



Heterogeneous investments promote cooperation in evolutionary public goods games

Qun Wang^a, Hanchen Wang^a, Zhuxi Zhang^c, Yumeng Li^{a,b,*}, Yu Liu^{a,*},
Matjaž Perc^{d,e,f,**}

^a School of Electronic and Information Engineering, Beihang University, Beijing 100191, PR China

^b Shen Yuan Honors College, Beihang University, Beijing 100191, PR China

^c National Transportation Preparation Office, Beijing 100036, PR China

^d Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, SI-2000 Maribor, Slovenia

^e Center for Applied Mathematics and Theoretical Physics, University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia

^f Complexity Science Hub, Josefstadtstraße 39, A-1080 Vienna, Austria

HIGHLIGHTS

- Heterogeneous investments promote cooperation.
- The microscopic mechanism behind the promotion of cooperation is revealed.
- Heterogeneous investments lead to more robust clusters of cooperators.
- Future research in terms of asymmetric influences on game dynamics is discussed.

ARTICLE INFO

Article history:

Received 11 January 2018

Received in revised form 8 February 2018

Available online 6 March 2018

Keywords:

Evolutionary games

Cooperation

Public goods

Heterogeneous investment

Pattern formation

ABSTRACT

The public goods game is widely accepted as a suitable theoretical paradigm for explaining collective cooperation. In this paper, we investigate the impact of heterogeneous investments on cooperation in groups, where the investment of one player to a particular group depends on the fraction of cooperators in that group. Our research reveals that the level of cooperation is significantly promoted as the level of heterogeneity in the investments increases. By studying the payoffs of players at the boundaries of cooperative clusters, we show that the positive effect on the evolution of cooperation can be attributed to the formation of clusters that are more robust against invading defectors. The presented results sharpen our understanding of cooperation in groups that are due to heterogeneity and related asymmetric influences on game dynamics.

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1. Introduction

The emergence of cooperation among selfish individuals contradicts Darwin's theory, which has attracted much attention in diverse disciplines [1–7]. To explain this challenging issue, researchers often resort to a powerful theoretical framework of evolutionary game theory [8–11]. Two simple games, the prisoner's dilemma game and the snowdrift game, are widely used as typical paradigms in this field. In a typical prisoner's dilemma or snowdrift game, each player can adopt two pure

* Corresponding authors at: School of Electronic and Information Engineering, Beihang University, Beijing 100191, PR China.

** Corresponding author at: Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, SI-2000 Maribor, Slovenia.

E-mail addresses: liyumeng@buaa.edu.cn (Y. Li), b16002@buaa.edu.cn (Y. Liu), matjaz.perc@uni-mb.si (M. Perc).

strategies: cooperate (C) or defect (D). Then, the players play the game in pairs to earn payoff. If they both take the same action for C or D, each will get a payoff of reward (R) or punishment (P). If they choose distinct strategies, the defecting player is tempted to achieve the maximum payoff (T), and the cooperating player receives the sucker payoff (S), with the precondition $2R > T + S$. The ranking of these four payoff for the prisoner's dilemma game is $T > R > P > S$, and it is $T > R > S > P$ for the snowdrift game. The main difference between the games is that the evolutionary stable strategy in the prisoner's dilemma game is mutual defection, whereas in the snowdrift game, mutual defection leads to the lowest payoffs for both players [12–15].

As the prisoner's dilemma game and the snowdrift game are generally used to characterize pairwise interactions, the public goods game (PGG) is used for group interactions [16]. Indeed, the PGG has received unprecedented attention in the physics community in the past decade [17–42]. In the original PGG model, all N participants simultaneously decide whether to contribute (cooperate) or not (defect) to a common pool. Then, the total investment of all cooperators in the public pool is multiplied by a factor r ($1 < r < N$) and contributes equally to all players, regardless of their contributions. Namely, the whole system can reach the best state when all participants invest with the maximum amount to the public pool. However, the Nash equilibrium in PGG is all defection. Participants are faced with the temptation of free riding because all players do better when contributing zero than when they contribute something, regardless of what anyone else does, which can also be called the “tragedy of the commons” [43].

Many real-world systems can be described as networks, in which nodes represent the interacting individuals and edges characterize their interactions. One interesting research direction is to study the evolutionary game dynamics on networks. It has been shown that cooperative behavior can emerge when individuals interact on networks, including regular lattices, small-world, scale-free and dynamical networks [44,2]. Recently, several mechanisms have been put forward to illustrate the evolutionary cooperation of PGG. Guan et al. found that the variation in strategy transfer capability can promote the cooperation level [45]. Segbroeck et al. studied the evolutionary dynamics of repeated group interactions, leading populations to engage in dynamics involving both coordination and coexistence [46]. Santos et al. found that social diversity can remarkably promote cooperation on heterogeneous graphs [47]. Szolnoki et al. focused on the PGG with delayed distribution and found decelerated invasion and waning-moon patterns [48], while in [49] it was even found experimentally that punishment diminishes the benefits of network reciprocity, to name just some examples.

