

Premature seizure of traffic flow due to the introduction of evolutionary games

Matjaž Perc

Department of Physics, Faculty of Education, University of Maribor,
Koroška cesta 160, SI-2000 Maribor, Slovenia

E-mail: matjaz.perc@uni-mb.si

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Abstract. We study the impact of evolutionary games on the flow of traffic. Since traffic participants do not always conform to the imposed rules, the introduction of games, i.e. set of strategies defining the behavioural pattern of agents on the road, appears justified. With this motivation, and the fact that individuals can change their strategy in the course of time, the evolutionary prisoner's dilemma game is introduced between neighbouring agents, enabling them to choose between cooperation and defection. Mutual cooperation enables forwarding to both agents for one step, while the defector is able to advance two steps when facing a cooperator, whereby the latter is forced to go one step backwards. Two defectors end up in a halt until the next iteration. Irrespective of their strategy, however, agents can move only if the road ahead is free. Jumps are never allowed. We show that this simple and plausible supplementation of the discrete cellular automaton Biham–Middleton–Levine (BML) model induces a traffic flow seizure by a substantially lower initial density of cars as in the absence of evolutionary games. The phenomenon is explained by studying the one-dimensional variant of the BML model with different advancement steps on the circular ring. In view of the proposed explanation, findings are generalized also to other types of games, such is the snowdrift game, and some statistical properties of gridlock formation in the presence of evolutionary rules are outlined. Our findings suggest that 'bending the law' results in a premature occurrence of traffic jams and thus unnecessarily burdens the transportation system.

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1. Introduction

The study of traffic flow within the framework of physics has a rich and fruitful history [1]–[8]. One of the most commonly used traffic flow models employed in this context is the discrete cellular automaton Biham–Middleton–Levine (BML) model [9] describing a two-dimensional (2D) traffic flow. Originally, the BML model consists of eastbound and northbound agents that are able to advance for one step at each even and odd iteration, respectively, provided the target destination is not occupied. Despite its simplicity, the model exhibits complex behaviour such as phase transitions and self-organization in dependence on the initial density of uniformly distributed agents on the spatial grid. The phase transition describes the passing from the state where all agents can move uninterruptedly to the state of complete gridlock. For low and intermediate initial densities of agents on the spatial grid the system exhibits various stable self-organized patterns, ranging from a free flowing state that is characterized by alternating stripes of eastbound and northbound bound agents to a global jam state with a characteristic jam length that is determined by the size of the spatial grid. By high initial densities of agents the system has no time to self-organize, which results in a collection of small random jams scattered across the spatial grid. Figure 1 captures all three described scenarios in the original BML model.

Importantly, the original BML model is, except for the initial distribution of agents on the spatial grid, fully deterministic. Although all real-life traffic systems are subject to a set of deterministic rules participants have to obey, a deterministic traffic flow is still more a desire rather than fact. In particular, variations in traffic flow emerge as a consequence of varying crowdedness of the roads, as appropriately described also by the original BML model in figure 1, but also due to the innate drive that is routed in many individuals forcing them to advance to their destination as quickly as possible, thus maximizing their efficiency and output. In particular, it is nearly an everyday experience for each individual actively participating in traffic that its advantage, given by the traffic rules, is taken away from him/her by an overly eager individual trying to reach a destination quicker by somewhat ‘bending the rules’. More often than not such instances provoke the desire for revenge in the betrayed individual, thus inducing a general tendency towards such a defecting behaviour. Similar scenarios are well known in the framework of game theory [10], where the choice of an optimal playing (or in our case driving) strategy plays a central role in assuring success. By acknowledging the fact that individuals might also opt to change their strategies in the course of time, either on the basis of past experience or comparisons with neighbours, the scenario is more precisely described by evolutionary game theory [11]–[13], where the problem of cooperation and defection has been introduced as a particular example of frequency-dependent interactions. Often, egoistic individuals compete to accumulate as much resources as possible in order to prosper, thereby not paying any attention

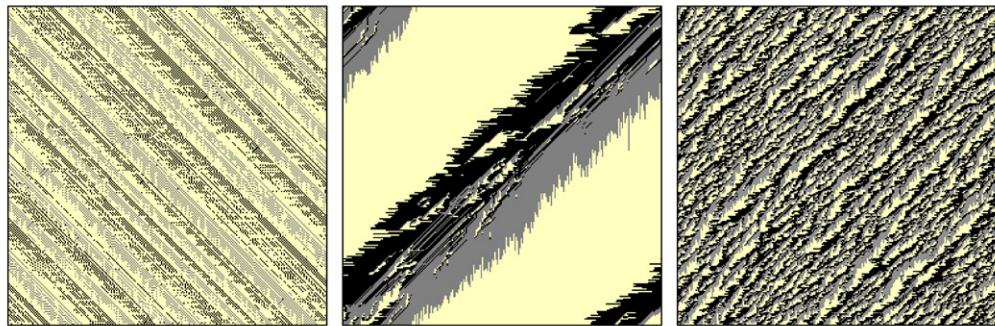


Figure 1. Characteristic snapshots of traffic flow for the original BML model by different initial densities of agents on the 256×256 spatial grid. Eastbound agents are black, while northbound agents are depicted grey. Yellow areas denote vacant sites. The left-hand side panel features a free-flowing state where all agents are able to advance uninterrupted. The density of agents on the spatial grid is 0.28. The middle panel features a complete gridlock where none of the agents can ever move again. Note that the gridlock is a consequence of a single global jam that has a characteristic length equalling $\sqrt{2n}$, n being the system size in one direction. The density of agents on the spatial grid in this case is 0.40. The left and middle panel feature typical examples of self-organization in the BML traffic model. The right-hand side panel also features a complete gridlock, which unlike in the middle panel is formed by several smaller jams. By high densities of agents, equalling 0.70 in the right-hand side panel, the system has no time to self-organize, which results in randomly scattered jams across the spatial grid.

to the harm inflicted on others or the society in general. In particular, the problem is well described by the prisoner's dilemma game [14]. In its original form the game consists of two agents who have to decide simultaneously whether they want to cooperate or defect. The dilemma is given by the fact that although mutual cooperation yields the highest collective income a defector will do better if the opponent cooperates. Since agents are aware of this fact they both decide to defect whereby none of them gets a profit. In view of above-outlined facts there appears to exist a good motivation to merge traffic flow modelling with evolutionary game theory.

