Phase transitions in models of human cooperation

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A R T I C L E   I N F O

A B S T R A C T

If only the fittest survive, why should one cooperate? Why should one sacrifice personal benefits for the common good? Recent research indicates that a comprehensive answer to such questions requires that we look beyond the individual and focus on the collective behavior that emerges as a result of the interactions among individuals, groups, and societies. Although undoubtedly driven also by culture and cognition, human cooperation is just as well an emergent, collective phenomenon in a complex system. Nonequilibrium statistical physics, in particular the collective behavior of interacting particles near phase transitions, has already been recognized as very valuable for understanding counterintuitive evolutionary outcomes. However, unlike pairwise interactions among particles that typically govern solid-state physics systems, interactions among humans often involve group interactions, and they also involve a larger number of possible states even for the most simplified description of reality. Here we briefly review research done in the realm of the public goods game, and we outline future research directions with an emphasis on merging the most recent advances in the social sciences with methods of nonequilibrium statistical physics. By having a firm theoretical grip on human cooperation, we can hope to engineer better social systems and develop more efficient policies for a sustainable and better future.

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1. Introduction

The human race is remarkable in many ways. We are champions of cooperation [1]. We sacrifice personal benefits for the common good, we work together to achieve what we are unable to achieve alone, we are compassionate, and we are social. And through this cooperation, we have had astonishing evolutionary success. We have conquered our planet, and today there is an abundance of technological breakthroughs and innovations that make our lives better. At the same time, our societies are home to millions that live on the edge of existence. We deny people shelter, we deny people food, and we deny people their survival. We still need to learn how to cooperate better with one another. The problem, however, is that to cooperate more or better, or even to cooperate at all, is in many ways unnatural. Cooperation is costly, and exercising it can weigh heavily on individual wellbeing and prosperity. If only the fittest survive, why should one perform an altruistic act that is costly to perform but benefits another? Why should we care for and contribute to the public good if freeriders can enjoy the same benefits for free? Since intact cooperation

forms the bedrock of our efforts for a sustainable and better future, understanding cooperative behavior in human societies has been declared as one of the grand scientific challenges of the 21st century [2].

In the past, Hamilton's kin selection theory has been applied prolifically to solve the puzzle of cooperation among simpler organisms [3], resting on the fact that by helping a close relative to reproduce still allows indirect passing of the genes to the next generation. Ants and bees, for example, are famous for giving up their own reproductive potential to support that of the queen [4]. Birds do cooperative breeding that prompts allomaternal behavior where helpers take care for the offspring of others [5]. Microorganisms also cooperate with each other by sharing resources and joining together to form biofilms [6]. But in nature cooperation is common not only between relatives. And this seems to be all the more true the more intelligent an organism is. Higher mammals, and humans in particular, are in this respect at the top of the complexity pyramid where one can distinguish a vast variety of prosocial and antisocial behavior.

Accordingly, many other mechanisms have been identified that promote cooperation, most famous being direct and indirect reciprocity as well as group selection [7]. Network reciprocity [8] has recently also attracted considerable attention in the physics community, as it became clear that methods of nonequilibrium statistical physics can inform relevantly on the outcome of evolutionary

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games in structured populations [9–13]. While the basic idea behind network reciprocity is simple — cooperators do better if they are surrounded by other cooperators — the manifestation of this fact and the phase transitions leading to it depend sensitively on the structure of the interaction network and the type of interactions, as well as on the number and type of competing strategies.

While the infusion of statistical physics to this avenue of research is still a relatively recent development, evolutionary game theory [14] is long established as the theory of choice for studying the evolution of cooperation among selfish individuals, including humans [15]. Competing strategies vie for survival and reproduction through the maximization of their utilities, which are traditionally assumed to be payoffs that are determined by the definition of the contested game. The most common assumption underlying the evolution in structured populations has been that the more successful strategies are imitated and thus spread based on their success in accruing the highest payoffs. Mutation has also been considered prominently, in that it can reintroduce variation into the population or represent cultural evolution and social learning, in which people imitate those with higher payoffs and sometimes experiment with new strategies. Evolutionary dynamics based on these basic principles has been considered as the main driving force of evolution, reflecting the individual struggle for success and the pressure of natural selection.

