



# Dynamic evolution of economic networks under the influence of mergers and divestitures



Yinhai Fang<sup>a,b</sup>, Haiyan Xu<sup>a,\*</sup>, Matjaž Perc<sup>b,c,d,\*\*</sup>, Qingmei Tan<sup>a</sup>

<sup>a</sup> College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, 211100, China

<sup>b</sup> Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, SI-2000 Maribor, Slovenia

<sup>c</sup> CAMTP – Center for Applied Mathematics and Theoretical Physics, University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia

<sup>d</sup> Complexity Science Hub Vienna, Josefstädterstraße 39, A-1080 Vienna, Austria

## HIGHLIGHTS

- An evolutionary model with mergers and divestitures on a network is proposed.
- Mergers and divestitures have neither good nor bad characteristics in their own right.
- The success depends on the development status of entities in the economic network.
- Specific guidelines for economic sectors developing specific merger and divestiture policies.

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## ABSTRACT

Mergers and divestitures are two major economic activities. In this article, an evolutionary model combined with mergers and divestitures is proposed from the perspective of complex network science. More specifically, the Axelrod model is introduced to present the identity of entities on the economic network, the Cobb–Douglas production function is used to calculate the power of each entity, and the probabilities of mergers and divestitures are determined by the Fermi function considering key economic indicators during the evolution. We find that mergers and divestitures have neither good nor bad characteristics by themselves, and that the key to the success of these activities is the ability and the development status of entities in the economic network. Identity dimension, learning ability, initial network size, and density usually play important roles in the progress under the influence of mergers and divestitures. Power, degree, age, identity distance, maximal ingredient, network size, and the frequency of mergers and divestitures likewise can give rise to very different processes in different economic environments. The presented results may have a high reference value when economic sectors develop specific merger and divestiture policies, and we also outline directions for future research that seem important and promising along the same lines.

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## 1. Introduction

During the last twenty years, not only did network science become a very popular topic in physics research, but also more and more other disciplines integrate network theory into their primary research [1–7] (e.g., computer science, biology, social science, economy). In the field of economics and management, various interactions and connections can

\* Corresponding author.

\*\* Corresponding author at: Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, SI-2000 Maribor, Slovenia.  
E-mail addresses: [xuhaiyan@nuaa.edu.cn](mailto:xuhaiyan@nuaa.edu.cn) (H. Xu), [matjaz.perc@um.si](mailto:matjaz.perc@um.si) (M. Perc).

be represented as networks [8–16]. Many researchers, in particular, focus on the supply chain network [17–20], global value network [21–24], input–output network [25–27] and financial network [28–33] in econophysics. Indeed, abstract models in existing studies have provided useful insights for the research of trade and business. However, many studies are based on the international perspective (e.g., global trade network) or industry perspective (e.g., supply chain network). Very little economic network research has been done from the meso perspective (e.g., regional economic network and its evolution) and the economic diversity among entities is seldom taken into account in evolution dynamics. An original model based on complex network theory and Axelrod model [34] is proposed in this paper, which focuses on merger & acquisition (M&A) and divestiture among economic entities in the economic system with specific scale, considering the economic diversity, degree, age and power of each entity at the same time during the whole evolution.

M&A is one kind of main business activities during enterprises' expansion, which have an impact on organizational performance [35,36]. R. Rao–Nicholson looked at M&A in countries from association of southeast Asian nations and found that M&A has a positive impact on economy during the financial crisis [37]. E. M. Fich studied the completed M&A deals which do good to shareholders' dollar wealth [38]. Others have investigated various factors that facilitate or hamper M&A including the significant heterogeneity in the total factor productivity, environmental conditions, firm specific circumstances, asymmetric information, laws and regulations and so on [39–42]. Frameworks are also exhibited to guide managers to take M&A strategies considering economy, finance, target, growth strategy, organizational structure and human resource [43,44]. Through literature review, we find that the research on M&A is mainly based on the perspective of business management. However, few people have discussed the impact of M&A on the operation of the entire economic system, which maybe especially important for the department of economic policy.

M&A is more and more popular among enterprises of emerging economy with the development of globalization in the last several years. Meanwhile, few companies devote considerable time and attention to divestiture, which usually taken as a method in response to pressure and seldom as a comprehensive strategy. L. Dranikoff showed that a well-thoughtout divestiture strategy is essential to the success of business [45]. L. Capron analyzed 253 horizontal acquisitions and found that asset divestiture is a logical consequence of reconfiguring the structure of resources supported by M&A [46]. M. Haynes examined the impact of divestiture on firm performance and find that divestiture has a positive, significant and substantial effect on raising the profitability of company [47]. D. Lee conducted a meta-analysis of the relation between divestiture and firm performance [48]. Also, current research mainly focuses on the importance of divestiture to the business of enterprises. Specifically, when and how enterprises should take divestitures to promote their own success. As the same with M&A, the impact of divestiture to the overall economic system has not been given enough attention in the existing literatures. Here, we think about M&A and divestiture based on the introduction of diversity, power, age and degree of each entity at the same time. In our model, M&A is abstracted as the consolidation of two or more entities on the network into a composite entity. Correspondingly, divestiture is the inverse process in which a composite entity will split into several entities. Interactions among entities on the network will produce collective phenomena which is intrinsically different from the behavior of individual entity. Therefore, it is of great practical significance to study the characteristics of economic evolution based on complex network model.

The remainder of this paper is arranged as follows. In the second section, we describe the details of M&A – divestiture evolution model. Then, we present the main results of the simulations and give some discussions in the third section. Finally, we show relevant conclusions and outlook of this work in the fourth section.