In many previous mechanisms, one cooperator will contribute the same value to the participating groups. However, in reality, the investment of each cooperator can be heterogeneous according to the environment. Thus, it is natural to consider investment heterogeneity in the PGG model. Cao et al. studied an unequal investment mechanism on a scale-free network, in which the investment of players is related to its degree [50]. Yuan et al. presented an investment heterogeneity mechanism in PGG on a square lattice, which allowed the investment of cooperators to be mapped to the fraction of cooperators inside [51]. Both works found that cooperation is promoted by the heterogeneous investment mechanism; however, the total investment of each player is rather different. To further explore the effect of heterogeneous investment on cooperation, we fix the total investment of all players to be 1. It is found that cooperation is still markedly enhanced, and the results are examined by the payoff differences along the boundaries of cooperative domains from a microscopic point of view.

2. The model

Here, each player is located on a site of 100×100 square lattice with periodic boundary conditions and interacts with its Von Neumann neighborhood. Initially, all players are designated as a cooperator (C) or a defector (D) with equal probability 0.5. Then, each player participates in $k_i + 1$ PGG groups, where one PGG group is centered around itself and the other k_i groups are correspondingly centered around their nearest neighbors [47]. The total investment of all players is set to 1. Here, we hypothesize that the investment of a cooperator in a PGG group depends on the proportion of cooperators inside that group. g_x^y is the investment of player x in the PGG group centered around y .

$$g_x^y = \frac{s_x(N_c)_y^\alpha}{\sum_{j \in \Omega_x} (N_c)_j^\alpha} \tag{1}$$

where N_c is the number of cooperators inside x -centered PGG group and Ω_x is the community composed of the neighbors of x and itself. $s_x = 1$ represents C and $s_x = 0$ represents D. All contributions are multiplied by the factor r and are then equally divided among all players. Under such a mechanism, the payoff of an individual x associated with the PGG group centered on individual y is given by

$$m_{x,y} = \frac{r}{k_y + 1} \sum_{i=0}^{k_y} g_i^y - g_x^y \tag{2}$$

where i represents the i th neighbors of player y . The total payoff of the player x can be expressed as

$$M_{x,y} = \sum_{y \in \Omega_x} m_{x,y} \tag{3}$$

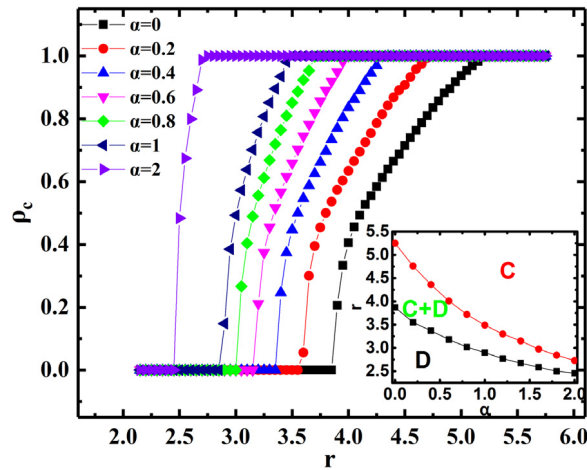


Fig. 1. The cooperation frequency ρ_c as a function of the synergy factor r for different values of α . Inset: The red line marks the transition position to pure C, and the black line marks the transition position to pure D, depending on the compassion parameter α . Noteworthy, these results are to a large independent of the structure of the interaction network, as long as r/G is considered as the dilemma strength [5], where $G = k_i + 1$ is the group size (see also [52]). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where Ω_x is the community made up of the neighbors of x and itself. In the process of evolution, player x randomly selects one of the neighbors to update the strategy and then adopts the strategy of y with a probability

$$W_{x \rightarrow y} = \frac{1}{1 + \exp[(M_x - M_y)/k]} \quad (4)$$

where k characterizes the uncertainty by strategy adoptions [52]. In this paper, we set $k = 0.1$, although other values do not significantly affect our results.

The cooperation frequency is obtained by averaging the last 2000 steps after a transient time of 20,000 steps. For each parameter setting, the final results are averaged over more than 50 independent runs.

3. Results

We first investigate the effect of heterogeneous investment on cooperation frequency ρ_c in Fig. 1. When $\alpha = 0$, the model reduces to the traditional PGG. Herein, cooperators emerge at approximately $r = 3.7$ and dominate the whole system when $r = 5.25$. With the increment of α , the cooperation frequency increases monotonously. When $\alpha = 2$, cooperators emerge even at $r = 2.48$, and defectors quickly go extinct at $r = 2.7$. We then examine the phase transition point, as shown in the inset of Fig. 1. With the increment of α , the range of both the pure D state and the mixed state is greatly reduced. Furthermore, the range of pure C state will become larger with a large value of α .