Importantly, this study is certainly not the first to intimately link evolutionary game theory with topics of physics. Nearly a decade ago Szabó and Töke [15] discovered that critical phase transitions belonging to the directed percolation universality class can be observed in a simple two-strategy evolutionary spatial prisoner's dilemma game on a square lattice. Related observations have been reported later on also for spatial public goods games [16]. More recently, the addition of stochasticity and other unpredictable factors to evolutionary games and population dynamics has emerged as being a very fruitful avenue of research [17]–[20]. In particular, reports of phenomena such as coherence resonance [21], previously reported mainly in the study of dynamical systems [22, 23], reveal fascinating new correlations between evolutionary game theory and topics of physics. An excellent introductory review on the interrelation between game theory and physics is given in [24].

Of particular importance for the present study are also previous works where traffic flow simulations and concepts similar to evolutionary game theory have been considered earlier.

In particular, [25] features an immensely interesting and generally applicable study where systems of driven entities, such as the presently studied BML model, tend to reach an optimal state associated with minimal interaction and dissipation based on principles of non-equilibrium thermodynamics and game theoretical ideas. In particular, cooperative trail formation in a 2D traffic dynamics based on interaction, imitation, and payoff matrix-like formulation has been investigated. Importantly, cooperative trail formation and coherent moving states have also been studied independently in [26] and [27], respectively. Utility-based decision models for achieving traffic optimization have been employed in [28], while the fact that individuals often react differently to the same situation in traffic, thus leading to the development of characteristic response patterns or roles, has been studied in [29]. The latter development of roles or individual strategies is directly linked to selfish routing [30, 31], which however fails to ensure shortest overall travel times. Instead, individuals end up in an equilibrium characterized with equal travel times irrespective of the chosen route leading from the origin to the goal. Very recently, the role of networks underlying the traffic flow has also been studied thoroughly within the framework of physics [32]–[34]. Finally, we would like to mention the study by Helbing *et al* [35], where perhaps the most similar concept to the one presented here has been employed. In particular, the authors of [35] study the emergence of cooperation and fair behaviour in a route choice game.

The present study aims at extending the apparently very fruitful combination of traffic flow modelling and game theoretical concepts. To this purpose, we supplement the BML traffic flow model, arguably representing the physicist's approach to modelling traffic, with the evolutionary prisoner's dilemma game. In particular, each agent on the spatial grid is able to play the prisoner's dilemma game with one neighbour that has the opposite direction of movement. If both decide to cooperate the classical BML model is regained, thus allowing each individual to proceed one step east (north) at each even (odd) iteration, respectively, provided the target destination is not occupied. If, on the other hand, one agent decides to defect, the cooperator is forced to go one step backwards, while the defector is allowed to advance two steps ahead. Note that as before, the actual moves take place only if the road is free. In this sense the necessary 'free road condition', introduced in the original BML model, is superior to the outcome of the game. If the road ahead is not free, agents stay on their initial sites irrespective of the outcome of the game. Finally, if both agents decide to defect they must stay put irrespective of whether the road ahead is free or not. Note that the classical payoff scheme of the prisoner's dilemma game is replaced by the number of steps each agent is allowed to make in the next relevant iteration. The payoff ranking of the prisoner's dilemma game is obeyed since the temptation to defect $T = 2$, corresponding to two forward steps if a defector is faced with a cooperator, is larger than the reward $R = 1$ of two cooperators, which is again larger than the punishment $P = 0$ of two defectors, which is finally larger than the sucker's payoff $S = -1$ corresponding to the one backwards step a cooperator has to make if facing a defector. We show that this simple and, as argued above plausible, supplementation of the paradigmatic BML model induces a traffic flow seizure by a substantially lower initial density of cars as in the absence of evolutionary games. Our findings thus suggest that 'bending the law' results in a premature occurrence of traffic jams and thus unnecessarily burdens the transportation system. The phenomenon is explained by studying the 1D variant of the BML model with different advancement steps on the circular ring. Ultimately, the premature seizure of traffic flow due to the introduction of the evolutionary prisoner's dilemma game is attributed to the disturbing effects of varying advancement steps on the self-organized free-flowing state, which induce small localized jams on the spatial grid that eventually extend to a complete global gridlock. In view of the proposed explanation, we

generalize our findings also to other types of games, such as the snowdrift game [36], where the punishment P and sucker's payoff S simply exchange their value with respect to the prisoner's dilemma game, and reveal differences between their effects. Finally, some statistical properties of gridlock formation in the presence of evolutionary rules are presented.

The paper is structured as follows. Section 2 is devoted to the accurate description of the BML traffic model that is supplemented by the evolutionary prisoner's dilemma and snowdrift game. Main findings are presented in section 3, while in the last section, we summarize the results and outline some possibilities for future research in the framework of merged traffic flow simulations and evolutionary game theory.

