of nodes on layer A (figure 6(c)), benefits of assortative matching ($\delta = 0.0$) can still be observed even though they are less significant. Looking at the probability of strategy changes in the right panel for layer B (figure 6(d)), however, one can observe a relatively small difference between different rewiring probabilities for each group of nodes. Taken together, these results corroborate and explain well the results presented in figures 2–4, thus confirming that assortativity can provide only a narrow margin for enhanced cooperation on multilayer networks.

4. Discussion

While the effects of mixing patterns on the evolution of cooperation on single scale-free networks have been studied more than a decade ago in a seminal paper by Rong *et al* [29], the same problem has remained open for multilayer networks until now. Of course, while Rong *et al* focused on assortativity between the hubs and low degree nodes in the single network, we here focus on assortativity in links between the two network layers. While research in 2007 has shown that assortativity patterns play an important role on isolated networks, in particular that assortative mixing destroys the sustainability of cooperators and promotes the invasion of defectors because it increases the vulnerability of cooperator hubs because it links them together, and vice versa that disassortative mixing promotes cooperation because it fosters the isolation of hubs and makes it even less likely for defectors to be able to invade successfully, we here observe a rather more marginal role on multilayer networks. As emphasized already in the title of this paper, assortativity provides a narrow margin for enhanced cooperation. Indeed, bias in the influence of payoffs from the two layers has a stronger overall impact on cooperation, while assortativity has a noticeable impact only in a rather narrow window of the bias and the multiplication factor values.

It is worth pointing out that, unlike on isolated networks, on multilayer networks assortativity promotes cooperation because the payoffs are composed from both layers. Hence, when hubs are linked together, they reinforce themselves, without thereby giving defectors a larger chance to invade, given that strategy changes across the layers are not permitted. Only the payoffs in one layer determine the evolutionary success in the other layer, and vice versa. If we would allow also for strategy transfer, the setup would effectively become quite identical as on a single and isolated network, and undoubtedly we would observe the same results as Rong *et al* [29].

Our observations can be explained well by the distributions of the two strategies across a particular network layer, where we have observed that assortative matching between the layers somewhat strengthens the positions of cooperative hubs, but in comparison to disassortative matching between the layers the difference is, expectedly given the stationary fractions of cooperators, very modest. This was also supported quantitatively by determining the probability of strategy changes for different groups of nodes, in particular hubs, high degree, intermediate-low, and low degree nodes, as was also done for isolated scale-free networks back in the day [9, 28].

Since multilayer networks go a step further in accurately reflecting the reality of human interactions, it would be fascinating to try and confirm these theoretical results with human experiments. We hope this work will stimulate further research to help uncover optimal conditions for public cooperation and promote related research along the fascinating interface of physics and societies [87].

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