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Resonance-like cooperation due to transaction costs in the prisoner's dilemma game

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HIGHLIGHTS

- Resonance-like cooperation due to transaction costs in the prisoner's dilemma game.
- Improved network reciprocity in the prisoner's dilemma game.
- Optimal heterogeneity for the evolution of cooperation in social dilemmas.
- Monte Carlo simulations and the pair-approximation analysis confirm the results.

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ABSTRACT

Cooperation is omnipresent in the evolution of social species. In human societies, people voluntarily associate and cooperate with each other to exchange payments, which results in the inclusion of transaction costs during the process. This paper applies transaction costs to the spatial prisoner's dilemma game to better understand the evolution of cooperation by introducing an active–passive mechanism. In particular, a player who actively proposes the game should pay a transaction costs, while the passive responder pays nothing. Using Monte Carlo simulations and pair-approximation analysis, we investigate the frequency of cooperators in the stationary state for different values of the transaction cost, which maintain the same trend. It is found that there exists an optimal value of the transaction cost at which cooperation is optimally promoted. For small or large values of the transaction cost, the promotive effect decreases, and the evolution of cooperation may be impaired if compared to the absence of the active–passive mechanism. Finally, we explain the emergence of large clusters and theoretically confirm the existence of the optimal transaction cost. The mechanism of transaction costs enhancing cooperation resembles a resonance-like phenomenon, which may be helpful in understanding the cooperative behavior induced by the different behaviors between individuals in society.

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1. Introduction

Cooperation behavior has been considered as an essential prerequisite for the maintenance of population diversity in biological systems. The emergence and persistence of cooperation among rational individuals contradicts natural selection theory. In such a puzzle, game theory provides a theoretical platform for different fields [1–5]. Two game models, the prisoner's dilemma game (PDG) and the snowdrift game (SG), are typical paradigms for addressing the cooperative phenomenon, which have drawn much attention [6]. For example, in a typical game of PDG, two players simultaneously decide whether to cooperate (C) or defect (D). If they both choose C (or D), each will receive a payoff of R (or P). If one of them chooses C while the other chooses D, the defector receives a maximum payoff of T, and the cooperator receives S, with the precondition of $2R > T + S$ for game theory. The rank of the four payoff values is $T > R > P > S$. Obviously, the entire population tends to defect within the PDG in well-mixed populations. Therefore, much research has been devoted to exploring mechanisms that can bring about the promotion of the cooperative strategy among selfish players.

To understand the existence of cooperative behavior, Nowak discussed five celebrated mechanisms, including kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection [7–16]. In particular, following the pioneering discoveries of Nowak and May that spatial structure can promote the evolution of cooperation [17], network reciprocity has become a popular theme in statistical physics research [8,18–25]. A series of research studies showed that spatial structure on PDG and other game models can help cooperators to survive on regular networks [26–30] and complex networks [31–38] and even dominate on multilayer networks [39–42]. Santos and Pacheco found the efficient effect of scale-free network structure on the promotion of cooperation frequency in PDG and SG [19,43,44]. In addition, several factors have been considered that influence the evolution of cooperation, such as noise [45], repeated interaction [6], reward [46–48], punishment [49–53], voluntary participation [54,55], compassion [56], and strategic complexity [57–60]. In the meanwhile, network science provides a novel path and also has be applied into the field of data communication [61,62].

In many previous studies, payoff redistribution methods have attracted more attention and brought about pivotal progress in evolutionary game theory, which has been mostly motivated by social economics [63]. Jiang et al. found that reducing the heterogeneity of payoffs is an effective way to promote cooperation in PDG [64]. Xu et al. applied a progressive tax mechanism in a spatial evolutionary game and showed that there existed a nonmonotonic influence in the fraction of cooperators [65]. However, we noticed that transaction costs, as a basic concept of social economics, is neglected. In economics and related disciplines, a transaction cost is a cost in making any economic trade when participating in a market, such as the poundage incurred in the process of stock trade, and bank transfers [66].