2. Traffic flow model with evolutionary rules

The original BML model consists of two types of agents that differ in the prescribed direction of movement, which is not allowed to change during the simulation. Some agents can advance only towards north, while others move only towards east. Initially, eastbound and northbound agents are uniformly distributed on the $n \times n$ spatial grid with periodic boundary conditions. The crucial parameter, determining the evolution of traffic flow in the BML model, is the initial density χ of agents on the spatial grid, whereby eastbound and northbound agents participate with equal probability. The discrete simulation of the traffic flow has two phases, given by the even and odd iteration steps, respectively. All eastbound agents move one step to the east every even iteration step, while all northbound agents move one step to the north every odd iteration step. Importantly eastbound agents are allowed to move only if the eastward site is empty, while northbound agents advance only if the northward site is empty. The BML model is, except for the initial distributions of agents on the spatial grid, fully deterministic. Figure 1 above captures possible scenarios in dependence on χ . By introducing the average velocity v_i , for each agent i , as the number of all successful moves divided by the number of all attempted moves in a given amount of time, a rather sharp phase transition can be identified in the system [9], separating the free-flowing state, characterized by $\Lambda = 1$ (left-hand side panel of figure 1), from the fully jammed state, characterized by $\Lambda = 0$ (middle and right-hand side panel of figure 1), where Λ is simply the average velocity of all agents in the system obtained by averaging v_i over all i . Importantly, however, a recent very interesting study by D'Souza [37] reveals that sharp phase transitions in dependence on χ are attainable only for fairly small system sizes of up to $n = 64$ in 1D, while in larger lattices jams and freely flowing traffic can coexist, forming intermediate stable phases that blur the transition from $\Lambda = 1$ to $\Lambda = 0$ as χ is increased. Additionally, [37] features a derivation of simple geometric constraints for which such intermediate phases can be observed. Other relevant studies, previously analysing the BML model either through numerical simulation or analytical approaches, are given in [38]–[48].

As already outlined in the introduction, we supplement the above-described original BML traffic model with the evolutionary prisoner's dilemma game. In particular, we allow neighbouring agents to decide whether to cooperate or defect, thus determining their fate with respect to further advancements on the spatial grid in dependence on the outcome of the game. Initially, all agents are designated either as cooperators or defectors, whereby the direction of movement on the spatial grid has no relevance by the initial assignment of strategies. We introduce a parameter κ , determining the initial density of cooperators among all agents. When iterating the BML model, every even iteration step the agents play the game with those nearest neighbours who are

prohibiting them in making one step forward. This is done in an alternate fashion for eastbound and northbound agents, meaning that by the first even iteration all eastbound agents play with the neighbour in their way, while by the next even iteration all northbound agents play with their disturbing neighbour. Such an alternate scheme is necessary since the outcome of each game determines forwarding of both agents participating in the game, which can be eastbound and northbound agents with equal probability, and thus the game can be played only every two iteration steps (every even or odd step) so that the imposed future steps, determined by the prisoner's dilemma game, can take effect on both, eastbound and northbound, types of agents. Importantly, if an agent does not have a neighbour blocking its way, it simply advances one step irrespective of its strategy. On the other hand, if the game takes place possible outcomes are the following. If both agents cooperate the classical BML model is regained, thus allowing each individual to advance one step east (north) at the following even (odd) iteration, respectively. If, on the other hand, one agent defects the cooperator is forced to go one step backwards, while the defector is allowed to advance two steps ahead. Finally, if both agents defect they must stay put. These evolutionary rules can be summarized succinctly by the so-called payoff matrix

i/j	C	D
C	1/1	2/-1
D	-1/2	0/0

(1)

where i and j mark any of the two involved agents, while C and D stand for cooperation and defection, respectively. Numbers for possible combinations of strategy pairs determine advancements on the spatial grid. The payoff ranking, payoffs being the permission to advance for a given amount of steps on the spatial grid, of the prisoner's dilemma game is obeyed since the temptation to defect $T = 2$ is larger than the reward $R = 1$, which is again larger than the punishment $P = 0$, which is finally larger than the sucker's payoff $S = -1$. Importantly, as in the original BML model, the actual moves, as determined by the prisoner's dilemma game, take place only if the road is free. In this sense the necessary 'free road condition', introduced in the original BML model, is superior to the outcome of the game. If the road ahead is not free agents stay on their initial sites irrespective of the outcome of the game. Finally, the evolution of the two strategies on the spatial grid takes place by allowing changes of strategies always after both, eastbound and northbound, agents have made their moves (e.g. every even iteration step), by comparing their average velocities v_i (replacing the number of successful moves with the sum of appropriate payoffs) with their nearest neighbours. In particular, the central agent always adopts the strategy of those nearest neighbour who has the largest average velocity up until that time. If the central agent itself has the largest average velocity, it simply retains its strategy, whereas if an agent does not have a nearest neighbour in any of the four directions its strategy is also preserved. This is the so-called best-takes-over strategy adoption rule that is well-known in the framework of evolutionary game theory [13]. Note that the average velocity v_i of each particular agent i is taken as the ultimate measure of success of a given strategy, which is reasonable since it uniquely determines how fast the agent moves through traffic.

Towards the end of the next section, we also consider the evolutionary BML model supplemented by the snowdrift game [36] in order to generalize our findings, whereby the description of the model is identical to the one presented above, only that the punishment $P = 0$ and the sucker's payoff $S = -1$ exchange their value. More precisely, if both agents cooperate again the classical BML model is regained, thus allowing each individual to advance one step

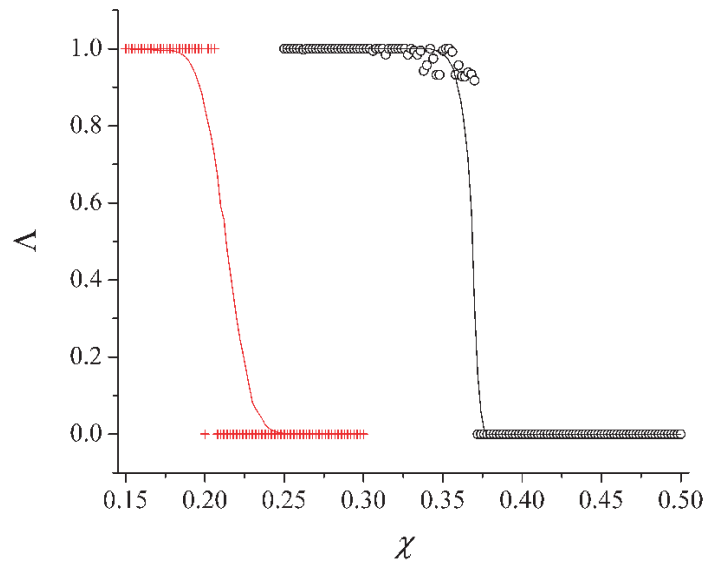


Figure 2. Phase transitions from the free flowing ($\Lambda = 1$) to the fully jammed ($\Lambda = 0$) state in dependence on χ in the original (black) and evolutionary prisoner's dilemma (red) BML traffic model. Symbols show results of individual runs at any given χ , while lines depict averages over 10^4 realization with different initial conditions. The initial fraction of cooperators among all agents on the 64×64 spatial grid in the evolutionary case equals $\kappa = 0.50$.

east (north) at the following even (odd) iteration, respectively. If, on the other hand, one agent defects the cooperator is forced to stay put in the next iteration, while the defector is allowed to advance two steps ahead. Finally, if both agents defect they must both go one step backwards.