Undoubtedly, traditional evolutionary game theory, as briefly outlined above, has provided fundamental models and methods that enable us to study the evolution of cooperation, and research along these lines continues to provide important proof-of-principle models that guide and inspire future research. But the complexity of such systems also requires methods of nonequilibrium statistical physics be used to better understand cooperation in human societies, and to reveal the many hidden mechanisms that promote it. In the continuation, we first present the public goods game on the square lattice as the null model of human cooperation [16]. We then proceed with representative extensions of the game involving punishment [17] and correlated positive and negative reciprocity [18], which deliver fascinating examples of phase transitions in the realm of this research. We conclude with an overview of important progress made in related fields, and we outline possible directions for future research in the realm of statistical physics of evolutionary games.

2. The null model

The public goods game is simple and intuitive. In a group of players, each one can decide whether to cooperate or defect. Cooperators contribute $c = 1$ to the common pool, while defectors contribute nothing. The sum of all contributions is multiplied by a multiplication factor $r > 1$, which takes into account synergistic effects of cooperation. In particular, there is an added value to a joint effort that is often more than just the sum of individual contributions. After the multiplication, the resulting amount of public goods is divided equally amongst all group member, irrespective of their strategy. In a group g containing G players the resulting payoffs are thus

$$\Pi_C^g = r(N_C + 1)/G - 1 \tag{1}$$
$$\Pi_D^g = r N_C/G, \tag{2}$$

where N_C is the number of cooperators around the player for which the payoff is calculated. Evidently, the payoff of a defector is always larger than the payoff of a cooperator, if only $r < G$. With a single parameter, the public goods game hence captures the essence of a social dilemma in that defection yields highest short-term individual payoffs, while cooperation is optimal for the group,

and in fact for the society as a whole. If nobody cooperates public goods vanish and we have the tragedy of the commons [19].

In a well-mixed population, where groups are formed by selecting players uniformly at random, $r = G$ is a threshold that marks the transition between defection and cooperation. If players imitate strategies of their neighbors with a higher payoff, then for $r < G$ everybody defects, while for $r > G$ everybody in the population cooperates. Interactions among humans, however, are seldom random, and it is therefore important for the null model to take this into account. The square lattice is among the simplest of networks that one can consider. Notably, previous research has shown that for games governed by group interactions using the square lattice suffices to reveal all feasible evolutionary outcomes, and moreover, that these are qualitatively independent of the details of the interaction structure [11].

For simplicity but without loss of generality, let the public goods game thus be staged on a square lattice with periodic boundary conditions where L^2 players are arranged into overlapping groups of size $G = 5$ such that everyone is connected to its $G - 1$ nearest neighbors. The microscopic dynamics involves the following elementary steps. First, a randomly selected player x with strategy s_x plays the public goods game with its $G - 1$ partners as a member of all the $g = 1, \ldots, G$ groups where it is member, whereby its overall payoff Π_{s_x} is thus the sum of all the payoffs $\Pi_{s_x}^g$ acquired in each individual group. Next, player x chooses one of its nearest neighbors at random, and the chosen co-player y also acquires its payoff Π_{s_y} in the same way. Finally, player y imitates the strategy of player x with a probability given by the Fermi function

$$W(s_x \rightarrow s_y) = \frac{1}{1 + \exp[(\Pi_{s_y} - \Pi_{s_x})/K]}, \tag{3}$$

where K quantifies the uncertainty by strategy adoptions [16]. In the $K \rightarrow 0$ limit, player y copies the strategy of player x if and only if $\Pi_{s_x} > \Pi_{s_y}$. Conversely, in the $K \rightarrow \infty$ limit, payoffs cease to matter and strategies change as per flip of a coin. Between these two extremes players with a higher payoff will be readily imitated, although under-performing strategies may also be adopted, for example due to errors in the decision making, imperfect information, and external influences that may adversely affect the evaluation of an opponent. Repeating these elementary steps L^2 times constitutes one full Monte Carlo step (MCS), which gives a chance to every player to change its strategy once on average.

This null model — the spatial public goods game — has been studied in detail in [16], where it was shown that for $K = 0.5$ cooperators survive only if $r > 3.74$, and they are able to defeat defectors completely for $r > 5.49$. Both the $D \rightarrow C + D$ and the $C + D \rightarrow D$ phase transition are continuous. Subsequently, the impact of critical mass [20], i.e., the evolution of cooperation under the assumption that the collective benefits of group membership can only be harvested if the fraction of cooperators within the group exceeds a threshold value, and the effects of different group sizes [21], have also been studied in the realm of this two-strategy spatial public goods game.