## 2. Mathematical model

Graph and network ( $G$ ) are mathematical concepts consisted of vertices or nodes ( $V$ ) connected by links or edges ( $E$ ), presented as  $G = (V, E)$  [49,50]. We consider evolutionary dynamics on a network with  $N$  nodes. Each node in the model represents an economic entity (e.g., bank, factory, airline, hospital). An edge between two nodes represents the business interactions or competition–cooperation relationship between them. An economic network can denote an economic system in a zone or a country. Different nodes on the network mean different entities with different business scope. As described above, some inherent properties could be used to label the entity. For example, the industry catalog is usually used to describe the characteristic of each entity in the real-world, (e.g., agriculture, forestry, fishing, mining, construction, manufacturing, transportation, communications, electric). In our model, identity is introduced as a quantity index for this purpose and each entity is given an identity that has a position in the dimensional space. Vector  $\mathbf{C}_i(t) = (c_{i,1}(t), c_{i,2}(t), c_{i,3}(t), \dots, c_{i,\omega}(t))$  with  $\omega$  attributes represents the identity of an independent economic entity  $i$  at time  $t$ .  $c_{i,k}(t)$  is the value of the  $k$ th attribute and  $k = 1, 2, 3, \dots, \omega$  represent the number of attribute. Given two independent entities  $i$  and  $j$ , the identity distance between them is defined as:

$$d_{i,j} = \sqrt{\sum_{k=1}^{\omega} (c_{i,k} - c_{j,k})^2} \quad (1)$$

Similar entities have little identity distance between pairs of individuals. At the same time, we introduce  $\mathfrak{C}_i(t) = (c_{i,1}(t), c_{i,2}(t), c_{i,3}(t), \dots, c_{i,\omega}(t))$  to represent the identity of entity  $i$  when it belongs to a composite entity ( $U$ ). Then we can calculate the expectation of identity distance among ingredients of the composite entity as follows:

$$\overline{d_i^U} = \frac{2 \times \sum_{i \neq j} \sqrt{\sum_{k=1}^{\omega} (c_{i,k} - c_{j,k})^2}}{h(h-1)} \quad (2)$$

Entities  $i$  and  $j$  are the ingredient of composite entity.  $h$  means the number of ingredient of composite entity. During the process of M&A and divestiture, the development level of entities is a key indicator. Here, the quantity index of power is introduced to be an important indicator to show the development of each entity in the economic system. For example, the power of a company may include research & development, preferential policy benefit, product competitiveness, market share, financial soundness, and so on. Larger value of power stands for a higher level of competitiveness. The power equation is defined according to the form of Cobb–Douglas production function [51]:

$$p_i(t) = p_{i,1}(t)^{\alpha_1} \times p_{i,2}(t)^{\alpha_2} \times p_{i,3}(t)^{\alpha_3} \times \dots \times p_{i,f}(t)^{\alpha_f} \tag{3}$$

$p_{i,k}(t)$  is the  $k$ th sub-power index of entity  $i$ .  $t$  refers to the evolution time.  $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_f$  are the output elasticities of  $p_{i,1}, p_{i,2}, p_{i,3}, \dots, p_{i,f}$  respectively. Here, we consider the constant returns to scale with  $\sum_{k=1}^f \alpha_k = 1$ .

In our model, we calculate the power of each entity by taking into account of age ( $a$ ), degree ( $k$ ), ingredient ( $I$ ) and diversity ( $D$ ). Age of an entity is the time interval from its birth to the present time when the entity is still alive. In particular, age is reset as zero if the entity is merged and reset as one while reinstated as an independent entity. Degree describes how many neighbors one entity can conduct interaction with, which is a key indicator that reflects the level of activity of entity in the economic system. Ingredient represents the number of entities merged by a composite entity. It indicates that there are giant enterprises in the economic system when the value of ingredient is large, which can partly reflects monopoly of the market. Diversity describes the characteristic of attribute of each entity defined as:

$$D_i = \sum_{k=1}^{\omega} c_{i,k} \tag{4}$$

So, the power of entity  $i$  can be defined as follows:

$$p_i(t) = a_i(t)^{\alpha_1} \times k_i(t)^{\alpha_2} \times I_i(t)^{\alpha_3} \times D_i(t)^{\alpha_4} \tag{5}$$

The difference of power between entity  $i$  and  $j$  is

$$\Delta p_{i,j} = |p_i - p_j| \tag{6}$$

Specially, it is need to be noted that the value of age, degree, ingredient and diversity may be different when calculate the expectation of difference of power among ingredients of composite entity. Age ( $a_i^l$ ), degree ( $k_i^l$ ), ingredient ( $I_i^l$ ) and diversity ( $D_i^l$ ) of ingredient  $i$  keep the same as the value before merged when calculate the expectation of difference of power according to Eq. (8). Firstly, we should calculate the diversity of ingredient  $i$  of composite entity:

$$D_i^l = \sum_{k=1}^{\omega} c_{i,k} \tag{7}$$

The power of ingredient  $i$  is given by

$$p_i^l(t) = a_i^l(t)^{\alpha_1} \times k_i^l(t)^{\alpha_2} \times I_i^l(t)^{\alpha_3} \times D_i^l(t)^{\alpha_4} \tag{8}$$

Then, the expectation of power difference inside composite entity ( $U$ ) can be calculated by

$$\overline{\Delta p_i^U} = \frac{2 \times \sum_{i,j \in U, i \neq j} |p_i^l(t) - p_j^l(t)|}{h(h-1)} \tag{9}$$

For the process of M&A, the new born composite entity ( $U$ ) has the same age as sponsor entity  $i$  of M&A and the ingredient of composite entity is:

$$I_i(t+1) = I_i(t) + I_j(t) \tag{10}$$

Here,  $i = U$ . Composite entity can learn from entities that merged by itself, and the identity of composite entity can be calculated as follows:

$$C_i(t+1) = C_i(t) + \rho C_j(t) \tag{11}$$

Here,  $\rho$  is the learning ability. Then, diversity of composite entity can be calculated by identity vector with Eq. (4). The degree of composite entity is given by:

$$k_i(t+1) = [k_i(t) - 1] + [k_j(t) - 1] - k_{com}(t) \tag{12}$$

Here,  $k_{com}$  is the number of shared neighbors of entity  $i$  and  $j$ . Obviously, M&A will change the topology of the economic network.