For this purpose, we apply transaction costs to the spatial prisoner's dilemma game with the goal of understanding the evolution of cooperation by introducing an active–passive mechanism. In particular, a player who actively proposes the game should pay the transaction costs c, while the passive responder pays nothing. Using Monte Carlo simulations and pair-approximation analysis, we investigate that the frequency of cooperators in the stationary state for different values of c maintains the same trend. It is found that the mechanism of transaction costs promoting cooperation resembles a resonance-like phenomenon, which means there exists an optimal value c enhancing the cooperation frequency. If the regulation is too strong, the cooperation will disappear compared to the absence of the active–passive mechanism. We also present an explanation for the emergency of large clusters and theoretically prove the existence of the optimal price c. This work may be helpful in understanding the cooperative behavior induced by different behaviors between individuals in society.

In the remainder of this paper, we first introduce the spatial game model and the active–passive mechanism. Subsequently, we investigate its effect on the evolution of cooperation in detail. In the last section, we summarize our conclusions.

2. Model

In this paper, we consider the PDG on a 100×100 two-dimensional square lattice network with periodic boundary conditions, choosing the von Neumann neighborhood as the elementary interaction neighborhood. That is, each player occupies a site in the network and is only allowed to interact with the four nearest players. Without loss of generality, we use the simplified version of the PDG to construct a model, where the temptation to defect $T = b$, reward for mutual cooperation $R = 1$, the punishment for mutual defection $P = 0$, and the sucker's payoff $S = 0$, whereby $1 \leq b \leq 2$ ensures a proper payoff ranking. This simplification reduces the variables in the game model while maintaining the underlying meaning of PDG; therefore, it is commonly used in many previous studies [17,26,38,67,68]. Therefore, the payoff matrix of PDG can be written as:

$$A = \begin{bmatrix} 1 & 0 \\ b & 0 \end{bmatrix} \tag{1}$$

The proposed model is constructed as follows. Player x can take one of two strategies, cooperation (C) or defection (D), which are described by the following:

$$s_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} or \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2}$$

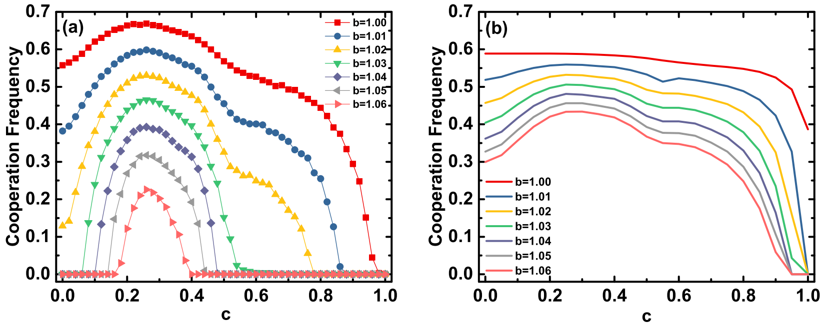

Fig. 1. (Color online) Cooperation frequency as a function of transaction cost c ($0 \leq c \leq 1$) under different values of b. Panels (a) and (b) are the results from the simulation and theoretical analysis based on pair-approximation theory, respectively.

At each time step, after player x interacts with their four neighbors, the player can collect a payoff P_x based on the payoff-matrix elements, which can be expressed as:

$$P_x = \sum_{y \in \Omega_x} s_x^T A s_y \tag{3}$$

where Ω_x represents the nearest neighbor set of node x and A is the payoff matrix. A tunable parameter of transaction cost c ($0 \leq c \leq 1$) is introduced in every generation. In each generation, for every pair of players, one will be randomly selected to be an active player who actively proposes the game. If player x is active in one pair of players, player x must pay a transaction cost c, while the passive responder pays nothing. The number of player x who acts as an active player in the four pairs of players is set as T_x. Therefore, the payoff P_x of player x can be replaced by fitness F_x as follows:

$$F_x = P_x - c \cdot T_x \tag{4}$$

Next, before the start of the next round, a synchronous updating is implemented where player x compares its total fitness F_x with a randomly chosen neighbor y among the four neighbors of player x. Player x will adopt its strategy with a probability H depending on their difference in fitness, which is determined by the Fermi function:

$$H_{s_x \to s_y} = \frac{1}{1 + exp[(F_x - F_y)/\kappa]} \tag{5}$$

where κ characterizes the uncertainty of the stochastic noise effects, including individual trials, fluctuations in payoffs, and errors in decision. Following previous studies, we set the noise level as $\kappa = 0.1$ [60].

