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Double resonance in cooperation induced by noise and network variation for an evolutionary prisoner's dilemma

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Abstract. We study effects of slowly varying small-world topology and additive spatiotemporal random variations, introduced to the payoffs of a spatial prisoner's dilemma game, on the evolution of cooperation. We show that there exists an optimal fraction of shortcut links, constituting the variable complex network of participating players of the game, for which noise-induced cooperation is resonantly enhanced, thus marking a double resonance phenomenon in the studied system. The double resonance is attributed to the time-dependence of the connectivity structure that induces a tendency towards the mean-field behaviour in the limit of random graphs. We argue that random payoff disturbances and complex network topology are two potent extrinsic factors able to boost cooperation, thus representing a viable escape hatch out of evolutionary stalemate.

Contents

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1. Introduction

The evolution of altruistic behaviour among selfish individuals in human and animal societies is an evergreen topic, fascinating evolutionists, biologists, mathematicians and physicists alike. Thereby, evolutionary game theory [1]–[3] presents a competent framework for studying the evolution of cooperation among selfish individuals. In particular the prisoner's dilemma [4] is one of the most commonly employed games for this purpose, originally consisting of two players who have to decide simultaneously whether they want to cooperate or defect [5]. While mutual cooperation yields the highest collective payoff, which is equally shared between the two players, individual defectors will do better if the opponent decides to cooperate. The two players reconcile the resulting dilemma so that they decide to defect, whereby both end up empty-handed instead of equally sharing the rewarding collective payoff received by mutual cooperation. However, since widespread cooperation is crucial for the prosperity of society and is therefore frequently encountered in real life [6]–[9], several mechanisms have been proposed to explain the emergence of cooperation in various types of games. Prominent examples include spatial extensions [10]–[20], direct and indirect reciprocity [21]–[24], and voluntary participation [25]–[28].

Recently, external stochastic influences have also emerged as being potent promoters of cooperation in the spatial prisoner's dilemma game. In particular, we showed that Gaussian payoff variations are able to boost cooperation in a resonant manner depending on the intensity of additive noise, thus indicating a classical coherence resonance scenario in the spatial prisoner's dilemma game [29], acknowledging the fact that enhanced cooperation is considered a constructive effect facilitating the overall welfare of the population. Previously, it has been discovered that stochastic influences play a vital part in evolutionary dynamics, affecting both the overall population gain [30] and equilibrium selection [31]–[33], or even the nature of phase transitions from one equilibrium towards the other [16].

In addition to external stochastic influences that usually enter the game under the assumption of environmental effects, small-world and random topologies of the spatial grid [34] have recently also been discovered as crucial factors affecting the dynamics of cooperation [35]–[40]. In particular, it has been argued [36] that small-world networks present an optimal environment for playing the prisoner's dilemma game due to a more rapid convergence to equilibrium states in comparison to regular lattices. Furthermore, Hauert and Szabó [38] have shown that less rigid neighbourhood structures of lattices maintain cooperation even for treason temptation values at which defection is the only strategy by a local spatial topology of players. Recently, a detailed study of effects of regular random graphs on the evolution of cooperation has also been performed by Vukov *et al* [40]. They showed that for sufficiently high noise levels, affecting the strategy adoption process of players on the spatial grid, the optimal topological environment is given by minimizing the number of loops in the connectivity structure. On the other hand, for low levels of noise the preferred structure is built up from randomly overlapping triangles that have only one common site [40].

Importantly, however, studies in the past have often focused explicitly on regular small-world and random graphs [38]–[40] that maintain not just the number of all links in the spatial grid but also the connectivity, i.e. number of connections, of each player. On one hand, the constant connectivity allows better interpretation of the results and the application of sophisticated analytical methods that provide insights into the effects of different network structures, while on the other hand, imposing restrictions that are unlikely to be fulfilled in real-life situations.

In particular, it is not difficult to imagine that some players, be it human individuals, firms or animals, interact with more neighbours or have more connections than others, thus essentially violating the requirement imposed by regular graphs. Moreover, it appears reasonable to assume that a player is not bound to its initially chosen co-players forever, but must be allowed to change its partners in time. In other words, exclusive fidelity and monogamy are rare, and thus should not be taken for granted when modelling real-life situations.

