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## Research



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# Synergistic third-party rewarding and punishment in the public goods game

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We study the evolution of cooperation in the spatial public goods game in the presence of third-party rewarding and punishment. The third party executes public intervention, punishing groups where cooperation is weak and rewarding groups where cooperation is strong. We consider four different scenarios to determine what works best for cooperation, in particular, neither rewarding nor punishment, only rewarding, only punishment or both rewarding and punishment. We observe strong synergistic effects when rewarding and punishment are simultaneously applied, which are absent if neither of the two incentives or just each individual incentive is applied by the third party. We find that public cooperation can be sustained at comparatively low third-party costs under adverse conditions, which is impossible if just positive or negative incentives are applied. We also examine the impact of defection tolerance and application frequency, showing that the higher the tolerance and the frequency of rewarding and punishment, the more cooperation thrives. Phase diagrams and characteristic spatial distributions of strategies are presented to corroborate these results, which will hopefully prove useful for more efficient public policies in support of cooperation in social dilemmas.

#### 1. Introduction

Cooperation is an ubiquitous phenomenon in social and biological communities [1–10]. However, individuals driven by selfishness to obtain higher personal pay-offs often defect, which can seriously undermine the level of cooperation and ultimately lead to the so-called tragedy of the commons [11]. For example, the emergence of climate warming [12], acid rain [13], ozone hole [14], soil erosion [15], excessive or suboptimal use of antibiotics [16] and imperfect vaccination plans [17], can all to various degrees be linked to cooperation and pose threats to the survival and development of humankind in the foreseeable future.

Theoretical and experimental studies have proposed effective approaches to alleviate the decline of cooperation in repeated interactions [18–25]. Researchers have considered various mechanisms, such as network reciprocity [19,26–34], tit-for-tat [35,36], reputation [37–39], extortion [40–43], migration [44–49] and diversity [50–55], to mention just some.

In parallel, rewarding for prosocial actions [56–62] and punishment of defection [63–71] have also been considered prominently, and have been proven to be to some degree effective in promoting cooperation. The drawback of punishment and reward, however, is that they are both costly actions, which will lead to the emergence of second-order free-riders, referring to players who choose to contribute to cooperation but abstain from punishing or rewarding others [72–77]. Recent research has shown, for example, that second-order free-riding on antisocial actions, so punishment aimed at cooperators or rewarding aimed at defectors, might restore the effectiveness of prosocial actions [78]. Sharing the efforts of costly actions, such as punishment [75], or allowing for adaptation and self-organization instead of constant adherence to costly actions [58,79] has in theory also proven to be effective towards resolving the second-order free-riding problem.

Traditionally, punishment and reward have been assumed to emerge from the behaviour of players within the game and were usually studied separately. However, in more realistic scenarios, punishment and reward are usually carried out simultaneously. Here, we expand the scope of punishment and reward rules in the public goods game by thus considering them as potentially concurrent actions, but conducted by a third party. In particular, we introduce a threshold parameter to decide whether to punish or reward, and the intensity factor that determines the strength of punishment and/or reward during each game. Moreover, we also examine the influence of the frequency of punishment and reward on the evolution of cooperation.

The remainder of this paper is arranged as follows. Section 2 provides a description of the spatial public goods game with third-party punishment and reward. The results are presented in §3, whereas the last section summarizes our main conclusions.

# 2. Spatial public goods game with the synergy effect of punishment and reward

In our model, individuals are randomly arrayed on a square lattice of size  $L \times L$  with periodic boundary conditions. Each player has G-1 direct neighbours and participates in G=5 public goods games. At first, every individual is designed as a cooperator  $(s_i=C)$  or a defector  $(s_i=D)$  with equal opportunity. Cooperators need to pay the same cost (c=1) to the common pool, while defectors contribute nothing.  $N_c$  is the number of cooperators in the game, and the final contribution in the group needs to multiply the enhancement factor R ( $R \in [1, G]$ ). In addition to cooperators and defectors who directly involved in the game, we introduce a third party to punish or reward the behaviour in the game. The total resource in the pool is the sum of final contributions from players and the rewards or punishment from the third party, and then distributes equally in the group regardless of the individual's specific strategy. Therefore, the player i obtains pay-off  $P_i^g$  from a group g expressed by the following mathematical formula:

$$P_{i}^{g} = \begin{cases} \frac{R \times N_{c} \times (1 + \Pi_{g})}{G} - 1 & \text{if } S_{i} = C\\ \frac{R \times N_{c} \times (1 + \Pi_{g})}{G} & \text{if } S_{i} = D \end{cases}$$
 (2.1)