Both the original as well as the evolutionary BML model are simulated until one convergent state, given either by $\Lambda = 1$ or $\Lambda = 0$, is reached, or up to 10^7 iteration steps for non-convergent states. Findings are presented separately for characteristic individual runs, obtained via a single simulation by a given initial density of agents χ and fraction of cooperators κ , and averaged runs, obtained via averaging the results over several thousand realizations with different initial conditions by a given χ and κ . We find that in this way results reveal interesting features that remain hidden if only individual or average results are considered.

3. Results

We start by studying Λ in dependence on χ for the original and evolutionary BML model of size $n = 64$ in 1D of the spatial grid. Note that $\Lambda = 1$ characterizes the free flowing state where all agents are always able to move when they intend to do so, while $\Lambda = 0$ characterises the fully jammed state where no agent can move ever again. For the original BML model, the critical initial density of agents where the traffic flow seizes completely equals $\chi = 0.375$ on average, as indicated by the black line in figure 2, whereby individual gridlocks can occur already by somewhat smaller χ (≈ 0.368), as indicated by the black circles showing results for a single realization at any given χ . Characteristic spatial portraits in figure 1 capture possible scenarios on both sides of the transition. Remarkably, the BML model supplemented by the evolutionary prisoner's dilemma game, with cooperators and defectors initially uniformly distributed on the

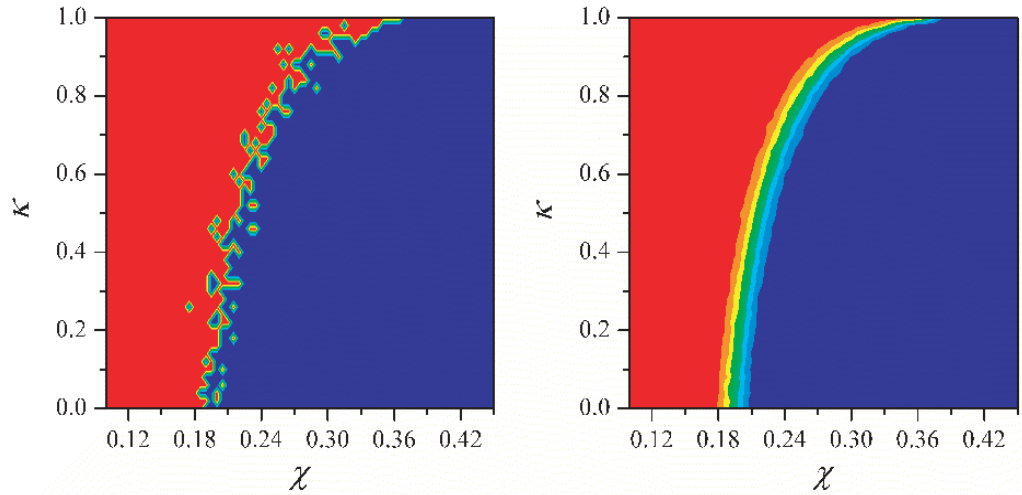


Figure 3. Premature seizures of traffic flow by decreasing the initial fraction of cooperators κ , participating in the evolutionary prisoner's dilemma game on the spatial grid. The colour profile encodes different values of Λ in a linear fashion, whereby blue corresponds to $\Lambda = 0$ (fully jammed state) and red to $\Lambda = 1$ (free flowing state). Both panels feature results for the 64×64 spatial grid, whereby the left-hand side panel shows results obtained by individual runs for a given combination of χ and κ , while the right-hand side panel features averages over 10^4 realization with different initial conditions.

spatial grid with equal probability ($\kappa = 0.5$), exhibits the phase transition already at $\chi = 0.249$ on average, as indicated by the red line in figure 2, whereby as before individual gridlocks can occur already by quite substantially smaller χ (≈ 0.198), as indicated by the red crosses showing results for a single realization at any given χ . The fairly large dissipation of χ for which $\Lambda = 1$ or $\Lambda = 0$ in the evolutionary BML model, indicated by the red crosses in figure 2, is the reason why the phase transition in that case is less sharp than in the classical BML model. Below, we will argue that this difference in the phase transition emerges because the mechanism of global gridlock formation in the evolutionary BML model differs substantially from the classical self-organizing scenario that leads to a single global jam, as shown in the middle panel of figure 1. In sum, results presented in figure 2 clearly show that the supplementation of the original BML model by evolutionary rules induces a premature seizure of traffic flow by a substantially lower initial density of agents on the spatial grid.