The aim of the present paper is therefore to study joint effects of extrinsic stochastic payoff variations as well as non-regular (the connectivity of each player is allowed to vary) slowly varying small-world topology of players on the evolution of cooperation in the spatial prisoner's dilemma game. We show that appropriately tuned payoff variations can revert the extinction of cooperators occurring by large defection temptation values. However, cooperators receive the biggest boost if, in conjunction with the appropriately adjusted noise level, also the rigidness of the regular spatial topology of players is optimally relaxed. Thereby, the equilibrium frequency of cooperators depends resonantly on both extrinsic parameters, thus marking a double resonance phenomenon in the studied system, or alternatively, a complex network enhanced coherence resonance [29]. Importantly, we note that by applying non-regular slowly varying small-world lattice configurations, we obtain qualitatively different results as reported previously for regular small-world and random networks [38, 40]. At present, the fraction of cooperators F_C depends resonantly on the fraction of rewired links, whilst in the case of regular small-world graphs F_C is of saturating nature when approaching the regular random graph limit. The discrepancy between the presently reported and previous results is attributed to the time-dependence of the connectivity structure that induces a tendency towards the mean-field behaviour of F_C in the limit when all links are rewired.

At the end, we point out interesting conceptual similarities between the presented results and noise-driven small-world-coupled spatially extended systems. Also, since uncertainties and long-range connections among distant players are common in everyday life, we suggest that both currently studied mechanisms are viable and potent promoters of cooperation.

2. Spatial prisoner's dilemma game

We consider an evolutionary two-strategy prisoner's dilemma game with players located on vertices of a two-dimensional square lattice of size $n \times n$ with periodic boundary conditions. Moreover, we introduce a probability p that an individual plays the game with a randomly chosen distant player instead of one of its four nearest neighbours. When the whole lattice is updated each player will have played eight games on average, irrespective of p. In the limit $p = 0$, the eight games per player per lattice update correspond to the fact that each player has four nearest neighbours plus it is itself a nearest neighbour four times to other players. Self-interactions are, however, always excluded. Importantly, by $0 < p \leqslant 1$ the connectivity of players is no longer constant, i.e. we consider a non-regular complex network of players. Thus, some players may interact more often than others in a given update of the spatial grid which, however, happens by chance and may affect cooperators or defectors with equal probability. Finally, we allow the connectivity to vary every 100 updates of the spatial grid, thus introducing a non-quenched complex network. Notably, the results below do not differ substantially if the connectivity of players is varied more or less often, as long as the refresh rate is fast in comparison with the total number of lattice updates. We will show that the presently applied non-regular slowly varying small-world topology affects the evolution of cooperation substantially differently than already thoroughly studied quenched regular small-world or random graphs [38]–[40].

The currently employed two-strategy prisoner's dilemma game comprises cooperators (C) and defectors (D), which are initially uniformly distributed on the square lattice. A player P_i can change its strategy after each full lattice update, whereby the performance of one randomly chosen nearest neighbour P_j is taken into account according to

$$W[P_i \leftarrow P_j] = \frac{1}{1 + \exp[(S_i - S_j)/K]}. \tag{1}$$

The cumulative payoffs of both players (S_i, S_j), acquired during each update of the spatial grid, are calculated in accordance with the payoff matrix [11]

P_i/P_j	C	D
C	$1+\xi_i/1+\xi_j$	$1+r+\xi_i/-r+\xi_j$
D	$-r+\xi_i/1+r+\xi_j$	$0+\xi_i/0+\xi_j$

(2)

In particular, two cooperators receive the reward $R = 1$, two defectors receive the punishment $P = 0$, while a cooperator and defector receive the sucker's payoff $S = -r$ and the temptation $T = 1 + r$, respectively, thus satisfying the prisoner's dilemma payoff ranking $T > R > P > S$ if the temptation to defect $r > 0$. The payoff matrix is subjected to temporally and spatially white additive Gaussian noise, satisfying the correlation function $\langle \xi_i(k)\xi_j(l)\rangle = \sigma^2\delta_{ij}\delta_{kl}$, whereby indexes ($i$, j) mark any of the two involved players, while k and l index two consecutive pair interactions. Moreover, σ^2 is the variance of payoff variations, whereas $K = 0.1$ in equation (1) is the uncertainty related to the strategy adoption process [33].