In general, it is important that in structured populations, due to network reciprocity, cooperators are able to survive at multiplication factors that are well below the $r = G$ limit that applies to well-mixed populations. The $r > 3.74$ threshold for cooperators to survive on the square lattice can be considered as a benchmark value, below and above which we have harsh and lenient conditions for the evolution of public cooperation, respectively.

3. Public goods game with punishment

Despite ample cooperation in human societies [1], and despite our favorable predispositions for prosocial behavior that are likely
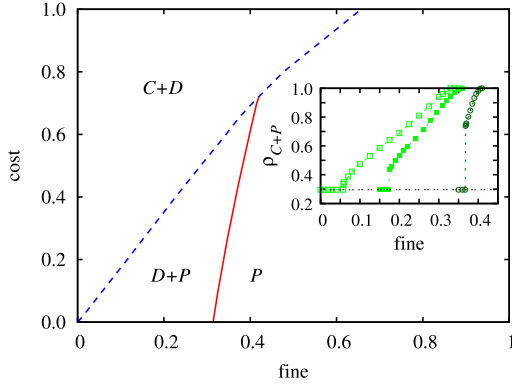
Fig. 1. Full fine–cost (β–γ) phase diagram, as obtained for $r = 3.8$ and $K = 0.5$. Solid red line denotes continuous phase transitions, while dashed blue line denotes discontinuous phase transitions. Different phases are denoted by the symbols of the strategies that survive in the stationary state. Inset shows the overall fraction of both cooperative strategies (ρ_{C+P}) in dependence on the fine β, as obtained for punishment costs $\gamma = 0.1$, 0.3, and 0.65 from left to right. Similar phase diagrams can be obtained for smaller values of r, where, however, only strategy D survives at small values of the punishment fine. We recall that $r = 3.74$ is the benchmark value beyond which a mixed $C + D$ phase is stable in the null model without punishment. These results are reproduced with permission from [17]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

rooted in difficulties of our pre-human ancestors in rearing offspring that survived [22], cooperation today is still subject to both positive and negative incentives [23]. Positive incentives typically entail rewards for behaving prosocially, while negative incentives typically entail punishing free-riding [24]. However, just like public cooperation incurs a cost for the wellbeing of the common good, so does the provisioning of rewards or sanctions incur a cost for the benefit or harm of the recipients. Individuals that abstain from dispensing such incentives therefore become second-order freeriders [25], and they are widely believed to be amongst the biggest impediments to the evolutionary stability of rewarding and punishing.

The null model introduced above can be easily upgraded to account for positive and negative incentives. Cooperators that punish defectors ($s_x = P$), for example, can be introduced as the third competing strategy. In this case, both cooperative strategies (C and P) contribute $c = 1$ to the common pool, while defectors contribute nothing. Moreover, a defector is fined with $\beta/(G-1)$ from each punishing cooperator within the group, which in turn requires each punisher to bear the cost $\gamma/(G-1)$ for each defector that is punished. A defector thus suffers the maximal fine β if it is surrounded solely by punishers [$N_P = G - 1$ in Eq. (4)], while a lonely punisher bears the largest cost γ if it is surrounded solely by defectors [$N_D = G - 1$ in Eq. (6)]. In agreement with these rules, the payoff values of the three competing strategies obtained from each group g are

$$\Pi_D^g = R(N_C + N_P)/G - \beta N_P/(G-1), \tag{4}$$

$$\Pi_C^g = R(N_C + N_P + 1)/G - 1, \tag{5}$$

$$\Pi_P^g = R(N_C + N_P + 1)/G - 1 - \gamma N_D/(G-1) \tag{6}$$

where N_{s_x} denotes the number of players with strategy s_x around the player for which the payoff is calculated.