Considering the activity of divestiture in the model, a composite entity with two or more ingredients selected by random will take divestiture under the specific constraint conditions. The set of new divestiture entities:

$$N_{nd} = \{\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3, \dots, \mathfrak{H}_s\} \tag{13}$$

In general, number of element of  $N_{nd}$  is randomly generated from two to  $i$  ( $2 \leq s \leq I$ ) satisfying constraint condition of the total number of ingredient of these new divestiture entities equal to  $i$  ( $I_1 + I_2 + I_3 + \dots + I_s = I$ ) and the number of ingredient of each new divestiture entity is also randomly assigned. Age of new divestiture entity is all set to one. Identity vector of new divestiture entity has the same dimension with the composite entity ( $U$ ). Every feature is determined at random and satisfies the constraint below:

$$\mathbf{C}_{\mathfrak{S}_1}(t+1) + \mathbf{C}_{\mathfrak{S}_2}(t+1) + \mathbf{C}_{\mathfrak{S}_3}(t+1) + \dots + \mathbf{C}_{\mathfrak{S}_s}(t+1) = \mathbf{C}_U(t) \quad (14)$$

Then, diversity of new divestiture entities can be calculated by identity vector with Eq. (4). The degree of the new divestiture entities calculated as the following two steps: Firstly, the newborn divestiture entities establish relationships with each other randomly with a constant probability  $p_{in}$ . Then, the new divestiture entities make relationships with the neighbors of the father composite entity randomly with a constant probability  $p_{out}$ . The power of new divestiture entities can be calculated by Eq. (5).

Entity  $i$  as the sponsor takes M&A with entity  $j$  with a probability  $W(i \leftarrow j)$  given by the Fermi function as follows:

$$W(i \leftarrow j) = \frac{1}{1 + \exp[(d_{i,j} - \overline{d_{i,j}})/K]} \times \frac{1}{1 + \exp[(p_j - p_i)/K]} \quad (15)$$

Here,  $\overline{d_{i,j}}$  is the average value of identity distance  $d_{i,j}$  ( $i \neq j$ , and  $i, j \in N$ ). Left part of Eq. (15) is the effect of identity distance between entity  $i$  and  $j$  on the probability of M&A and the right part is the effect of the difference of power between entity  $i$  and  $j$  on the probability of M&A.  $K$  denotes the amplitude of noise, the so-called intensity of selection. Composite entity ( $U$ ) take divestiture with the probability  $W(U \rightarrow)$  also given by Fermi function as follows:

$$W(U \rightarrow) = \frac{1}{1 + \exp[(\overline{d_i^U} - \overline{d_i})/K]} \times \frac{1}{1 + \exp[(\overline{\Delta p_i^U} - \overline{\Delta p_i})/K]} \quad (16)$$

Here,  $\overline{d_i}$  is the average value of  $\overline{d_i^U}$  (all the  $U \in N$ ) and  $\overline{\Delta p_i}$  is the average value of  $\overline{\Delta p_i^U}$  (all the  $U \in N$ ). The left part of Eq. (16) is the influence of identity distance among ingredients of composite entity ( $U$ ) on the divestiture probability and the right part is the influence of the difference of power among ingredients of composite entity ( $U$ ) on the divestiture probability.

The initial condition of simulation is that the economic network has  $N$  ( $N = 50, 100, 200$ ) entities randomly distribute on the ER network with connecting to each other with a constant probability  $p_{ori} = 0.25$  and  $0.75$ . The output elasticities satisfy  $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$ . The initial age of each entity is  $a = 1$ , and the initial ingredient  $I = 1$ . Other parameters are set as  $p_{in} = p_{out} = 0.4$  and  $0.9$ ,  $C_i(t=0) = c_i(t=0)$ ,  $\alpha_i'(t=0) = \alpha_i(t=0)$ ,  $k_i^l(t=0) = k_i(t=0)$ ,  $I_i^l(t=0) = I_i(t=0)$ ,  $D_i^l(t=0) = D_i(t=0)$  at the beginning of the simulation. Economic network with the M&A and divestiture evolves according to the following steps:

(1) Select entity  $i$  randomly as the sponsor of M&A or divestiture. If the ingredient of the chosen entity  $i$  is one. Then turn to step (2). If the ingredient of entity  $i$  is  $N$ . Then turn to step (3). If the ingredient of entity  $i$  is among numerical interval of  $[2, N-1]$ . Then turn to step (4).

(2) Choose a neighbor  $j$  of entity  $i$  at random. Calculate the identity distance ( $d_{i,j}$ ) and power difference ( $\Delta p_{i,j}$ ) between entity  $i$  and  $j$  with equation ((1), (2), (4), (5), (6)). Entity  $i$  will merge entity  $j$  with the probability as Eq. (15).

(3) Calculate the expectation of identity distance ( $\overline{d_i^U}$ ) and power difference ( $\overline{\Delta p_i^U}$ ) among the ingredients of composite entity with equation ((2), (7), (8), (9)). Composite entity  $U$  will take divestiture with the probability as Eq. (16).