Here,  $\Pi_g$  represents the synergy effect of reward and punishment with the consideration of the intensity factor  $(\alpha)$  and the threshold of punishment and reward  $(\beta)$  in our model. The individuals in the group will get punishment when the number of cooperators  $(N_c)$  in the group satisfies  $N_c < \beta$ . By contrast, individuals will receive rewards. The strength of reward or punishment will become stronger as the gap between  $N_c$  and  $\beta$  becomes wider. Intensity factor reflects the maximal possible strength of punishment  $(-\alpha)$  and reward  $(\alpha)$ . Also, parameter  $\tau$  is granted to indicate the time interval for the third party to take punishments and rewards, and  $t_i$  represents the successive steps of evolution. The synergy effect can be calculated as below:

$$\Pi_{g} = \begin{cases}
\alpha \times \frac{(N_{c} - \beta)}{G} & \text{if } t_{i}\%\tau = 0 \\
0 & \text{if } t_{i}\%\tau \neq 0
\end{cases}$$
(2.2)

Since each player participates in five (G=5) public goods games, accordingly, the total payoff is  $P_i = \sum_{g \in G} P_i^g$ . And then, individual i will imitate the strategy of individual j with the probability as follows:

$$W(S_i \leftarrow S_j) = \frac{1}{1 + \exp[(P_i - P_j)/K]},$$
(2.3)

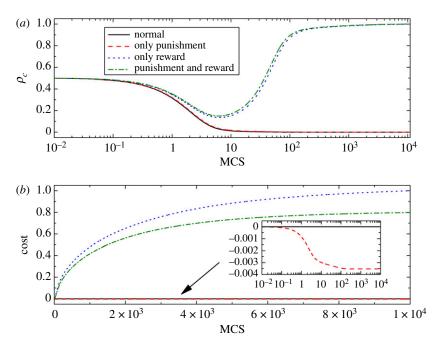
where K portrays the noise introduced to permit irrational choices. We set the noise level to be K = 0.5 according to many previous studies [9,80,81].

The numerical results of Monte Carlo simulations are obtained on  $200 \times 200$  to  $800 \times 800$  periodical boundary square lattice. There are  $L \times L$  games during one full Monte Carlo step (MCS) to make sure each player has a chance to conduct a game in one MCS. The key quantity of the cooperation ( $\rho_c$ ) is got on the steady state. Furthermore, each data are obtained by the averaging over 10 different realizations in order to ensure suitable accuracy.

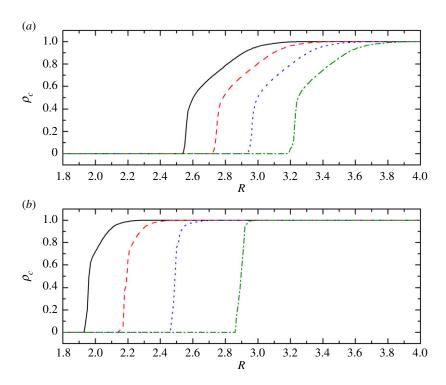
## 3. Results

Before presenting the main results of the synergy effect of punishment and reward on the cooperation, we first compare the evolution of cooperation in four different situations: neither punishment nor reward (default case), only punishment, only reward, punishment and reward. In figure 1a, the presented results indicate that the fraction of cooperators declines rapidly during the early evolution (i.e.  $MCS \le 10$ ), when the third party does nothing or only adopts a punishment mechanism. To the opposite, with the intervention of only reward or punishment and reward, the system tends to be an all-C state. In addition, figure 1b shows the variance of third-party intervention costs in the four scenarios mentioned above. The third-party costs nothing in the default case and can have a little income in the punishment only case. The cost of only reward intervention is the highest, but the positive effect on cooperation is almost identical to the punishment and reward intervention. These results thus propose that, with the same cost, the synergy effect of punishment and reward has the strongest positive impact on the evolution of cooperation.