3. Simulation results

It is very important to quantify the cooperation frequency ρ_c for studying cooperative behavior. All of our simulation results are the average of the last 1000 of the 20,000 total Monte Carlo (MC) time steps with 20 runs, where the system reaches a steady state. Additionally, the cooperators and defectors are randomly distributed on the nodes of the network at the beginning of the evolution.

For different values of b, the relationship between cooperation frequency and transaction cost c is featured in Fig. 1(a), which shows that cooperation frequency decreases monotonically with increasing b, no matter the value of c. The model is equivalent to a standard PDG when $c = 0.0$; the cooperators become extinct at $b = 1.03$. One can find that the transaction cost c plays an important role in promoting the emergence of cooperators. Among different values of b, the most powerful promotion occurs at $b = 1.03$, and the increment of the cooperation level can be up to 0.4651 when c is 0.26. However, the variation of cooperation frequency as a function of c presents a inverted-U effect phenomenon. It is worth noting that cooperation reaches the highest level regardless of b when c is 0.26, which is a resonance-like cooperation promotion. In other words, there is at least one optimal value of c; a larger or smaller c value will cause a decrease in the cooperation frequency. It is clearly observed that when c increases from 0, cooperation will be enhanced prominently and reaches its maximum at the optimum value ($c = 0.26$); after that, cooperation frequency will begin to decline until there are no more cooperators on the entire network. Herein, one could conclude that the 'institution of transaction cost' can greatly improve the cooperation level, but it fails when the values of transaction cost are set too high.

The dependence of cooperation frequency on c can be qualitatively predicted analytically through a pair-approximation analysis. To verify the relationship between cooperation frequency and transaction cost c (solid curves) in Fig. 1(b), we give an analytical approximation of the spatial dynamics based on the pair approximation [69,70]. Unlike the mean-field theory, which considers the frequency of strategies as well-mixed populations, the pair approximation tracks the frequencies of all possible strategy pairs in the game. In pair approximation, $p_{s,s'}$ indicates the probability of finding an individual adopting strategy accompanied by a neighbor adopting s', where $s, s' \in \{c, d\}$ in the two-strategy PDG. Additionally, c and d indicate the strategies of cooperators and defectors, respectively. To track the time development of the frequencies of all possible strategy pairs in the two-strategy PDG, we thus must determine the following: $\dot{p}_{c,c}$, $\dot{p}_{c,d}$, $\dot{p}_{d,c}$ and $\dot{p}_{d,d}$. Because of the symmetry condition $p_{c,d} = p_{d,c}$ and the obvious constraint $p_{c,c} + p_{c,d} + p_{d,c} + p_{d,d} = 1$ the dynamics of the two-strategy game can be described by the following two differential equations:

$$\begin{aligned}\dot{p}_{c,c} = \frac{2p_{c,d}}{\rho_c^3\rho_d^3}\Bigg\{&\sum_{x,y,z}[n_c(x,y,z)+1]\,p_{d,x}p_{d,y}p_{d,z}\\ &\sum_{u,v,w}p_{c,u}p_{c,v}p_{c,w}H\left[P_c(u,v,w)\rightarrow P_d(x,y,z)\right]\\ &-\sum_{x,y,z}n_c(x,y,z)p_{c,x}p_{c,y}p_{c,z}\\ &\sum_{u,v,w}p_{d,u}p_{d,v}p_{d,w}H\left[P_d(u,v,w)\rightarrow P_c(x,y,z)\right]\Bigg\}\end{aligned} \tag{6}$$

$$\begin{aligned}\dot{p}_{c,d} = \frac{2p_{c,d}}{\rho_c^3\rho_d^3}\Bigg\{&\sum_{x,y,z}[1-n_c(x,y,z)]\,p_{d,x}p_{d,y}p_{d,z}\\ &\sum_{u,v,w}p_{c,u}p_{c,v}p_{c,w}H\left[P_c(u,v,w)\rightarrow P_d(x,y,z)\right]\\ &-\sum_{x,y,z}[2-n_c(x,y,z)]\,p_{c,x}p_{c,y}p_{c,z}\\ &\sum_{u,v,w}p_{d,u}p_{d,v}p_{d,w}H\left[P_d(u,v,w)\rightarrow P_c(x,y,z)\right]\Bigg\}\end{aligned} \tag{7}$$