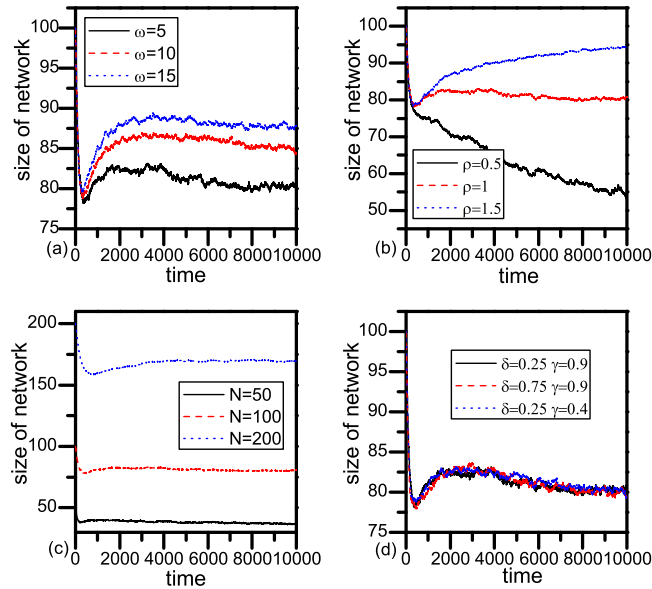
(4) Choose a neighbor  $j$  of entity  $i$  at random. Calculate the identity distance ( $d_{i,j}$ ) and power difference ( $\Delta p_{i,j}$ ) between entity  $i$  and  $j$  with equation ((1), (2), (4), (5), (6)) and expectation of identity distance ( $\overline{d_i^U}$ ) and power difference ( $\overline{\Delta p_i^U}$ ) among ingredients of the composite entity ( $i = U$ ) with equation ((2), (7), (8), (9)). If  $W(i \leftarrow j) \geq W(U \rightarrow)$ , the entity  $i$  will merge entity  $j$  with the probability as Eq. (15). If  $W(i \leftarrow j) < W(U \rightarrow)$ , the entity  $U$  will take divestiture with the probability as Eq. (16).

(5) Repeat from (1) up to a limited number of time steps elapsed.

### 3. Results

#### 3.1. Size evolution of the economic network

Size of economic network is usually considered as the barometer of economic prosperity when we talk about the development of regional economy. Fig. 1 shows the evolution of network size under the influence of different parameters. In Fig. 1(a), the increase of identity dimension will significantly affect the size of network during the whole evolution process. The size of network decreases rapidly to 80% of the initial size at the first 700 time steps no matter what the identity dimension is. Then the increase of identity dimension has a positive impact on the size of network, which means that an entity in an economic system have more attributes (e.g., supply more kinds of products and services) will lead to more prosperous. Fig. 1(b) presents the impact of learning ability on size evolution. It is obvious that the high learning ability will contain the demise of the nodes on the economic network. An economic system with poor learning ability



**Fig. 1.** Size evolution of economic network with different initial situation and parameters. Fig. 1(a) shows the impact of identity dimension ( $\omega$ ) on the evolution of network size. Presented results are obtained for  $N = 100$ ,  $\rho = 1$ ,  $\delta = 0.25$  and  $\gamma = 0.9$ . Fig. 1(b) shows the impact of learning ability on network size. Presented results are obtained for  $N = 100$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$  and  $\omega = 5$ . Fig. 1(c) shows the impact of initial size on the evolution of network size. Presented results are obtained for  $\omega = 5$ ,  $\rho = 1$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$ . Fig. 1(d) shows the impact of initial network density and the connect probability of ingredients after the divestiture activities happens ( $\gamma$ ) on the network size. Presented results are obtained for  $N = 100$ ,  $\rho = 1$  and  $\omega = 5$ . To improve accuracy, the final results are averaged over 100 independent simulations for each set of parameter values.

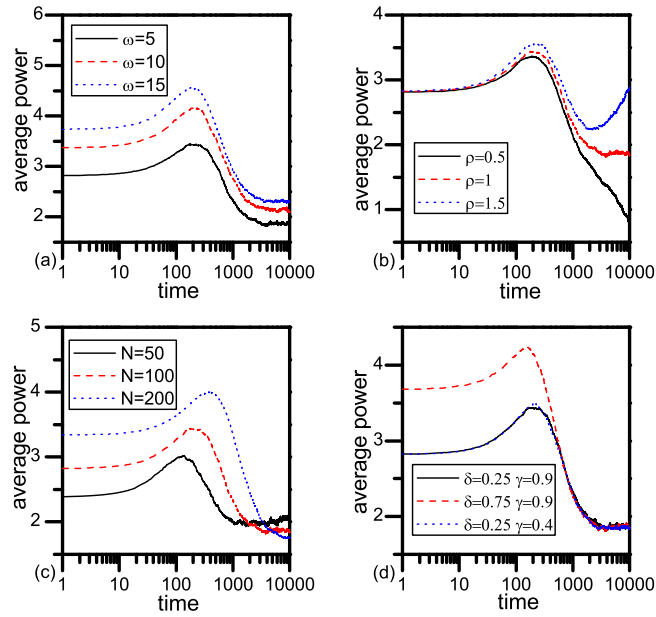
will have a large number of extinctions with the influence of the M&A and divestiture. From the patterns in Fig. 1(c) we can see that the economic network will stay at a state with more entities when the initial size of network is higher from the perspective of absolute number. It is need to be noted that the proportion of extinction is 25.42%, 19.74%, 15.08% when initial size of network is  $n = 50$ ,  $n = 100$ ,  $n = 200$  respectively. It means that big size of initial network really has a positive inhibition of the extinction of nodes. Fig. 1(d) focuses on the initial network density ( $\delta$ ) and the connect probability of ingredients after the divestiture happens ( $\gamma$ ) and find that these two parameters have no significant impact on the evolution of network size.