The studied spatial prisoner's dilemma game is iterated forward in time using a synchronous lattice update scheme [20, 41], thus letting all individual interact pairwise with their four nearest neighbours if $p = 0$ or with all their co-players as determined by the imposed complex network topology if $p > 0$. After every such lattice update, all players simultaneously update their strategy according to equation (1) and reset their cumulative payoffs to zero. For a large enough number of game iterations ($\geqslant 10^5$) and large system sizes ($n \geqslant 400$), the frequencies of cooperators F_C and defectors F_D approach an equilibrium value irrespective of the initial conditions and lattice topology realizations, provided long enough discard times are taken into account.

3. Results

By non-varying payoffs ($\sigma = 0$) and regular topology of players ($p = 0$) cooperators are able to survive only for small-enough values of r since then the risk of cooperation is low in comparison to possible losses. In particular, cooperators survive by forming clusters so as to protect themselves against being exploited by defectors. Thereby, cooperators located in the interior of such clusters enjoy the benefits of mutual cooperation and are therefore able to survive despite the constant exploitation by defectors along the cluster boundaries. However, as r exceeds a threshold value, equalling $r_{\mathrm{tr}} = 0.00634$ for the currently applied iteration scheme and player adoption rule, cooperators die out, whereby the transition from the mixed to the homogenous state pertains to the directed percolation universality class [42]–[44] since $F_C \propto (r_{\mathrm{tr}} - r)^{\beta_r}$ by $\beta_r \approx 0.58$. This is a well-known result for the current and similar game settings reported recently in [16, 29, 33].