To further illustrate the mechanism of cooperation enhancement through heterogeneous investment, we investigate the time evolution of cooperation frequency for different values of the parameter α (Fig. 2(a)). In the initial stage of evolution, C and D are evenly distributed in the network. Cooperators are easily invaded by defectors, leading to a decrease in ρ_c . For a standard PGG ($\alpha = 0$), cooperators cannot form clusters and will ultimately go extinct. As the heterogeneous investment mechanism works ($\alpha = 0.2, 0.4, 0.6$), cooperators can survive and form robust clusters (Fig. 2(b)–(d) shows the distribution of C and D at 1000 generations for different values of α). The stationary state of the whole system steps into a mixed C + D state and the size of cooperator clusters' mushrooms (Fig. 2(b)–(d)). When $\alpha = 0.8$, cooperators occupy relatively the whole sites at $T = 1000$ (Fig. 2(e)). When α grows even larger ($\alpha = 1, 2$), the whole system will rapidly transfer to the pure C state.

To explain the promotive impact of the parameter α on the evolution of cooperation, we show a toy model of the investment distribution to examine the interaction for boundary players (Fig. 3(a)). The central cooperator (marked in black) will participate in its 5 non-Neumann neighborhood groups $\{\eta, \delta, \gamma, \theta, \varepsilon\}$. In the traditional PGG ($\alpha = 0$), the investment of the central C player to different groups is unified to 0.2, and its total payoff is 1.516. For the central D player (marked in black), its total payoff is 1.332. When $\alpha = 2$, the central C invests 0.06 to group $\{\gamma, \delta\}$, 0.13 to group $\{\eta\}$ and 0.37 to group $\{\theta, \varepsilon\}$. The total payoff of the central C increases to 1.866, and the payoff of the central D player is reduced to 0.675. The difference between C and D is markedly narrowed. With the increment of α , cooperators tend to contribute more to groups with higher cooperation frequency and from which they can get more benefits. To quantify the ability of parameter α to facilitate and maintain cooperation, we examine the transition probability between the central C and D players (marked

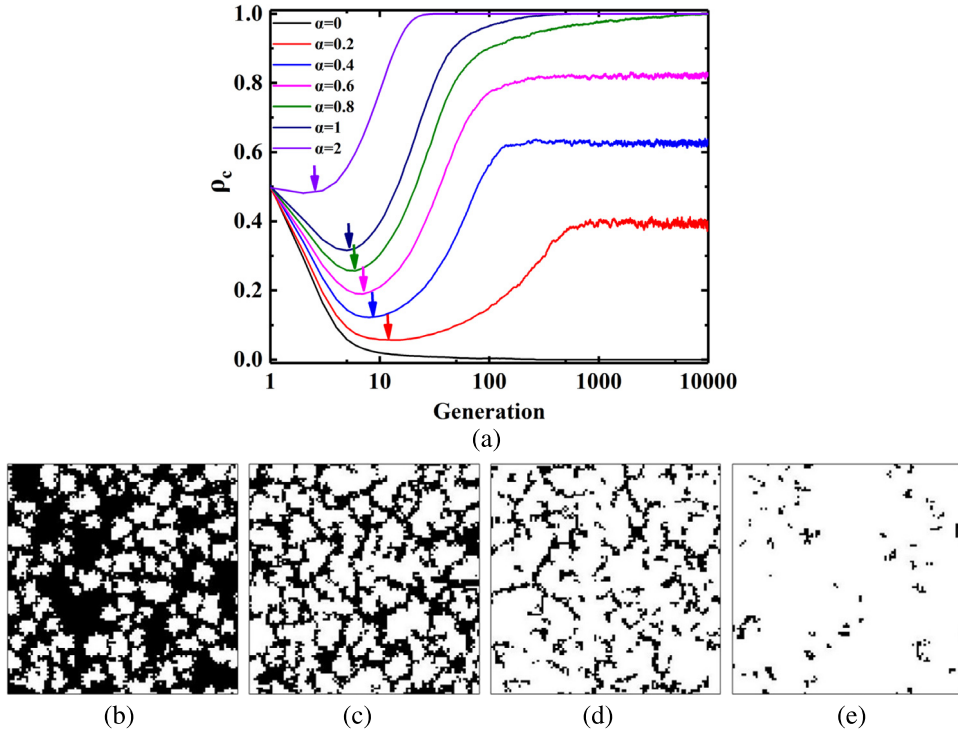


Fig. 2. The top panel depicts the evolution of cooperation of time courses for different values of α . Panels (b)–(d) show a series of distribution snapshots of the cooperators (white) and the detectors (black) on a 100×100 square lattice. The simulations were obtained for $r = 3.7, T = 1000$ and four values of α : (b) $\alpha = 0.2$, (c) $\alpha = 0.4$, (d) $\alpha = 0.6$ and (e) $\alpha = 0.8$.