### 3.2. Evolution of average power on the economic network

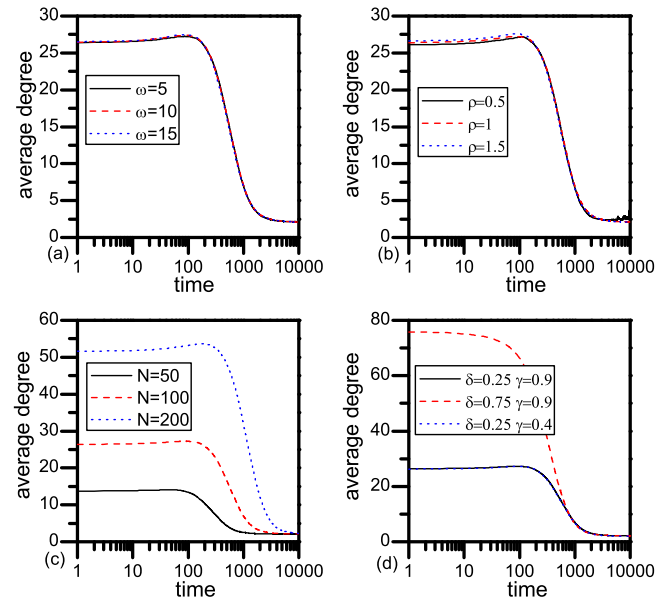
Average power is one of the key indicators reflecting the developing status of the economic system. Fig. 2(a) demonstrates the impact of identity dimension on the average power. It is obvious that the network on which its entities with large range of identity dimensions will get a higher average power. As presented in Fig. 2(b), average power may be greatly improved with the high learning ability. Especially, different level of learning ability lead to different evolution directions after the first 2000 steps. For example, the average will go through a rising process when  $\rho > 1$ . On contrast, it will decrease gradually when  $\rho < 1$ . Fig. 2(c) shows that the effect of the initial network size on the average power is divided into two completely different phases. Large network size has a positive influence on the average at the first period when steps is less than 2000 and then turns into a negative influence when steps is more than 4000. Fig. 2(d) shows that the initial network density has an obvious positive effect on the average power during the first 500 steps and then tend to be the same evolution pattern during the following 9500 steps. On the other hand, the connect probability of ingredients after the composite node takes divestiture has nothing to do with the average power.

### 3.3. Evolution of average degree of the economic network

Degree is one of the simplest and most important concepts for the characterization of nodes on the network. Higher average degree means that entities in the economic system have more relationships with other entities, and the economic system is more stable and secure. In Fig. 3(a, b), it can be seen that the pattern of curve is quite same with different identity dimensions and learning abilities. Fig. 3(c) shows the initial network size has a positive effect on the average degree, but all the economic systems will end at a low average degree. Then, in Fig. 3(d), it is easy to see that the average degree is much higher when the initial network density is high during the beginning of the evolution (i.e., steps  $\leq 800$ ) whereas the role of connect probability of ingredients after the composite node takes divestiture is still weak. In general, the average degree will decrease into a low level under the effect of M&A and divestiture activities no matter what the initial situations and parameters.

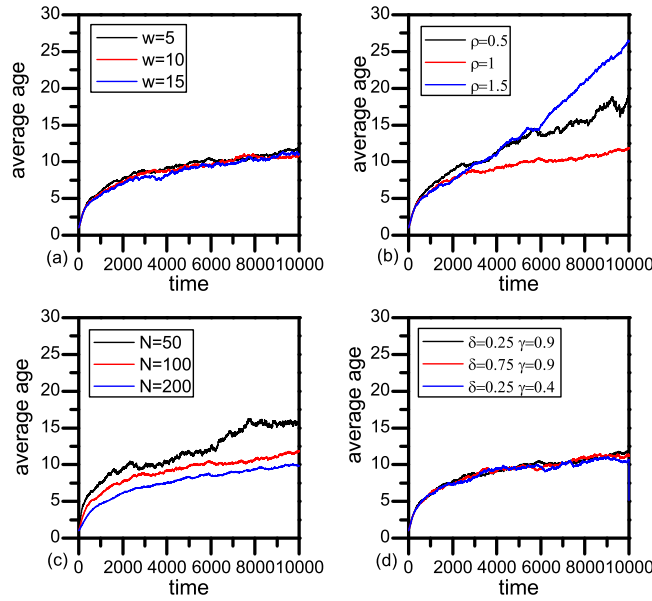


**Fig. 2.** The evolution of average power with different initial situations and parameters. Fig. 2(a) shows the impact of identity dimension ( $\omega$ ) on the evolution of average power. Presented results are obtained for  $N = 100$ ,  $\rho = 1$ ,  $\delta = 0.25$  and  $\gamma = 0.9$ . Fig. 2(b) shows the impact of learning ability ( $\rho$ ) on average power. Presented results are obtained for  $N = 100$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$  and  $\omega = 5$ . Fig. 2(c) shows the impact of initial size ( $N$ ) on the evolution of average power. Presented results are obtained for  $\omega = 5$ ,  $\rho = 1$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$ . Fig. 2(d) shows the impact of initial network density ( $\delta$ ) and the connect probability of ingredients after the divestiture happens ( $\gamma$ ) on the average power. Presented results are obtained for  $N = 100$ ,  $\rho = 1$  and  $\omega = 5$ . To improve accuracy, the final results are averaged over 100 independent simulations for each set of parameter values.



**Fig. 3.** The evolution of average degree with different initial situations and parameters. Fig. 3(a) shows the impact of identity dimension ( $\omega$ ) on the evolution of average degree. Presented results are obtained for  $N = 100$ ,  $\rho = 1$ ,  $\delta = 0.25$  and  $\gamma = 0.9$ . Fig. 3(b) shows the impact of learning ability ( $\rho$ ) on average degree. Presented results are obtained for  $N = 100$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$  and  $\omega = 5$ . Fig. 3(c) shows the impact of initial size ( $N$ ) on the evolution of average degree. Presented results are obtained for  $\omega = 5$ ,  $\rho = 1$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$ . Fig. 3(d) shows the impact of initial network density ( $\delta$ ) and the connect probability of ingredients after the divestiture activities happens ( $\gamma$ ) on the average degree. Presented results are obtained for  $N = 100$ ,  $\rho = 1$  and  $\omega = 5$ . To improve accuracy, the final results are averaged over 100 independent simulations for each set of parameter values.