A representative phase diagram for the spatial public goods game with cooperators that punish defectors is presented in Fig. 1. The inset, on the other hand, shows how the overall cooperation level increases monotonously with the fine for three different values of the punishment cost γ. It can be observed that punishing cooperators always prevail for a sufficiently large fine, independently of the punishment cost γ. If the cost is lower than a critical

value ($\gamma \approx 0.65$ at $r = 3.8$), the application of a sufficiently large fine will lead to a discontinuous $C + D \rightarrow D + P$ phase transition, where punishing cooperators replace pure cooperators in the two-strategy phase. The occurrence of this discontinuous phase transition is rooted in an indirect territorial competition between strategies C and P, which compete independently against D on the square lattice. It is important to note that, in the beginning of the evolutionary process, C and P players may form mixed cooperative islands. However, when defectors are not in the neighborhood, the two strategies have identical payoffs and thus become equivalent, and the strategy update dynamics defined by Eq. (3) results in logarithmic coarsening that is otherwise characteristic for the voter model [26]. Although the coarsening is logarithmically slow, C and P players in these islands segregate quickly, given that their size is typically very small. After this segregation, homogeneous clusters of pure cooperators and punishing cooperators compete separately against the defectors. When the punishment fine is sufficiently large, punishing cooperators suddenly become more effective against defectors than pure cooperators, so that eventually the later are crowded out and replaced by the former. It is worth noting that discontinuous phase transitions due to indirect territorial competition appear to be common in evolutionary games in structured populations, as they have been observed before also in the public goods game with pool punishment [27] and in the public goods game with correlated positive and negative reciprocity [18] (see also [11,28] for reviews).

4. Public goods game with positive and negative reciprocity

While the results presented in Section 3 can serve as an introduction to phase transitions in models of human cooperation, it is of course possible to devise more elaborate models, which accordingly also give rise to more complex spatiotemporal dynamics between the competing strategies. An example is the public goods game with positive and negative reciprocity [18], where instead of the original two strategies making up the null model, we initially have defectors ($s_x = D$), cooperators that punish defectors ($s_x = P$), cooperators that reward other cooperators ($s_x = R$), and cooperators that both punish defectors as well as reward other cooperators ($s_x = B$) occupying the vertices of the square lattice with equal probability. As in Section 3, all three cooperative strategies (P, R and B) contribute $c = 1$ to the public good, while defectors contribute nothing. Moreover, a defector is fined with $\beta/(G-1)$ from each punisher (P or B) within the group, which in turn requires each punisher to bear the cost $\gamma/(G-1)$ for each defector that is punished. Similarly, every cooperator is given the reward $\beta/(G-1)$ from every R and B player within the group, while each of them has to bear the cost of rewarding $\gamma/(G-1)$ for every cooperator that is rewarded. In agreement with these rules, the payoff values of the four competing strategies obtained from each group g are

$$\Pi_D^g = r(N_P + N_R + N_B)/G - \beta(N_P + N_B)/(G-1),$$

$$\Pi_P^g = r(N_P + N_R + N_B + 1)/G - \gamma N_D/(G-1)$$
$$\quad + \beta(N_R + N_B)/(G-1),$$

$$\Pi_R^g = r(N_P + N_R + N_B + 1)/G - \gamma(N_P + N_R + N_B)/(G-1)$$
$$\quad + \beta(N_R + N_B)/(G-1),$$

$$\Pi_B^g = r(N_P + N_R + N_B + 1)/G - \gamma + \beta(N_R + N_B)/(G-1),$$

where N_{s_x} denotes the number of players with strategy s_x around the player for which the payoff is calculated.

An analysis of this model reveals that discontinuous phase transitions dominate, which has to do with the spontaneous emergence of cyclic dominance between strategies D, P and B. In particular, within the three-strategy $D + P + B$ phase strategy
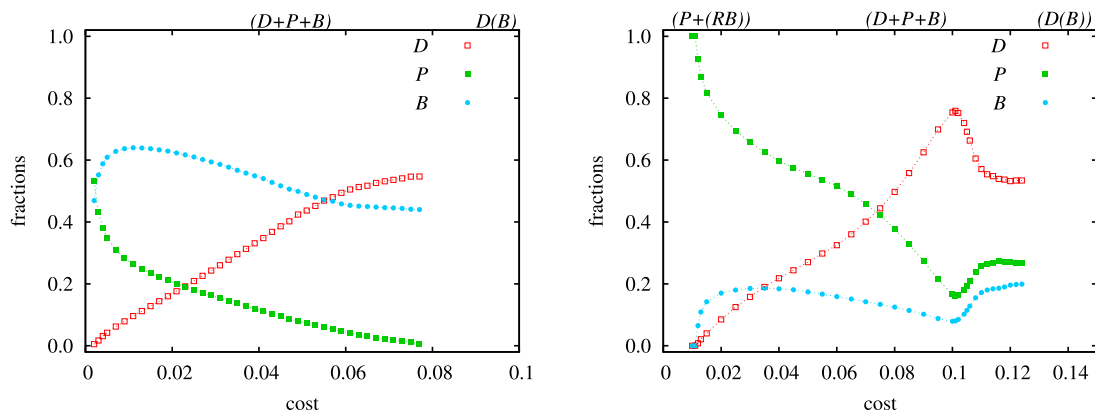
Fig. 2. Cross-section of the phase diagram, as obtained for $\beta = 0.37$ (left) and $\beta = 0.55$ (right). Depicted are stationary fractions of the four competing strategies in dependence on cost γ. Stable solutions are denoted along the top axis. Here $D(B)$ indicates that either a pure D or a pure B phase can be the final state if starting from random initial conditions. On the left, the $D + P + B \rightarrow D(B)$ phase transition is continuous because the average fraction of strategy P gradually decays to zero as γ increases. On the right, however, the $D + P + B \rightarrow D(B)$ phase transition is discontinuous because the amplitude of oscillations diverges independently of the system size as γ increases. These results are reproduced with permission from [18].