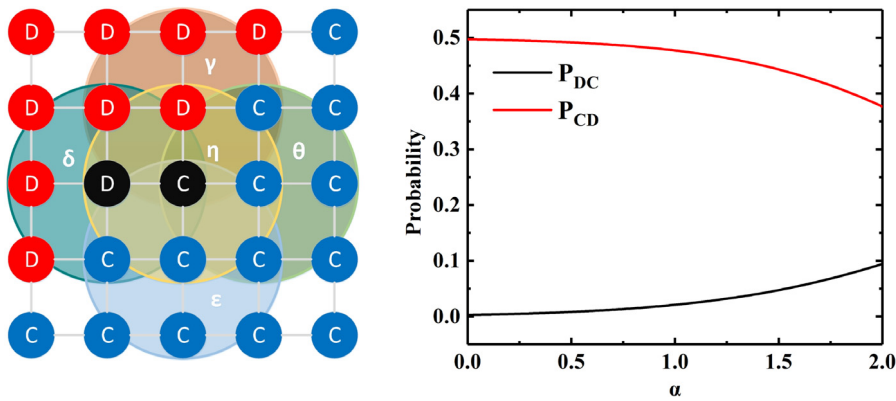


Fig. 3. Left panel shows a typical pattern of the situation of the boundary players in a 5×5 square lattice with a periodic boundary condition. A red circle indicates a defector, and a blue circle indicates a cooperator. The central C (marked in black) participates in 5 PGG groups $\{\eta, \delta, \gamma, \theta, \varepsilon\}$. Right panel is the transition probability between the central C–D pair (marked in black) for different values of α . P_{DC} denotes the probability of the central D transfer to C, and P_{CD} denotes the probability of the central C transfer to D. The depicted result were obtained for $r = 3.7$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

in black) for different values of α . The results presented in Fig. 3(b) indicate that α with the increment of α , the cooperator is more capable to resist the invasion of a defector.

Finally, we investigate the average payoff of boundary cooperators and defectors for different values of α . As shown in Fig. 4, the average payoff of the boundary defectors is always larger than that of boundary cooperators. However, the payoff difference between boundary cooperators and defectors decreases with the increase of α , indicating that the introduction of the heterogeneous investment mechanism weakens the free-rider phenomenon of the boundary defectors and makes the cooperation cluster more powerful against the invasion of defectors.

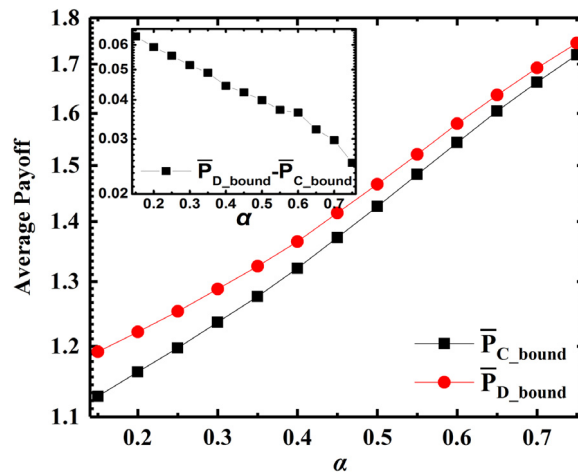


Fig. 4. The average payoff of boundary C and D players (\bar{P}_{C_bound} and \bar{P}_{D_bound}) as a function of α for $r = 3.7$. The inset shows the average payoff gap between boundaries C and D (\bar{P}_{C_bound} and \bar{P}_{D_bound}).

4. Conclusions

In summary, we investigate the evolution of cooperation in evolutionary public goods games by introducing a heterogeneous investment mechanism. In our model, the total investment in a group is mapped to the fraction of cooperators the group contains, and subsequently adjusted by a single parameter α . In this way, cooperators are prone to share much into the groups with higher cooperation levels and will obtain more benefits, encouraging a greater number of players to be cooperators. Focusing on the effect of α on the cooperation level in the population, we give a typical toy model of the boundary situation and quantitatively analyze the transfer frequency between different players. Our study may be helpful for understanding the significant effect of investment heterogeneity on the evolutionary cooperation in the spatial public goods game and related effects [53–55].

Acknowledgments

We thank X. B. Cao, W. B. Du and F. Ye for useful discussions. This paper is supported by the National Key Research and Development Program of China (Grant No. 2016YFB1200100), the National Natural Science Foundation of China (Grant Nos. 61425014, 61521091, 91538204, 61671031 and 61722102), and by the Slovenian Research Agency, Slovenia (Grants Nos. J1-7009 and P5-0027).

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