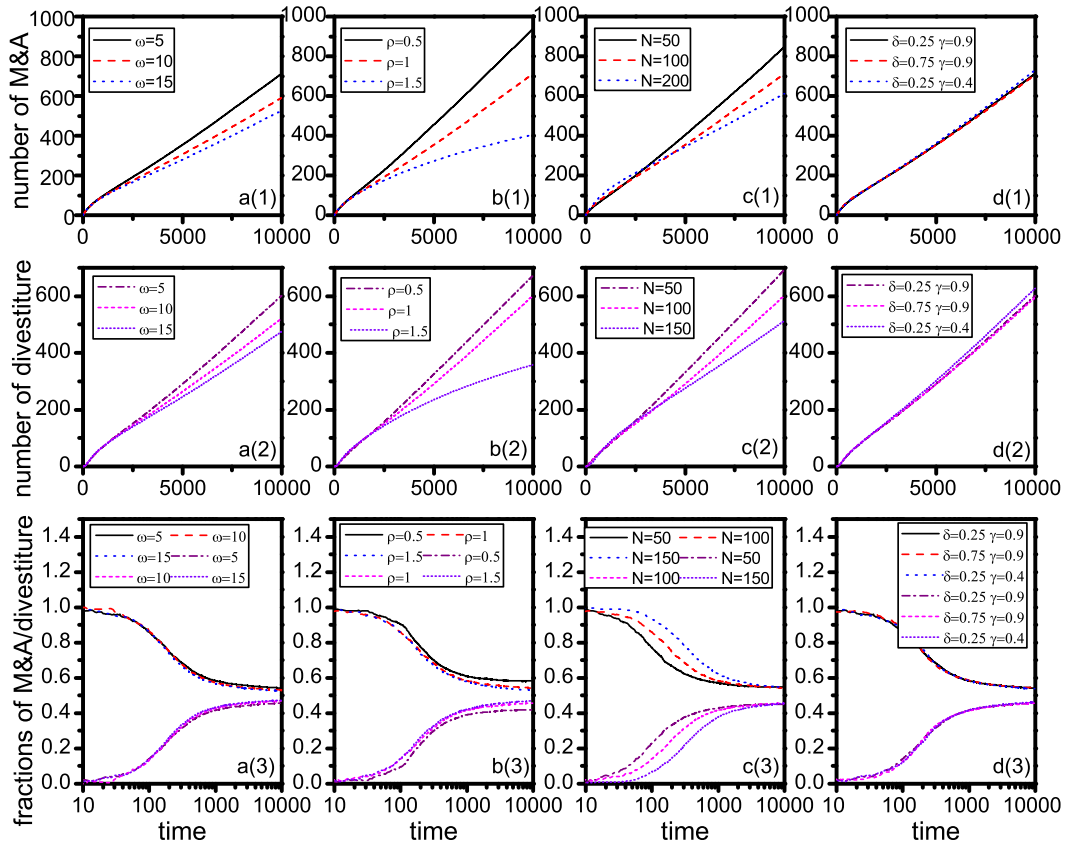
**Fig. 4.** The evolution of average age with different initial situations and parameters. Fig. 4(a) shows the impact of identity dimension ( $\omega$ ) on the evolution of average age. Presented results are obtained for  $N = 100$ ,  $\rho = 1$ ,  $\delta = 0.25$  and  $\gamma = 0.9$ . Fig. 4(b) shows the impact of learning ability ( $\rho$ ) on average age. Presented results are obtained for  $N = 100$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$  and  $\omega = 5$ . Fig. 4(c) shows the impact of initial size ( $N$ ) on the evolution of average age. Presented results are obtained for  $\omega = 5$ ,  $\rho = 1$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$ . Fig. 4(d) shows the impact of initial network density ( $\delta$ ) and the connect probability of ingredients after the divestiture activities happens ( $\gamma$ ) on the average age. Presented results are obtained for  $N = 100$ ,  $\rho = 1$  and  $\omega = 5$ . To improve accuracy, the final results are averaged over 100 independent simulations for each set of parameter values.

### 3.4. Evolution of average age of the economic network

In real-life society, average age is an important indicator measuring life expectancy of enterprises in the economic system. Here, we consider five different factors may affect average age of the economic network. As shown in Fig. 4(a, d), the identity dimension and initial network size have a little influence on the average age. In general, average age will increase from 1 to around 10 during the whole evolution. However, in Fig. 4(b), we observe that the effect of learning ability on average age is not monotonic (e.g., the network has higher average age in the case of  $\rho = 0.5$  and  $\rho = 1.5$  than in the case of  $\rho = 1$ ). But the oscillation of the curve is large during the ascent when the learning ability is low ( $\rho = 0.5$ ), which means that the economic system has a more uncertainty. While the learning ability is high ( $\rho = 1.5$ ), we can see an approximate stable rise curve. In Fig. 4(d), on the one hand, we can find that the initial network density has a negative effect on the average age (e.g.,  $\delta = 0.25$  has a higher average age than  $\delta = 0.75$  during the whole evolution). On the other hand, the connect probability of ingredients after divestiture happens has a positive influence on the average age (e.g.  $\gamma = 0.4$  has a lower average age than  $\gamma = 0.9$  during the whole evolution).