D outperforms strategy P, strategy P outperforms strategy B, while strategy B again outperforms strategy D. As is frequently the case [28], here the spontaneous emergence of cyclic dominance brings with it fascinating dynamical processes that are driven by pattern formation, by means of which this phase may terminate. Fig. 2 features two characteristic cross-sections of the phase diagram, which reveal two qualitatively different ways for the $D + P + B$ cyclic dominance phase to give way to the $D(B)$ phase. The process depicted in the left panel of Fig. 2 is relatively straightforward. Here the average fractions of strategies P and B decay due to the increasing cost γ, which ultimately results in the vanishing average value of the fraction of strategy P. The closed cycle of dominance is therefore interrupted and the $D + P + B$ phase terminates.

The situation for $\beta = 0.55$ in the right panel of Fig. 2 is more peculiar and interesting. Here the average values of all three strategies remain finite. Hence, the termination of the $D + P + B$ phase has a different origin than at $\beta = 0.37$. In fact, for $\beta = 0.55$ it is the amplitude of oscillations that increases with increasing values of γ. And it is the increase in the amplitude that ultimately results in a uniform absorbing phase regardless of the system size. Here it is crucial to emphasize that the increase of the amplitude of oscillation is not a finite-size effect. Although in spatial systems with cyclic dominance it is typical to observe oscillations with increasingly smaller amplitude as the system size is increased, this does not hold in the present case, where in fact we have divergent fluctuations in the stationary state.

5. Progress in related fields

Recent progress in three related fields is of particularly importance for the further development of models of human cooperation. In the first place, network science has been going from strength to strength during the past decade and a half, delivering inspirational results, models, and methods, that have revived not just statistical physics, but many other fields of natural and social sciences. The field of network science has definitively come of age [29], and it now allows us to study game-theoretical models on more realistic interaction networks. Changes in our interactions over time can be studied in the realm of temporal networks [10,30], while the interactions not just among us but also among different groups, institutions, and societies, can be accommodated by means of multilayer or interdependent networks [31,32]. Much has also been done recently to identify central individuals in networks, who might be particularly likely to exercise influence on others and promote different behavioral norms [33–35]. These advances shall definitively help us develop better models for describing the rise and fall of cooperation in human societies.

Secondly, with the coming of age of network science, there is also the maturing of computational social science [36], which flourishes with the availability of an ever increasing amount of digital data that the way we are living our lives is leaving behind. While social experiments in the past typically involved one-shot self-reported data on relationships and their outcomes in a small sample of people, the approach today is to mine massive amounts of digitized data for both the structure and the content of relationships [37]. This might include anything from the examination of group interactions through e-mail data to the tracking of movement of people in different environments. While inferring who cooperates and who not, when, why, and under what circumstances, from such digital traces is still a formidable challenge, insights from computational social science will certainly play a prominent role in expanding our understanding of human cooperation. Also important, the field itself fosters awareness and acceptance of computational models for the better understanding of our societies, which is very important for the promotion of this type of interdisciplinary research.