In these two equations, the sums run over all the possible strategies under consideration. Where $n_s(x, y, z)$ is the number of players adopting strategy s among the players x, y and z, while $\rho_s = \sum_{s'} p_{s,s'}$ is the frequency of each particular strategy s, whereby s' again runs over the set of all possible strategies. Importantly, we note that ρ_s establishes the formal connection between the mean-field theory and the pair approximation by converting pair configuration probabilities $p_{s,s'}$ to the approximations of the configuration probabilities of large clusters. Therefore, the qualitatively identical results of both approaches are warranted. Furthermore, $H\left[P_d(u, v, w) \rightarrow P_c(x, y, z)\right]$ is the strategy adoption function, while $P_s(x, y, z)$ denotes the player who adopts strategy s interacting with the neighbors who adopt strategies x, y, z and a player with strategy s'. Analogously, $P_{s'}(u, v, w)$ denotes the player who adopts strategy s' interacting with the neighbors who adopt strategies u, v, w and a player with strategy s. For details regarding the derivation of individual differential equations, we refer the reader to [69] and the seminal work of Matsuda et al. [70], where the pair approximation method is accurately described.

Note that the fitness difference between players with strategy s and s' in the strategy adoption function should be fully considered because of the introduction of transaction costs. Therefore, the strategy adoption function in Eqs. (6) and (7) can be replaced by the adjusted function as follows:

$$\begin{aligned}H\left[P_s \rightarrow P_{s'}\right] &= \frac{1}{1+exp[(F_s - F_{s'})/\kappa]}\\ &= \sum_{i=0}^{4}\sum_{j=0}^{4} Prob(T_s = i, T_{s'} = j)\\ &\frac{1}{1+exp[((P_s - c\cdot T_s) - (P_{s'} - c\cdot T_{s'}))/\kappa]}\end{aligned} \tag{8}$$

where $Prob(T_s = i, T_{s'} = j)$ denotes the probability that the player with strategy s and s' has to pay i and j transaction costs, respectively.

To explain how and why the different values of c influence cooperation level, we first investigate the time evolution of cooperation frequency for different transaction costs c. The time series of different c values with fixed $b = 1.02$ are shown in Fig. 2. In the inset of the figure, one can find that the ρ_c decreases sharply and that the larger c brings about the higher ρ_c for the first few generations. Subsequently, along with the evolution, the cooperation level increases to a steady state, and cooperators can survive and coexist with the defectors when c is not too large. Among them, moderate c ($c = 0.26$, the blue line) outperforms others and reaches a high cooperation state ($\rho_c = 0.54$). However, when c is too large ($c = 0.85$), the system will be occupied by defectors and falls into the pure D state.

After observing the changing of the lowest point in Fig. 2 (arrows), we investigated the cooperation frequency of the lowest point in the first 1500 generations of the evolution for different values of c and fixed $b = 1.02$ (Fig. 3). The generation

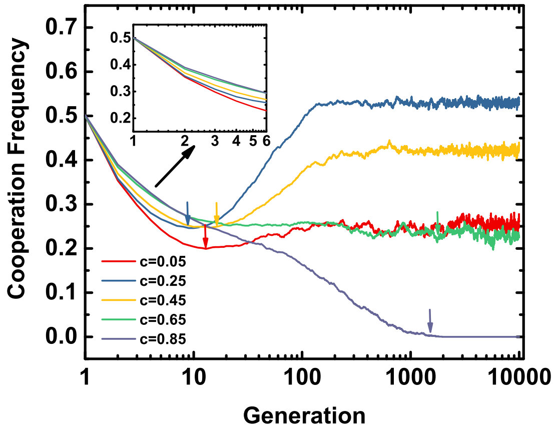

Fig. 2. (Color online) The panel depicts the time evolution of the cooperation frequency for $b = 1.02$, $\kappa = 0.1$ and different values of c. The time series were obtained by averaging 20 independent runs. The inset shows that the enlargement from the first to the sixth time step.