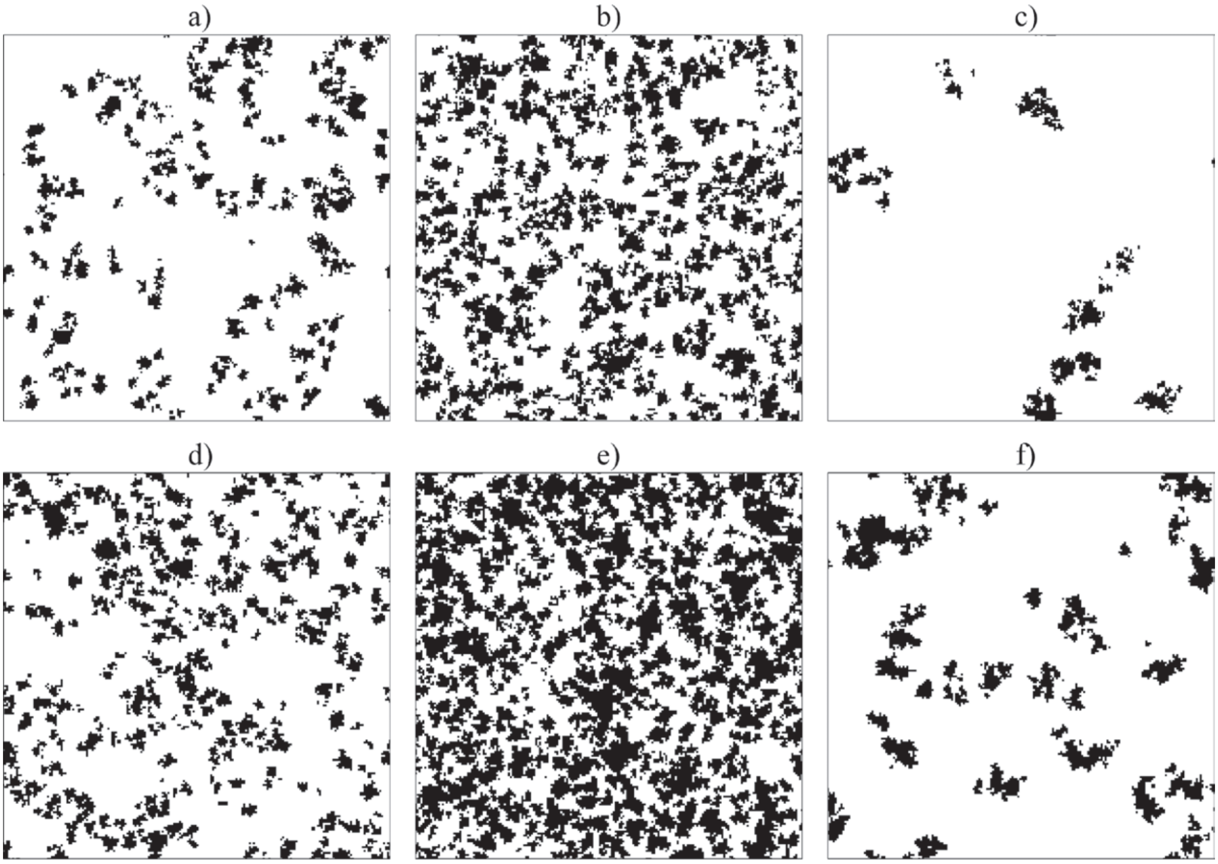

Figure 1. Characteristic equilibrium spatial distributions of cooperators (black) and defectors (white) obtained by $r = 0.03$ and various σ and p. (a) $\sigma = 0.14$, $p = 0.0$, $F_C = 0.11$. (b) $\sigma = 0.25$, $p = 0.0$, $F_C = 0.31$. (c) $\sigma = 0.36$, $p = 0.0$, $F_C = 0.05$. (d) $\sigma = 0.14$, $p = 0.02$, $F_C = 0.21$. (e) $\sigma = 0.14$, $p = 0.3$, $F_C = 0.48$. (f) $\sigma = 0.14$, $p = 0.6$, $F_C = 0.11$. All panels are depicted on a 200×200 spatial grid.

In the following, we will show that nonzero values of σ and p maintain cooperation in the studied prisoner's dilemma game even for r substantially exceeding r_{tr}, whereby cooperators obtain the biggest boost only if both parameters determining extrinsic influences are fine-tuned.

We start by visually inspecting six characteristic spatial distributions of cooperators and defectors obtained by $r = 0.03$ and various σ and p. Note that by $\sigma = 0$ and $p = 0$, defection is the only strategy for $r = 0.03$. The upper row of figure 1 clearly shows that even by $p = 0$, there exists an optimal nonzero value of σ for which cooperation is best enhanced. By considering the noise-induced maintenance and facilitation of cooperation a constructive effect, results presented in the upper panel of figure 1 thus indicate a typical coherence resonance scenario [29]. However, by introducing also nonzero values of p, the constructive effect of noisy payoff variations can be enhanced even further, as evidenced in the bottom row of figure 1. Importantly, similarly as by σ, there also exists and optimal value of p for which cooperators thrive best. Thus, results in figure 1 indicate a doubly resonant dependence of F_C on both σ and p.

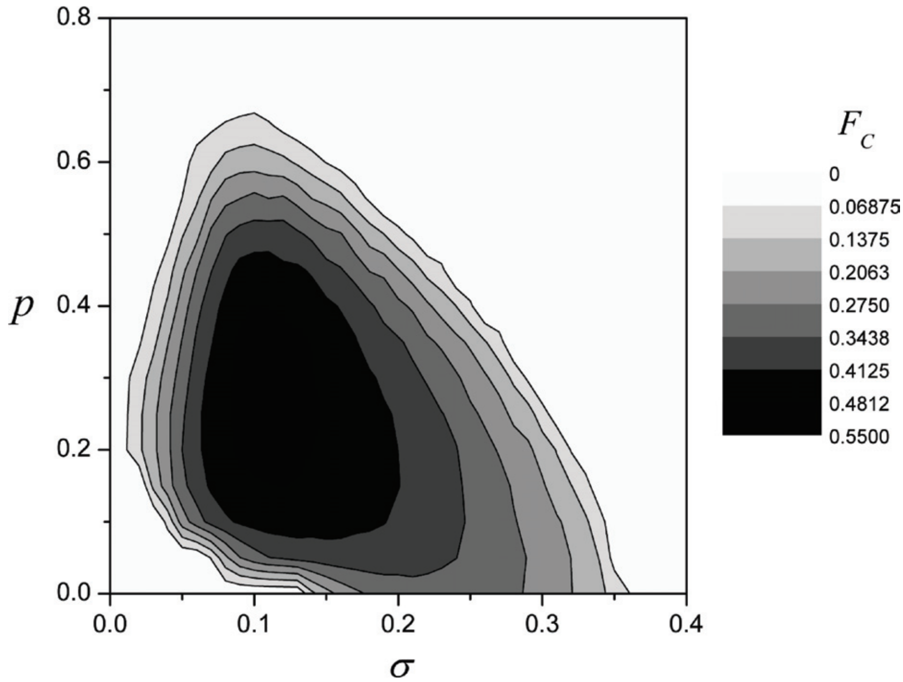

Figure 2. Double resonance, induced by noise and network variation, in the studied spatial prisoner's dilemma game in dependence on σ and p.