To study the effect of defectors on the traffic flow more precisely, we calculate Λ for various χ and κ . Note that by $\kappa = 1$, corresponding to the fully cooperative state of all agents, the original BML model is regained, despite the prisoner's dilemma game. Results in figure 3 clearly show that already a small initial fraction of defectors substantially hinders smooth traffic flow on the spatial grid. Moreover, the effect is of saturating nature as κ decreases further below 1. The small isolated islands of traffic flow seizures in the left panel of figure 3 are a consequence of chance related to the initial distributions of agents and their strategies on the spatial grid. These isolated islands, however, vanish and transform into smooth transitions if averages over realizations with different initial conditions are considered, as shown in the right-hand side panel of figure 3. Interestingly, the width of the coloured stripe (rainbow) in the right-hand side panel, indicating the sharpness of transition from $\Lambda = 1$ (red) to $\Lambda = 0$ (blue) at any given κ increases

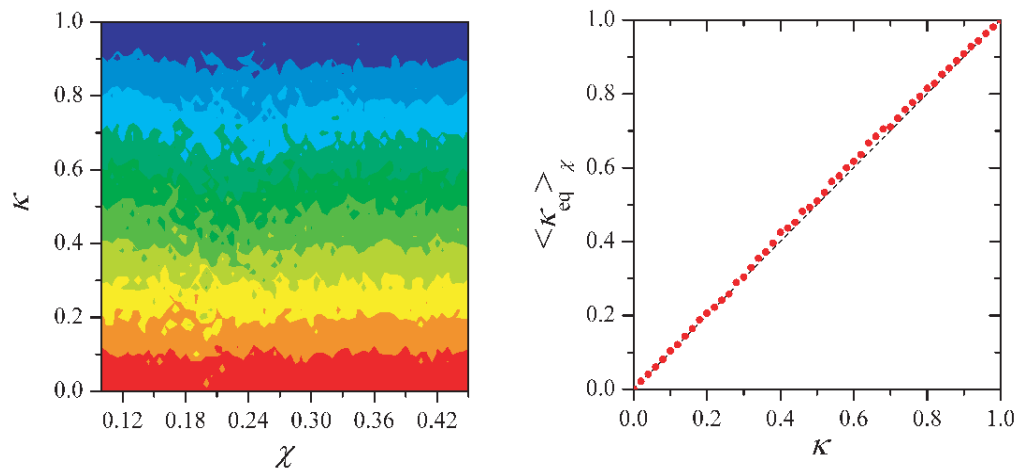


Figure 4. Dependence of equilibrium fractions of cooperators κ_{eq} for various χ and initial fractions of cooperators κ , participating in the evolutionary prisoner's dilemma game on the 64×64 spatial grid. In the left-hand side panel, the colour profile encodes different values of κ_{eq} in a linear fashion, whereby red corresponds to $\kappa_{\text{eq}} = 0$ (all defectors) and blue to $\kappa_{\text{eq}} = 1$ (all cooperators). If κ_{eq} is averaged over all χ it becomes evident that $\langle \kappa_{\text{eq}} \rangle_{\chi} \approx \kappa$ (red dots in the right-hand side panel), which suggests that defectors advance through traffic with virtually identical average velocities as cooperators (see text for details).

rapidly as κ drops below 1. This trend was pointed out already by figure 2, and similarly as the premature traffic flow seizure, is of saturating nature as κ decreases. Below, we will argue that this is intimately related with the mechanism of global gridlock formation as evolutionary rules take effect. The main trend, being that global gridlocks occur by substantially smaller χ if κ decreases, however, is evident and robust irrespective of the presentation of results.

It is interesting to see how the equilibrium fraction of cooperators κ_{eq} , obtained by averaging the fraction of cooperators at each particular iteration over a long time span after the initial transients have been discarded, depends on their initial fraction κ . Results in figure 4 show that, remarkably, the initial fraction is largely preserved in spite of the imposed deterministic best-takes-over strategy adoption rule, allowing agents to change their strategy in accordance with the performance, i.e. average velocity v_i , of their neighbours. This fact leads to the interesting conclusion that defectors do not pass through traffic quicker than cooperators. If they would, cooperators would adopt the defecting strategy until there were none left. The fact that cooperators and defectors have, on average, virtually identical v_i suggests that rude and selfish behaviour on the road not only induces premature traffic jams, but also does nothing good for the defectors either. This is a result that can be very well corroborated by intuitive experience. In particular, although defectors might temporarily gain an advantage over cooperators by escaping a potential slow passage, the self-organizing nature of the traffic flow prevents them from gaining a substantial advantage since they quickly join the tail of the next jam. In everyday terms, one can drive over a red light in one crossing only to join the tail of the bulk at the next. Then, however, the defecting strategy is pointless since the road ahead is not free, which ultimately hinders the success of intent. Importantly though, result in figure 4 should not be understood as if strategies of agents on the spatial grid do not change at all. In fact, strategy adoptions occur frequently

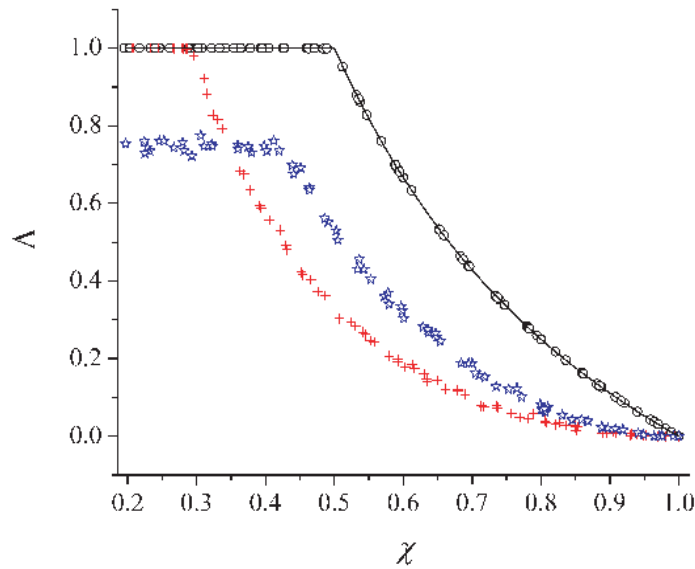


Figure 5. Λ of agents in the 1D BML model in dependence on χ and various advancement steps. Black circles and the solid line feature numerical results and the analytical prediction, respectively, for the original case where each agent attempts to move one step at every iteration. Red crosses show numerical results for the case where all agents attempt to advance two steps at every iteration. Blue stars depict results for the case where half of the agents on the circular ring attempt to advance one step ahead, while the other half attempts to advance two at every iteration. Results were obtained by employing a circular ring of size $n = 10^4$.

throughout the simulation, thus warranting an evolutionary process, but indeed the cooperative strategy is adopted just as frequently as the defecting strategy.