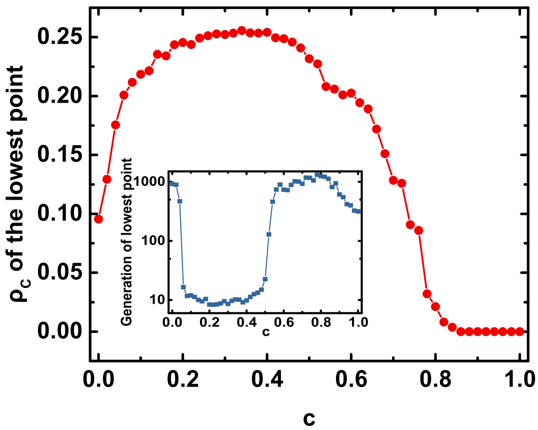

Fig. 3. (Color online) The cooperation frequency and generation number of the lowest point in the first 1500th time steps as a function of transaction cost c are shown in the panel and inset, respectively. Notation b is set as 1.02 and κ is 0.1 as above.

number of the lowest point have also been featured in the inset. One can find that there is some agreement between the trend of the lowest cooperation frequency and cooperation frequency of the steady state. Moreover, the higher the cooperation frequency in the steady state is, the earlier the lowest point is reached, as shown in the inset of Fig. 3. However, after cooperators' extinction in the system ($c > 0.86$), the generation number of the lowest point ($\rho_c = 0.0$) will decrease with increasing transaction cost c. One can conclude that the introduction of transaction cost will help cooperators form the clusters earlier and enhance their abilities to resist the defectors in the early state of the evolution. This particular mechanism somehow relates to the model of payoff matrix noise by Ref. [71–82]. Conversely, if transaction cost is too heavy for the cooperators, the defectors' invasion of cooperation clusters will be accelerated, and cooperators will go extinct faster.

Next, to demonstrate the underlying mechanism behind the above observations, we inspect the snapshots of strategy evolutions from the microscopic point of view at the 10th, 100th, and 1000th steps. In the first 10 steps, cooperators are under the exploitation of the defectors, and the system is dominated by the defectors, causing a decrease in the cooperation level. From Fig. 4(a)/(d)/(g)/(j)/(m), one can observe that the surviving cooperators gather together and form the clusters to resist the defectors. When the time series goes to the 100th step, different values of transaction cost c have different influence on the survival of the cooperators. As shown in Fig. 4(b)/(e)/(h)/(k)/(n), the large difference of the cooperation level can be found even though the number of cooperators does not differ significantly during the early stages of evolution. It has been confirmed that an intermediate c value promotes cooperation, while a smaller or larger value of c will cause the decrease in the cooperation frequency. When c is 0.25, the introduction of a transaction cost has a powerful promotive effect on the formation of cooperation clusters (Fig. 4(f)). Conversely, when c is too large, the transaction cost creates a great convenience for defectors to invade the small cooperation clusters. Cooperators cannot survive under the exploitation of defectors and the heavy transaction cost. Herein, cooperators will ultimately become extinct, and the whole network will be occupied by defectors (Fig. 4(o)). Except for the optimal value of transaction cost, the others can also give enhancements for the ability of cooperation clusters being able to survive (Fig. 4(c)/(i)/(l)). These typical snapshots are displayed to show