### 3.5. Evolution of M&A and divestiture

To find out the influence of different parameters in our model on the M&A and divestiture, we study the evolution of the number and frequency of M&A and divestiture in different situations. As shown in Fig. 5(a(1)–c(1)) and (a(2)–c(2)), the increase of identity dimension, learning ability and initial network size has a negative effect both on the M&A and divestiture. It means that, M&A and divestiture in the economic system with less identity dimension, low learning ability and little initial network size will occur more frequently. It is need to be noted, the different parameters have different influence degree during the evolution process. For example, the number of M&A and divestiture in an economic system with high learning ability is about half of the low learning ability system. From Fig. 5(d(1), d(2)), we can find that the initial network density and connect probability of ingredients after divestiture happens has a limited impact on M&A and divestiture. Based on this, It is obvious that M&A and divestiture are mainly affected by identity dimension, learning ability and initial network size instead of the structure of network. The fraction of M&As and divestitures are presented in Fig. 5(a(3)–d(3)). In general, the fraction of M&A is higher than divestiture during the whole evolution process. Especially, M&A is the main activities at the beginning of the evolution and then decrease gradually to a stable state. In contrast, the fraction of divestiture increases form 0 to a stable state. Fig. 5(b(3)) shows that low learning ability has a little positive influence on the fraction of M&A and a little negative influence on divestitures. Fig. 5(c(3)) shows that the large initial network size has an obvious positive effect on the fraction of M&A and negative effect on divestiture when steps less



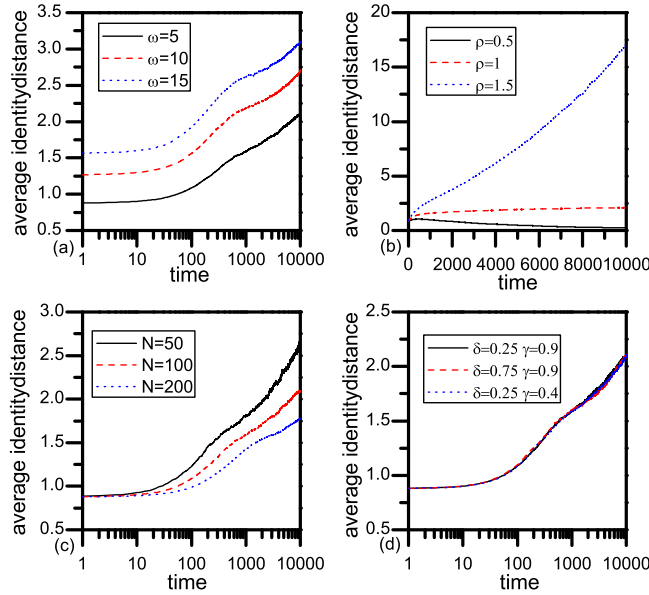
**Fig. 5.** The evolution of the quantitative characteristics of M&A and divestiture with different initial situations and parameters. Fig. 5(a(1)–d(1)) and (a(2)–d(2)) show the evolution of number of M&A and divestiture respectively. Fig. 5(a(3)–d(3)) present the evolution of the fraction of M&A (black solid line, red dashed line and blue dotted line) and divestiture (purple dash-dotted line, magenta short-dashed line and violet short-dotted line). From left to right, the influence of identity dimension ( $\omega$ ), learning ability ( $\rho$ ), initial network size ( $N$ ) and initial network density ( $\delta$ ) and the connect probability of ingredients after the divestiture activities happens ( $\gamma$ ) are considered respectively. To improve accuracy, the final results are averaged over 100 independent simulations for each set of parameter values.

than 3000. The identity dimension, initial network density and the connect probability of ingredients after the divestiture happens cannot change the evolution style of the fraction of M&A or divestiture.

### 3.6. Evolution of average identity distance

Average identity distance is an important indicator for measuring the economic diversity within the economic system. Fig. 6(a) shows that the increase of identity dimension has an obvious positive influence on average identity distance (e.g., the system has a much lower average identity distance when  $\omega = 5$ , and has higher average identity distance while  $\omega = 15$ ). In general, the average identity distance is low at the beginning of the evolution and increases gradually before 100 steps. Then the growth rate will speed up during the following evolution process for different value of the identity dimensions. the pattern shows that the increase of identity dimension will increase the differences between enterprises, which means that the increase of identity dimension will increase the diversity of economy on the network. Fig. 6(b) shows that learning ability has an obvious influence on the average identity distance. Average identity distance will decrease gradually when the value of learning ability is less than 1. While the value of learning ability is larger than 1, the decreasing style turns into increasing style (e.g., average identity distance decreases into 0 when  $\rho = 0.5$ , increases quickly when  $\rho = 1.5$  and basically will not change when  $\rho = 1$ ). Fig. 6(c) shows that the initial network size has a negative effect on the average identity distance. the average identity distance in small-scale economy is much larger than the large economy. It is need to be noted that the large value of average identity distance cannot represent the large value of the total identity distance for different network size. Although the average identity distance is low in networks with large size, but the total identity distance may still be very high after considering the impact of network size. The impact of initial network density and the connect probability of ingredients after the divestiture happens have little influence on the average identity distance as shown in Fig. 6(d).