The above finding, indicating that the average payoff of cooperators and defectors in the long run is the same, can be nicely supported by acknowledging the fact that this is a necessary condition for fixed points (equilibria) in replicator dynamics [12, 13], which usually describes well-mixed populations. Since presently agents move around and thus essentially interact randomly with one another on the spatial grid one can argue that the system setting actually corresponds to the well-mixed case, similarly as argued in [49]. Thus, the presently reported equilibration at equal fitness can be interpreted also as a necessary condition for the equilibrium rather than a surprising fact.

To gain an understanding of the premature traffic flow seizure due to the introduction of the prisoner's dilemma game, we study the 1D BML model in some detail. The 1D model is a complete analogue of its spatial counterpart, only that agents move along a closed ring in a single (e.g. clockwise) direction. The original BML model, where each agent can advance one step at each iteration, can be solved analytically by considering the vacant sites as anti-clockwise moving agents, with an exchange dynamics such that the number of clockwise and anti-clockwise agents moving at each iteration are the same [9, 50]. In particular, the analytical result predicts that $\Lambda = 1$ if $\chi < 1/2$, while for $\chi \geq 1/2$ Λ decreases to zero according to $\Lambda = (1 - \chi)/\chi$. The black line in figure 5 shows that this result is in excellent agreement with numerical calculations. Analogously,

we consider the case where all agents attempt to advance for two steps at every iteration. It is straightforward to reckon that $\Lambda = 1$ (representing only the success rate of attempted moves) if $\chi < 1/3$, while for $\chi \geq 1/3$ Λ decreases to zero continuously, similar to the former case. In particular, the critical initial density of agents on the circular ring $\chi = 1/3$ is obtained simply by acknowledging the fact that two forward steps of all agents on the circular ring require $2/3$ of the ring to be free in order for $\Lambda = 1$. If less than $2/3$ of the ring are free some agents will not have a free path, while trying to advance. Thus, if $\chi \geq 1/3$ Λ starts to decrease as denoted by the red crosses in figure 5. Finally, we consider the case where half of the agents on the circular ring attempt to advance one step ahead, while the other half attempts to advance two at every iteration. The two types of agents are uniformly distributed along the circular ring. Via the same reasoning as outlined above, we find the critical initial density of agents on the circular ring, where Λ starts to decrease continuously, to equal $\chi = 5/12$ (one half of agents require $1/2$ and the second $1/3$ of the circular ring to be free). However, numerical results, denoted by the blue stars in figure 5, reveal an interesting and unexpected phenomenon. Namely, $\Lambda < 1$ even for $\chi < 5/12$. This suggests that as soon as some agents try to move faster than others a new mechanism emerges, preventing the system to settle onto the self-organized free flowing state. The mechanism can be explained precisely by considering only two agents on the circular ring, whereby one always attempts to move one step and the other two steps ahead. Even in this simple case Λ will be smaller than one since the faster advancing agent will sooner or later bump into the slower advancing agent due to the periodic boundary condition. From that point on, the individual advancing two steps will be able to advance only every two iteration steps, thus splitting its success of attempted moves in half. Λ of the two agents combined will thus equal 0.75, which is exactly the plateau displayed by the blue stars in figure 5 for all $\chi < 5/12$. Only in the special case where a single agent occupies the circular ring the described mechanism, obviously, does not take effect.

By extending the above-outlined reasoning to the 2D BML model with evolutionary rules, we argue that the premature seizure of traffic flow emerges due to the disturbing effects of varying advancement steps on the self-organized free-flowing state, as similarly explained for the 1D model. Importantly, as the heterogeneity in advancement steps appears crucial for the occurrence of premature jamming, it seems straightforward to generalize above findings also to other types of evolutionary games, as they all introduce diversity in how individuals advance on the spatial grid. In particular, the snowdrift game [36] differs from the prisoner's dilemma game only in that the punishment $P = 0$ and the sucker's payoff $S = -1$ exchange their value (see section 2). Results in figure 6 feature Λ for various χ and κ . Note that by $\kappa = 1$, corresponding to the fully cooperative state of all agents, the original BML model is again regained, despite of the introduction of the snowdrift game. Results in figure 6 clearly show that already a fairly small initial fraction of defectors substantially hinders smooth traffic flow on the spatial grid. As by the introduction of the prisoner's dilemma game the small isolated islands of traffic flow seizures in the left panel of figure 6 are a consequence of chance related to the initial distributions of agents and their strategies on the spatial grid. These isolated islands vanish and transform into smooth transitions if averages over realizations with different initial conditions are considered, as shown in the right-hand side panel of figure 6. As in figure 3, the width of the coloured stripe (rainbow) in the right panel, indicating the sharpness of transition from $\Lambda = 1$ (red) to $\Lambda = 0$ (blue), increases rapidly as κ drops below 1.

Aside from obvious similarities between results in figures 3 and 6, there also exist some rather subtle differences that emerge due to the exchanged payoffs S and P . In particular, as κ decreases below 1 (e.g. $\kappa = 0.8$) premature jamming occurs by somewhat larger values of χ