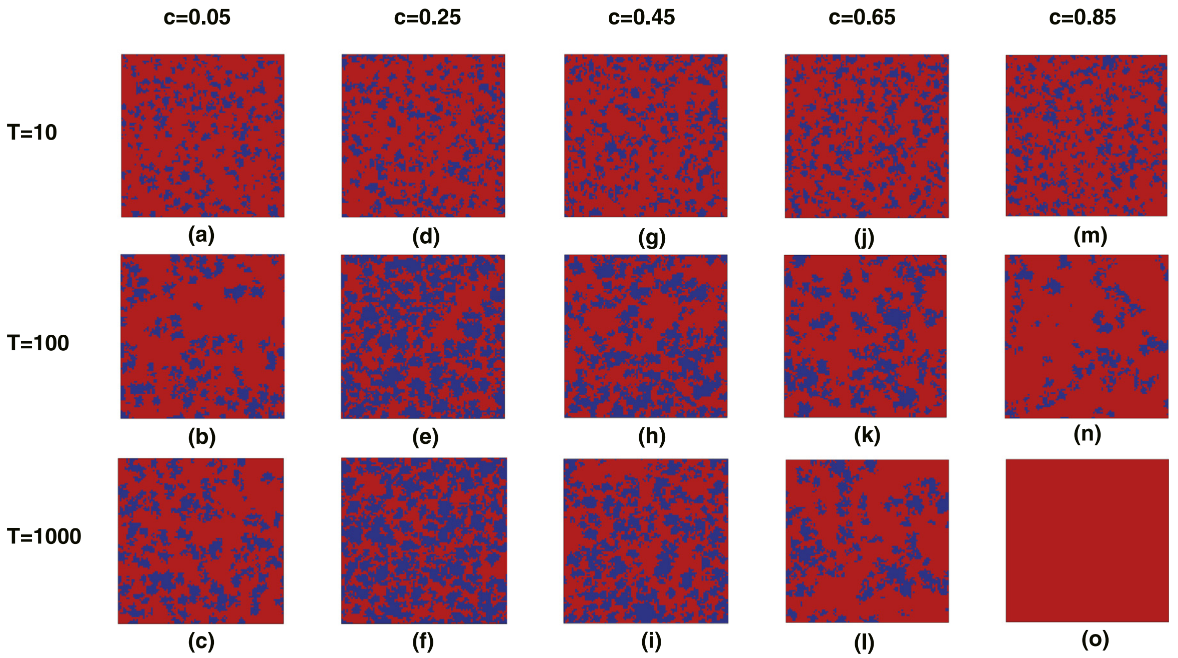

Fig. 4. (Color online) Characteristic snapshots of cooperators (blue) and defectors (red) for different transaction cost parameter c and time T. Columns from left to right: $c = 0.05$, $c = 0.25$, $c = 0.45$, $c = 0.65$, and $c = 0.85$, and Rows from top to bottom: $T = 10$, $T = 100$, $T = 1000$. Depicted results in all panels were obtained for $b = 1.02$ on a 100×100 square lattice . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

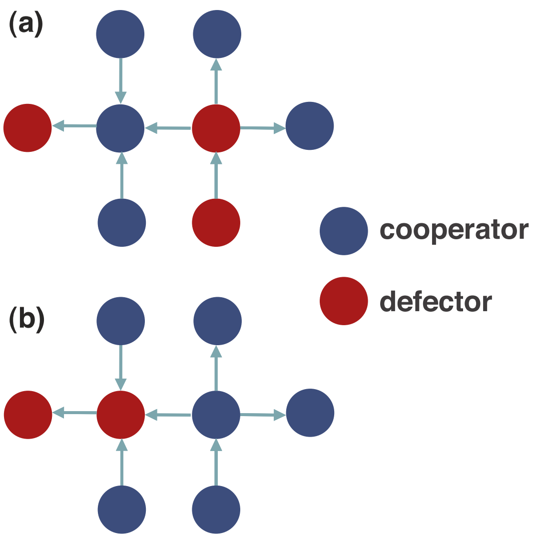

Fig. 5. (Color online) The panels show two typical subpatterns in evolutionary games on square lattices where cooperators and defectors are randomly mixed among the boundary players. A red circle indicates a defector, and a red circle indicates a cooperator. In these patterns, the individuals who are selected to pay the transaction cost are fixed in each pair of players. The arrows from the selected to the others are used to show transaction initiation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

how the system will operate when c is small, intermediate, and large enough. Obviously, most of the cooperators are not distributed in isolation but form some clusters.

To further illustrate the influence of the introduction of transaction cost on cooperators and defectors, we use two typical subpatterns to give a quantitative analysis of the transition probability. Fig. 5(a) denotes a typical subpattern in evolutionary games on square lattices where cooperators and defectors are randomly mixed. Without loss of generality, (each player pays two transaction costs among its four pair of neighbors), we let the center player C have to pay only one transaction cost ($T_C = 1$), and let the center player D have to pay three transaction costs ($T_D = 3$) for $c = 0.25$ and $b = 1.02$. For the standard PDG ($c = 0.0$), C's payoff is equal to 2, and D's payoff is $3 \times b = 3.06$. Based on Eq. (5), the strategy for updating the probability of C changing into D can be easily obtained, which is 0.999975, but the strategy of updating the probability of D changing into C is 0.000025. However, considering the transaction cost into the evolution, the probability of C changing into D is 0.996316, which reduces to 0.996431 times as much as it used to be. The probability of D changing into C is 0.003684, which becomes 148 times larger than that without the transaction cost. From this set, one can find that the introduction of transaction cost makes it difficult for defectors to exploit cooperators in the randomly distributed players during the early stages of evolution, which is beneficial to the formation of cooperative clusters. Fig. 5(b) shows the process of invading a cluster with heavy transaction cost ($c = 0.75$). However, in this set, we let the center player C pay three transaction costs ($T_C = 3$) and let the center player D pay one transaction cost ($T_D = 1$) also with fixed $c = 0.75$ and $b = 1.02$. For $c = 0.0$, C's payoff is 3, and D's payoff is $2 \times b = 2.04$. Similarly, we can easily obtain the strategy updating probability of C changing into D is 0.645656, while the probability of D changing into C is 0.354344. Under the heavy transaction cost, the transition probability of C into D is 0.9999998, which is 1.548811 times larger than it used to be. The transition probability of D into C is 4.73785E−07, which is reduced by seven orders of magnitude. In such a situation, the cost not only increases the difficulty of the cooperators against the exploitation of the defectors but also can introduce a large number of vulnerable points at the edge of the cooperative clusters, making the invasion easier. Therefore, excessive transaction costs are detrimental to the cooperators' survival throughout the system.