To quantify the results outlined above more precisely, we calculate F_C in dependence on various σ and p. Results presented in figure 2 clearly evidence that there indeed exists an optimal level of additive spatiotemporal noise as well as a fraction of shortcut links among distant players for which F_C is maximal, thus indicating the existence of double resonance in the studied spatial prisoner's dilemma game. Notably, by the presently applied temptation to defect, the slowly varying small-world topology alone is not sufficient for cooperation promotion in the studied system. However, there exists a range of $r > r_{\rm tr}$ for which cooperators can thrive exclusively on nonzero values of p even by $\sigma = 0$. Results reported in figure 2 are robust against the player adoption rule as well as the game iteration scheme. However, there always exists an upper bound $r_{\rm ub} > r_{\rm tr}$ for which the joint action of nonzero σ and p is still able to prevent extinction of cooperators. After extensive calculations in the three-dimensional parameter space spanned over r, σ and p, we found that for the presently applied game setting $r_{\rm ub} \approx 0.05$, which is roughly an order of magnitude larger than $r_{\rm tr}$.

Importantly, at first glance the above results may appear in contradiction with previous studies analysing effects of regular small-world or random graphs on the evolution of cooperation [38]–[40]. In particular, it is well known that regular random graphs are at least just as effective in maintaining cooperators alive as regular small-world graphs. Thus, the currently reported resonant dependence of F_C on p appears contradictive. We emphasize, however, that this discrepancy emerges because we consider non-regular slowly varying small-world networks. A key feature of such networks is that they reproduce the mean-field behaviour of players on the spatial grid as $p \to 1$. Thus, since the mean-field result predicts the extinction of cooperators, the decrease of the success of the cooperative strategy by large p on the spatial grid is not surprising. On the other hand, the constructive effect of the complex connectivity on the evolution of cooperation by small and intermediate values of p is in agreement with results reported previously, and can be attributed to the destruction of short loops of the regular lattice [40], as well as the addition of shortcuts that enhance the probability of cooperation between distant cooperative players which would not be able to collaborate in rigid nearest-neighbour lattice configurations.

A more precise explanation of the mechanism underlying the facilitation of cooperation by the joint action of payoff variations and non-regular slowly varying small-world topology can be provided by studying the noise-induced payoff ranking violations of the prisoner's dilemma game. Since average additions to the payoffs of each player due to noise equal zero ($\langle \xi_i \rangle_{\text{time}} = 0$ for all i), the payoff ranking $T > R > P > S$ is preserved on average over time. However, since the absolute magnitude of noise is allowed to exceed r or 1 locally, i.e. whenever two neighbours on the spatial grid interact, violations of the payoff ranking are possible at every instance of the game. The impact of these so-called local violations of the payoff ranking on the evolution of cooperation is profound. We define two possible types of payoff ranking violations that can occur whenever two individuals interact. First, let v_a denote the frequency of how often $T > R$ and $P > S$ rankings are violated. Note that both inequalities differ by r. Second, let v_b denote the frequency of how often the $R > P$ ranking is violated. Note that this inequality will be violated less often by a given σ than the former two since R and P differ by 1, which is substantially larger than r. The red and green lines in figure 3 show how v_a and v_b vary in dependence on σ. It is evident that the constructive effect of noise starts as soon as $v_a > 0$. On the other hand, the constructive effect seizes as soon as $v_b > 0$. This suggests an elegant explanation for the reported phenomenon. In particular, as soon as noisy payoff variations become large enough so that $T > R$ and $P > S$ inequalities are violated, i.e. v_a starts to exceed 0, two cooperators might end up receiving a larger payoff each than a defector facing a cooperator. Also, a cooperator facing a defector might be better off than two defectors. These two facts obviously favour the cooperative strategy since they potentially nullify the advantage defectors have over cooperators. However, as soon as noisy variations become so large that even the $R > P$ ranking is violated, i.e. v_b starts to exceeds 0, then two defectors might be better off than two cooperators, which again gives the winning edge to the defecting strategy, and hence results in a resonant dependence of cooperation fitness. Although being fairly simple, the described explanation outlines a general mechanism of cooperation promotion in the spatial prisoner's dilemma game. This effect can be resonantly pronounced by the non-regular slowly varying complex network topology due to the destruction of short loops of the regular lattice and the addition of shortcuts that link distant cooperators that would otherwise be unable to engage in cooperative alliances, as well as its unique feature of reproducing the mean-field behaviour as $p \to 1$.