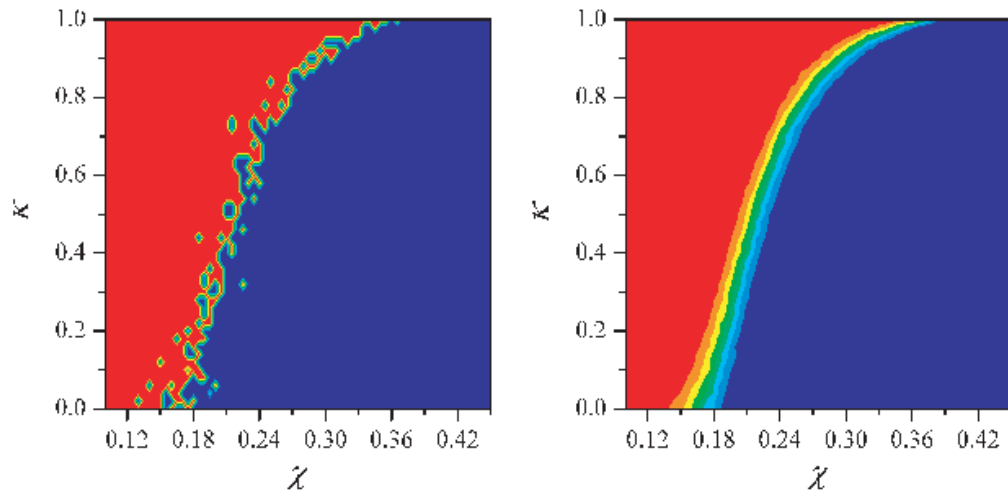


Figure 6. Premature seizures of traffic flow by decreasing the initial fraction of cooperators κ , participating in the evolutionary snowdrift game on the spatial grid. The colour profile encodes different values of Λ in a linear fashion, whereby blue corresponds to $\Lambda = 0$ (fully jammed state) and red to $\Lambda = 1$ (free flowing state). Both panels feature results for the 64×64 spatial grid, whereby the left-hand side panel shows results obtained by individual runs for a given combination of χ and κ , while the right-hand side panel features averages over 10^4 realization with different initial conditions.

as by the introduction of the prisoner's dilemma game (compare figures 3 and 6). This is simply due to the fact that a cooperator–defector pair in the snowdrift game has a smaller absolute difference in the desired advancement steps ($T - S = 2$) than a cooperator–defector pair in the prisoner's dilemma game ($T - S = 3$). Thus, a small fraction of defectors in the BML model supplemented by the evolutionary prisoner's dilemma game has a more disturbing effect on the free flowing state than in the snowdrift case. On the other hand, as κ approaches 0 (defectors are the dominant strategy) the snowdrift game appears to induce jamming earlier, i.e. by smaller χ , than the prisoner's dilemma game. Again, this is due to the fact that two defectors, receiving the punishment $P = -1$ by the snowdrift game, evoke a larger heterogeneity in the advancement steps than two defectors engaging in a prisoner's dilemma game where $P = 0$ (to see the difference in heterogeneity P should be compared to the one step ahead if a defector is isolated). Finally, we mention that in the BML model supplemented by the evolutionary snowdrift game defectors also do not advance faster than cooperators, and thus initial fractions of the two strategies on the spatial grid are preserved, implying $\langle \kappa_{\text{eq}} \rangle_{\chi} \approx \kappa$, identical as presented in the right-hand side panel of figure 4 for the BML model supplemented by the evolutionary prisoner's dilemma game.

Finally, it remains of interest to study the formation of gridlocks in the two studied evolutionary BML models more precisely in order to shed light on the features outlined by interpreting results presented in figures 3 and 6. In particular, we want to clarify why already a fairly small initial fraction of defectors substantially hinders smooth traffic flow on the spatial grid, and why the transition from $\Lambda = 1$ to $\Lambda = 0$ loses sharpness as soon as κ drops below 1. We will constrain the following treatment only to the BML model supplemented by the prisoner's

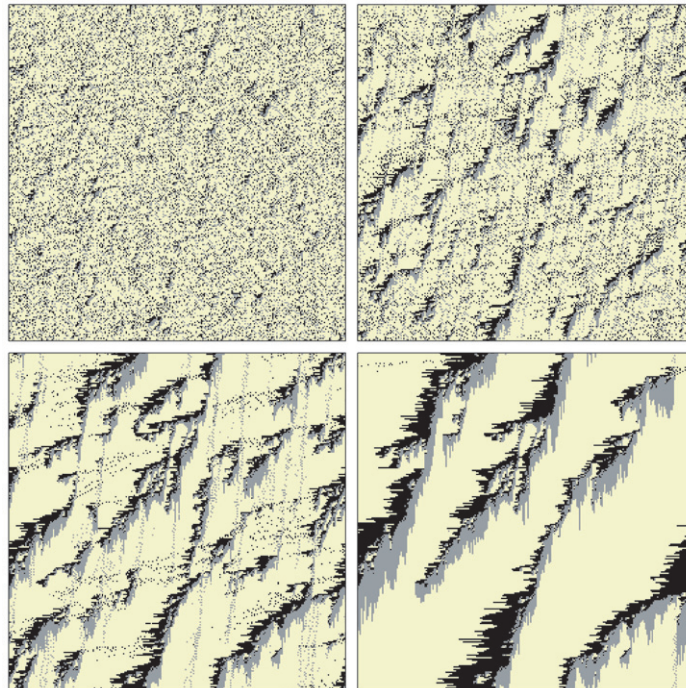


Figure 7. Premature development of a gridlock due to the introduction of evolutionary rules. Traffic portraits feature characteristic snapshots of the spatial grid at various times advancing from the top left towards the bottom right panel. The density of agents on the spatial grid is equal as in the left-hand side panel of figure 1, as is the colour coding. The initial fraction of cooperators participating in the evolutionary prisoner's dilemma game on the 256×256 spatial grid equals $\kappa = 0.50$

dilemma game since the presented findings equally apply also to other games, as the mechanism of premature jamming is always related to the introduced heterogeneity in advancement steps of agents on the spatial grid.

We argue that the premature seizure of traffic flow emerges due to the disturbing effects of varying advancement steps on the self-organized free-flowing state, which induce small localized jams on the spatial grid that eventually extend to a complete global gridlock as nicely presented in the panels of figure 7. In particular, note how the system fails to self-organize into a free-flowing state due to the emergence of small localized jams that emerge already at an early stage of the simulation, as presented in the top left-hand side panel of figure 7. These localized jams do not dissolve but grow, eventually leading to a premature traffic jam at a global scale, as depicted in the bottom right-hand side panel of figure 7. It is crucial to note that the global gridlock presented in the bottom right-hand side panel of figure 7 is structurally very different than the one presented in the middle panel of figure 1. The difference emerges due to the lack of self-organization in the evolutionary BML model. Therefore, instead of a single global jam with nearly constant width, there emerges a branched jamming pattern that apparently lacks any particular order or structure.