Conceptually, coherence resonance within the framework of noise-driven dynamical systems can be identical to this kind of phenomenon [83,84]. Perc has also found similar results that an intermediate intensity of payoffs' noise can mostly promote cooperation [85]. In our case, there always exists an intermediate intensity of transaction cost for which cooperation is mostly maintained. To further understand the resonance-like behavior induced by the transaction cost and validate the above simulation results, the mean-field method is also utilized to have a more detailed investigation of the motion of the cooperation frequency. $W_{C\to D}$ represents the transition probability of cooperators changing into defectors, while $W_{D\to C}$ is denoted as the transition probability of defectors changing into cooperators. In our analysis, under this updating rule, $W_{C\to D}$ and $W_{D\to C}$ can be respectively calculated as [19,86]:

$$W_{C\to D} = \frac{1}{N_C}\sum_{s_i=C} W_i \tag{9}$$

$$W_{D\to C} = \frac{1}{N_D}\sum_{s_i=D} W_i \tag{10}$$

where the N_C and N_D ($N_C + N_D = N$) are the number of the cooperators and defectors in the entire network, respectively. Then, the motion of ρ_c can be approximatively described as:

$$\frac{\partial \rho_c}{\partial t} = (1-\rho_c)W_{D\to C} - \rho_c W_{C\to D} \tag{11}$$

When the system has reached the steady state, $\frac{\partial \rho_c}{\partial t} = 0$. Consequently, cooperation frequency in the steady state can be written as:

$$\rho_c = \frac{W_{D\to C}}{W_{D\to C} + W_{C\to D}} = \frac{1}{1 + W_{C\to D}/W_{D\to C}} \tag{12}$$

As shown in Fig. 6(a), we can obtain the corresponding results based on the above analysis ($W_{C\to D}$ and $W_{D\to C}$) from the simulation. Then, the value of $W_{C\to D}/W_{D\to C}$ can be calculated based on the inset of Fig. 6(a). The transition probability $W_{C\to D}$ is larger than $W_{D\to C}$ for $c < 0.16$ and $c > 0.36$, while $W_{C\to D}$ becomes smaller than $W_{D\to C}$ for $0.16 < c < 0.36$. As a result, the $W_{C\to D}/W_{D\to C}$ is large for $c < 0.16$ and $c > 0.36$ and obtains the smallest value when the transaction cost c is approximately 0.26. By utilizing Eq. (12), we can also understand the changes of ρ_c for c. In particular, when c is approximately 0.26, we can find that $W_{C\to D}/W_{D\to C}$ is greater than those for other values of c. Therefore, the cooperation level obtains its optimal value at approximately $c = 0.26$ for $b = 1.02$. Additionally, based on Eq. (12), the comparison between the simulation results and the theoretical analyses of the cooperation frequency are shown in Fig. 6(b). From the comparison, one can find that the theoretical analyses are consistent with the results of the numerical simulations. The effect of c on cooperation is directly related to $W_{D\to C}$. The described explanation is consistent with the general mechanism of cooperation in the spatial prisoner's dilemma game.