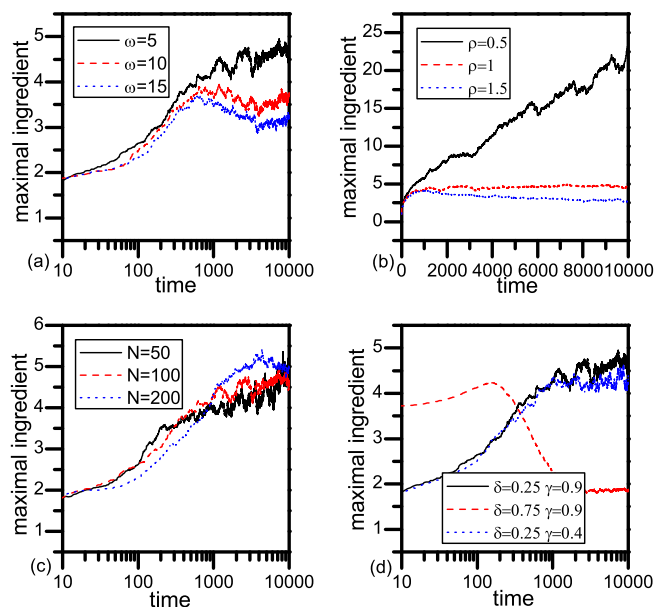
**Fig. 6.** The evolution of average identity distance with different initial situations and parameters. Fig. 6(a) shows the impact of identity dimension ( $\omega$ ) on the evolution of average identity distance. Presented results are obtained for  $N = 100$ ,  $\rho = 1$ ,  $\delta = 0.25$  and  $\gamma = 0.9$ . Fig. 6(b) shows the impact of learning ability ( $\rho$ ) on average identity distance. Presented results are obtained for  $N = 100$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$  and  $\omega = 5$ . Fig. 6(c) shows the impact of initial size ( $N$ ) on the evolution of average identity distance. Presented results are obtained for  $\omega = 5$ ,  $\rho = 1$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$ . Fig. 6(d) shows the impact of initial network density ( $\delta$ ) and the connect probability of ingredients after the divestiture activities happens ( $\gamma$ ) on the average identity distance. Presented results are obtained for  $N = 100$ ,  $\rho = 1$  and  $\omega = 5$ . To improve accuracy, the final results are averaged over 100 independent simulations for each set of parameter values.

### 3.7. Evolution of the maximal ingredient

Maximal ingredient on the economic network can be used as an indicator to reflect largest enterprises in the economy. Fig. 7(a) shows that the small identity dimension has a positive effect to form a larger economic entity. The difference of this influence between small dimension and large dimension is weak at the beginning of evolution. Then as the M&A and divestiture go on, the difference is obvious. Fig. 7(b) shows that the maximal ingredient experiences a rapid increase during the evolution when learning ability is low (e.g.,  $\rho = 0.5$ ). Initial network size has no obvious effect on the maximal ingredient as presented in Fig. 7(c). As shown in Fig. 7(d), it is interesting to see that the maximal ingredient experiences a slow increase and rapid decrease process, and then stays at a very low value level when initial network density is high (e.g.,  $\delta = 0.75$ ). On the other hand, maximal ingredient experiences a long-time increase and then oscillates around 4.5, which means that the initial network density has an obvious on maximal ingredient. In general, the value of maximal ingredient varies between 2 and 5. But economic systems with lower learning ability have contributed to the formation of giant companies. It is likely to eventually lead to the formation of monopoly, thereby reducing the efficiency of economic operations.

## 4. Discussion

In summary, we propose a new M&A – divestiture evolution model, which considers a new mechanism that focuses on the impact of M&A and divestiture on economic network. Axelrod mode is introduced as a quantity index to describe the inherent properties of different entities. Power of entities is got by the Cobb–Douglas production function with the consideration of age, degree, ingredient and diversity of entities. The difference of power and identity between two entities determines the probability of M&A and the difference of power and identity among ingredients of composite entity determines the probability of divestiture. The specific values of both probability can be calculated by Fermi function. We investigated the impact of identity dimension, learning ability, initial network size, initial network density and the connection probability of ingredients after the divestiture happens on the evolution of economic network with M&A and divestiture go on. Identity dimension has positive effects on network size, average power, average identity distance and negative effects on number of M&A, divestiture and maximal ingredient. Learning ability has positive effects on network size, average power, average identity distance and negative effect on maximal ingredient. It is need to be noted that the effect of learning ability on average age experiences a negative effect to positive effect during the increase of learning ability. Initial network size has a positive effect on the network size, average power (positive effect at the beginning 2000 steps and then turns into negative effect), average degree, and negative effect on the average age, the number of M&A



**Fig. 7.** The evolution of maximal ingredient with different initial situations and parameters. Fig. 7(a) shows the impact of identity dimension ( $\omega$ ) on the evolution of maximal ingredient. Presented results are obtained for  $N = 100$ ,  $\rho = 1$ ,  $\delta = 0.25$  and  $\gamma = 0.9$ . Fig. 7(b) shows the impact of learning ability ( $\rho$ ) on maximal ingredient. Presented results are obtained for  $N = 100$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$  and  $\omega = 5$ . Fig. 7(c) shows the impact of initial size ( $N$ ) on the evolution of maximal ingredient. Presented results are obtained for  $\omega = 5$ ,  $\rho = 1$ ,  $\delta = 0.25$ ,  $\gamma = 0.9$ . Fig. 7(d) shows the impact of initial network density ( $\delta$ ) and the connect probability of ingredients after the divestiture activities happens ( $\gamma$ ) on the maximal ingredient. Presented results are obtained for  $N = 100$ ,  $\rho = 1$  and  $\omega = 5$ . To improve accuracy, the final results are averaged over 100 independent simulations for each set of parameter values.

and divestiture, average identity distance. Initial network density has a positive effect on average power, average degree, and can lead to different evolution style of maximal ingredient. It is interesting to note that the connect probability of ingredients after the divestiture happens has little effect on the evolution progress.

With regard to these results, we emphasize the important role of M&A and divestiture in the development of the economy. Proper M&A and divestiture are conducive to economic development, while improper M&A and divestiture will have a negative impact on the economy. Enterprise decisions whether to take M&A or divestiture must be based on their own specific state of development. Also, our conclusions might help the government formulate effective economic policies and promote the development of the economic system. The present model can be extended to different interaction network topologies, and we hope it can be sustained in many real systems, and we also hope it can provide useful insights to resolve economic deadlock in general. Furthermore, there are also open research directions in terms of economic diversity and in terms of considering the identity distance among entities on different networks. It may also be possible to simulate different initial conditions [52], different factors of influence [53], or even scenarios of evasion [54], all of which have shown to have an impact on cooperation in general [55,56].

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