In order to analyse this difference statistically in dependence on κ , we calculate the average width of jams ψ by states of complete gridlock, whereby χ for the calculations is always set minimal where Λ still equals 0 (along the blue border in the right-hand side panel of figure 3).

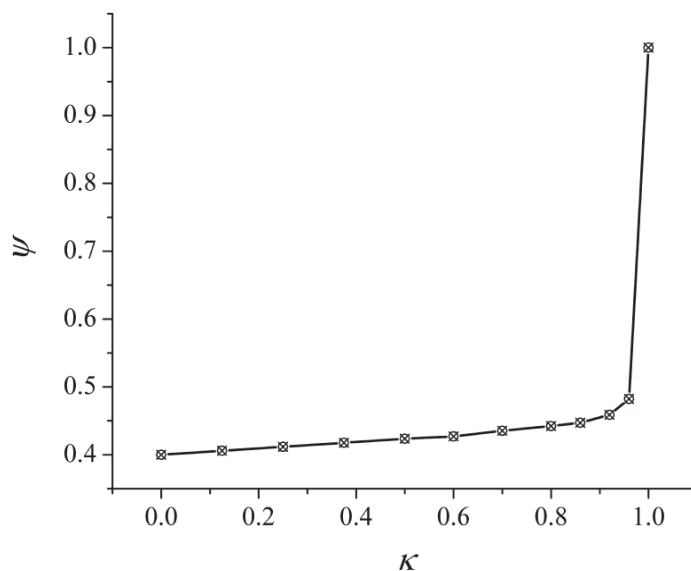


Figure 8. Average width of jams ψ in dependence on the initial fraction of cooperators participating in the evolutionary prisoner's dilemma game on the 256×256 spatial grid. The final value of ψ was obtained by averaging over 50 different jam widths across the spatial grid.

In particular, ψ is obtained by averaging the typical width of jams across the spatial grid, whereby the width at each instance is measured perpendicular to the direction of the jam. We argue that although ψ statistically assesses only the final state of the system (being the global gridlock), it is intimately related with the way gridlocks emerge on the spatial grid, and thus faithfully characterizes also the jam formation itself. Results presented in figure 8 are normalized with the width of the jam emerging in the absence of evolutionary rules (or equivalently if all agents are cooperators) presented in the middle panel of figure 1, thus yielding $\psi = 1$ by $\kappa = 1$. It is evident that there exists a sharp transition, splitting the average width of jams in half already by $\kappa = 0.97$, thus indicating that even a minute fraction of defectors on the spatial grid has nearly the same impact on the flow of the traffic as widespread defection. Note that $\psi = 0.4$ if $\kappa = 0$, which is only a marginal drop in comparison to $\psi = 0.5$ that is obtained by $\kappa = 0.97$. Results in figure 8 thus suggest that only a few defectors on the spatial grid immensely effectively hinder self-organization of the system towards a free-flowing state. This fact effectively explains why already a small initial fraction of defectors substantially hinders smooth traffic flow on the spatial grid, as presented in figures 3 and 6. Also, since virtually by all $\kappa < 1$ the self-organization is lost (indicated by the sudden drop of ψ already by $\kappa = 0.97$), and thus jams occur randomly throughout the spatial grid, the transitions from $\Lambda = 1$ to $\Lambda = 0$ are subjected to chance by a larger extent than in the absence of evolutionary games. Thus, phase transitions lose sharpness as soon as $\kappa < 1$, as indicated by the instant broadening of the rainbow region between the red and blue shades in figures 3 and 6.

4. Summary

In sum, we show that a simple and plausible supplementation of the original BML model with evolutionary games induces a premature seizure of traffic flow by a substantially lower initial density of agents as in the original case. Importantly, already a minute initial fraction of defectors among the participating agents substantially hinders smooth traffic flow on the spatial grid. Interestingly, rude and selfish behaviour on the road not only induces premature traffic jams, but also does nothing good for the defectors either, as they advance with virtually identical average velocities as cooperators. Intuitively, this fact can be directly related to everyday experience, where it is often the case that an individual can drive over a red light in one crossing only to join the tail of the bulk at the next. In view of insights gained from the analysis of the 1D BML model with different advancement steps, we argue that the premature seizure of percolation occurs due to the disturbing effects of varying advancement steps on the self-organized free-flowing state, which induce small localized jams on the spatial grid that eventually extend to a complete global gridlock. Due to the simplicity of the underlying mechanism of premature jamming, results presented for the prisoner's dilemma game can be extended also to other types of games, such as the snowdrift game, thus indicating the general validity of the presented results. In sum, we conclude that any individual strategies of agents on the road, exploiting other participants or trying to bend the rules, ultimately result in premature jamming and thus unnecessarily burden the transportation systems. In other words, it seems best to simply stick with the rules of the road and exclude individually-motivated decisions.

The present study introduces a viable approach by supplementing traffic flow simulations with evolutionary rules. In particular, the approach seems viable since participants of traffic are often human individuals who, unlike mindless post or e-mail packages, do not always simply obey the rules, but every now and then succumb to their innate drive to outperform their 'rivals', thus trying to reach the final destination quicker. Notably, the outlined approach opens up several interesting questions and challenges that have yet to be addressed. Particularly, the supplementation of a continuous traffic flow model [51] with evolutionary rules would enable a more precise study of the effects of various defection temptation values, possibly revealing a critical value at which the influence becomes decisive. Also, a more detailed statistical analysis of mechanisms leading to global jams in the presence of evolutionary rules requires a separate and more extensive study. Finally, it would be interesting to study the effect of evolutionary games in other cellular automaton based models of traffic flow, which are often presented and devised in the framework of physics [52]. We hope that the present study will be a source of inspiration, spawning new studies in the apparently very fruitful and interesting combination of traffic flow simulations and evolutionary game theory.

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