We also investigate the number, the maximal and the average size of the cooperator clusters (N_C, S_C^{max}, and $\overline{S_C}$) and as a function of c, as shown in Fig. 7. When c increases from 0.0, the S_C^{max} and $\overline{S_C}$ are not very large, while when c further increases, more players become cooperators; the cooperator clusters are interconnected with each other and become increasingly larger. When c reaches approximately 0.26, the cooperation frequency and the maximal and average size of cooperator clusters reach their peak value. However, with the continued increase in c, the promotion effect of transaction cost begins to decrease, namely, the cooperation level starts to go down. It is increasingly difficult for cooperators to resist the invasion of the defectors. The cooperator clusters become steadily smaller because of the erosion of defectors until they finally

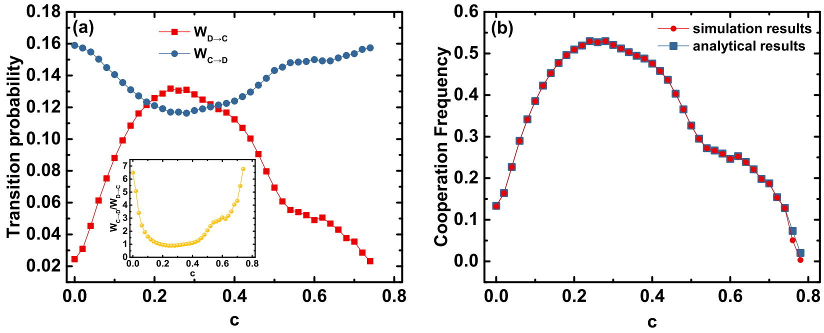

Fig. 6. Panel (a) presents the transition probability $W_{C\to D}$ and $W_{D\to C}$ as a function of the transaction cost c with fixed $b = 1.02$. The inset in Panel (a) shows the value of $W_{C\to D}/W_{D\to C}$ as a function of c. Panel (b) demonstrates the simulation results and the corresponding theoretical analyses of cooperation frequency.

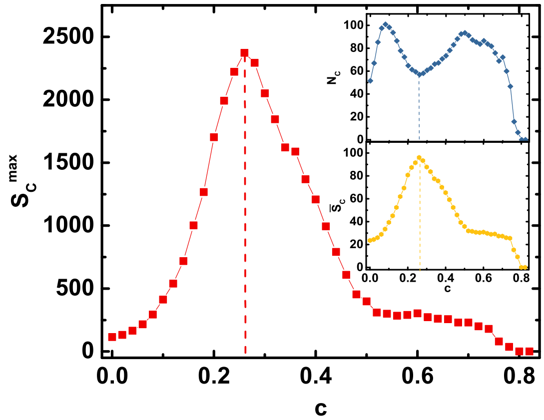

Fig. 7. (Color online) This panel shows that the size of the largest clusters (S_C^{max}) formed by cooperators for different values of c, and the inset presents the number of clusters N_C (upper) and average size of clusters formed by cooperators $\overline{S_C}$ (lower) when $b = 1.02$.

disappear. From the above simulation and analysis results, we can assert that the trend of cooperation frequency is consistent with the maximal and average size of cooperator clusters. It appears that the emergence of the larger cooperator clusters causes the promotion of cooperation. However, for the number of cooperation clusters shown in the inset of Fig. 7, one can find that, when c reaches approximately 0.26, the N_C becomes a valley point between two peaks rather than a peak similar to $\overline{S_C}$ and S_C^{max}. The reason why the number of cooperator clusters decreases sharply when c approximately 0.26 is that the clusters merge into each other and surround the defectors. However, smaller or larger values of c will break up the larger cooperator clusters into smaller ones, causing that the formation of the two peaks. In the previous studies, cooperators survive by forming tight clusters, and cooperators who resist defectors along the boundary can be enhanced by heterogeneous structures [87], neighbor attractiveness [88], and stochastic interactions [89]. Our assertion is consistent with them.

4. Conclusion

In summary, we have investigated the influence of an active–passive mechanism on the evolution of cooperation in the prisoner's dilemma game, where the player's payoff was adjusted by the transaction cost parameter c. The simulation results showed that the cooperation frequency induced by c was resonance-like phenomenon, where an optimal value existed for c. Using Monte Carlo simulations and pair-approximation analysis, we have observed that the frequency of cooperators in the stationary state for different values of c maintained the same trend. Focusing on the effect of c on the system, we have provided the time evolution of cooperation frequency and inspected the snapshots of strategy evolutions. We have further examined typical subpatterns to give a quantitative analysis of the transition probability. Finally, we have proven that the theoretical analysis of the cooperation level is consistent with the numerical simulations of the cooperation frequency. Since transaction costs are common in social economics, we hope that our study to be more useful to understanding the emergence of cooperation in modern society.

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