Thirdly, recent years have seen the advent of large-scale behavioral experiments [23], which are made easier by ready-made software designed specifically for such purposes [38], as well as by online recruitment of participants through platforms such as Mechanical Turk [39]. These experiments can target specific aspects of human cooperation, and as such can serve both as validation as well as guide for computational modeling. Also, they may help to pinpoint which parameters affect cooperative behavior and how. For example, experiments have shown that the benefit of cooperation has a positive effect on cooperative behavior [40], and that the group size can have several different effects as well [41], depending on the way the benefit of cooperation increases with the size of the group (see also [20,21]). Another important parameter has been found to be the cognitive effort spent in making a decision. While standard models assume that people have enough cognitive resources to evaluate all possible alternatives before making a decision, this is in fact not always the case. Several studies have found that the amount of cognitive resources spent on making a decision significantly affects cooperative behavior [42,43]. In general, combining evolutionary models with behavioral experiments can generate deep insights into human cooperation, and this is certainly a prospect that is worth exploring to the fullest in the future.

6. Future research

Over the past decade, the application of statistical physics to evolutionary games has delivered a great deal of insight into which strategies, mechanisms, and external factors promote cooperation in game-theoretical models, and why this is case. The time is thus ripe to try and integrate these models together into a more cohesive and relevant theoretical framework, so that it will not look like we are considering just some little fraction of the problem at a time, but that the resulting models will become more relevant for actual human cooperation. For this task, one may find inspiration and guidance in the many works on the subject stemming from anthropology, psychology, and sociology, which are still insufficiently integrated into theoretical research.

There are also several hypotheses available to explain why cooperation thrives in human societies, none of which have yet been integrated into game-theoretical models aimed at addressing the puzzle of human cooperation. There is the "heart on your sleeve" hypothesis, which holds that humans are cooperative because they can truthfully signal cooperative intentions. Most recent research indicates, for example, that third-party punishment is likely a costly signal of trustworthiness among humans [44]. Moreover, cultural group selection hypotheses argue that the importance of culture in determining human behavior causes selection among groups to be more important for humans than for other animals. And there are the moralistic reciprocity hypotheses, which assert that greater human cognitive abilities and advanced language allow us to manage larger networks of reciprocity. These and related hypotheses could all be verified in the realm of evolutionary games in structured populations. In particular, information sharing and the reliability of shared information across different networks representing different groups or populations could reveal whether honest signalling is indeed crucial. In terms of the integration of cognitive abilities, just recently Bear and Rand [45] have introduced a new modeling paradigm for looking at the evolution of cognition (intuition versus deliberation) as well as the evolution of behavior (cooperation versus defection) in an evolutionary setting. Their model is an important first step in taking into account advanced cognitive abilities when studying the evolution of cooperation, and there are many ways in which this model can be extended and studied in the realm of statistical physics of evolutionary games. Network effects, time variations in interactions and interdependencies across different networks, and many other game specific could be considered.

Another important but significantly underexplored subject concerns the differences among us in terms of what we are striving for, and in terms of our personal success and status. All these factors condition and affect our behavior. To illustrate the point, imagine two individuals using public transport. One is wealthy and the other is poor. One might assume that the wealthy individual is less likely to defect by using the service without paying for the ticket. The temptation to defect is higher for the poor individual. This is to remind us that each time we are faced with the choice of either cooperating or defecting, we are likely to perceive differently what we might gain or lose by choosing to cooperate. Here the so-called evolutionary multigames [46–49] provide an apt upgrade to the theoretical framework for properly addressing precisely such situations.

Looking forward, physicists should team up more closely with social scientists, and with their help merge, refine, and upgrade game-theoretical models so that they will become more widely acceptable for describing cooperation in human societies. The hope is to clearly identify strategies and factors that promote human cooperation, and of course, no less importantly, to identify everything that works in the opposite direction. We should utilize methods of statistical physics and network science, and in particular to extend the concepts of phase transitions and universality, for describing and explaining cooperation in human societies, and to come up with useful models that will help guide our efforts towards a sustainable, better future. This line of research has the potential to have a deeply positive impact on pressing challenges that we are facing today, many of which rely on large-scale cooperative efforts. And not just cooperative efforts that have a positive outcome, such as mitigation of social crisis or the preservation of natural resources for future generations, but to understand also those cooperative efforts that have very negative, adverse outcomes, such as acts of terror and oppression. Ultimately, we must learn how to create organizations, governments, and societies that are more cooperative, more productive, and more egalitarian. A predictive, computational theory of human behavior could help us engineer better social systems and inform relevantly on the policies and incentives that could mitigate crisis, inequality, and government failure.

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