4. Summary and discussion

In summary, we show that appropriately tuned Gaussian payoff variations and non-regular slowly varying complex network topology facilitate and maintain cooperation in the spatial prisoner's dilemma game for defection temptation values substantially exceeding the threshold marking the transition point to traitorous homogeneity in a sterile and regularly coupled environment. Importantly, the fraction of cooperators depends resonantly on both extrinsic factors, thus marking the existence of a double resonance phenomenon in the studied system. The double resonance is attributed to the joint effect of noise and network variation in the evolutionary process.

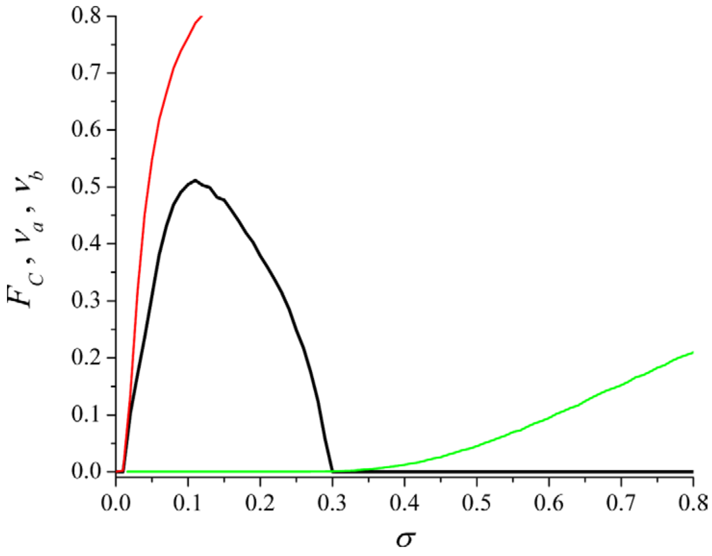

Figure 3. Promotion of cooperation by the joint action of noise and varying complex network topology for $r = 0.03$, $p = 0.3$ and different values of σ (black line). The red and green lines show the dependence of v_a and v_b, respectively. The promotion of cooperation starts when $v_a > 0$ and seizes when $v_b > 0$.

Interestingly, there exist great conceptual similarities between the currently presented results and noise driven spatially extended dynamical systems. In particular, it has been reported that noise alone can constructively influence the dynamics of spatially extended systems [45]–[47], but also that the noise-induced temporal or spatiotemporal order can be greatly enhanced by an appropriately pronounced small-world connectivity of coupled units [48]–[51]. In this sense the presented results uncover exciting correlations between evolutionary game theory [1]–[3] and stochastically driven spatially extended systems in various fields of research, ranging from chemistry, neurophysiology, cardiology to laser optics [52]. Indeed, by devising a mean-field-like or pair approximation of the spatial prisoner's dilemma game [16, 19, 38, 53], we end up with a set of differential equations exhibiting rich dynamical behaviour depending on the approximation rules and the game under consideration, thus indicating the first step towards further explorations of the outlined correlations between the two seemingly disparate disciplines.

Finally, we argue that random payoff variations and variable small-world connectivity are common in real life, either in human and animal societies or economic cycles, thus presenting two viable extrinsic mechanisms for cooperation facilitation. Especially among humans, the small-world is more of a rule rather than an exception due to the ever-increasing sophistication of communication techniques and available information sources. Moreover, it is reasonable to believe that the reproductive success of a species is affected by numerous unpredictable factors, arising either from the complex environment or the players themselves, whereby the interaction phase between two individuals, each trying to make the best out of the encounter, is the most likely part of the evolutionary process for uncertainties to take effect. Thus, both currently applied extrinsic factors appear to be viable and potent impeders of evolutionary stalemate.

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