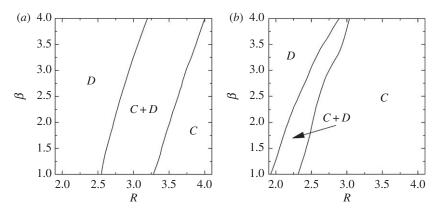
Next, we will further analyse the synergy effect of punishment and reward on the cooperation under different values of the threshold of punishment and reward ( $\beta$  = 1, 2, 3 and 4) with the low- and high-intensity factor ( $\alpha$  = 0.1 and 0.2). As depicted in figure 2, at first glance, we find a general rule that the smaller value of  $\beta$ , the lesser value of R is required for the system to get rid of the all-D state no matter the intensity factor is high or low. Also, it is clear to be observed that the positive influence decreases as  $\beta$  increases. Notably, we heed that  $\rho_c$  is strongly affected by the intensity factor as it shows that  $\rho_c$  has a big boost as  $\alpha$  increases. For example, by fixing R = 2.05 and  $\beta$  = 1, the  $\rho_c$  can be improved from 0 to 83.9% when the value of  $\alpha$  jumps from 0.1 to 0.2. Moreover, as the value of  $\alpha$  increases, the changes in the fractions of cooperation are more sensitive to R (i.e. we fix  $\beta$  = 2, the  $\rho_c$  increases from 0 to 100% in the range R  $\in$  [2.72, 3.50] with  $\alpha$  = 0.1, and in the range R  $\in$  [2.18, 2.51] with  $\alpha$  = 0.2), as we can see in figure 2a,b. The similar conclusions we can draw from scenarios of other different values of L and R, which means that the conclusions mentioned above are robust in populations with different sizes.



**Figure 1.** (*a*) The time evolution of the cooperation level among the populations, and panel (*b*) depicts the cost level of the third party in four different cases including the normal spatial public goods game without punishment or reward (black solid line), only punishment (red dashed line), only reward (blue dotted line), punishment and reward (olive dash-dotted line). The result is acquired with the combination of  $\alpha = 0.1$ ,  $\beta = 2.5$  and R = 3.5. (Online version in colour.)



**Figure 2.** Fractions of cooperation  $(\rho_c)$  as a function of enhancement factor (R) in the stationary state. Four different values of the threshold of reward and punishment  $(\beta)$  are considered in each panel.  $\beta=1$  (black solid line),  $\beta=2$  (red dashed line),  $\beta=3$  (blue dotted line),  $\beta=4$  (olive dash-dotted line). (a) and (b) are obtained by  $\alpha=0.1$  and  $\alpha=0.2$ , respectively. (Online version in colour.)

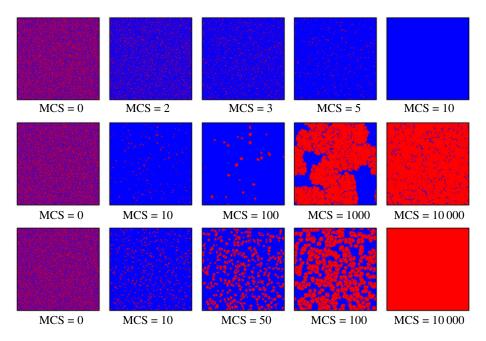


**Figure 3.** (a,b) Full  $R-\beta$  phase diagram obtained for  $\alpha=0.1$  and  $\alpha=0.2$ , respectively. Different phases are denoted by the symbols of the strategies that survive in the final strategy distribution. In detail, C denotes full cooperators, D denotes full defectors and C+D denotes the coexistence of cooperators and defectors. (Online version in colour.)

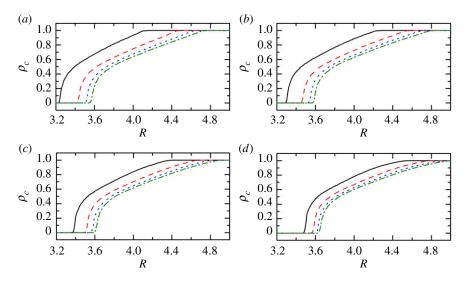
Then we depict the  $R-\beta$  phase diagrams to explore the behaviour of cooperation in dependence on the synergy effect of punishment and reward in figure 3. The area of each panel in figure 3 is evidently divided into three different phases as C, D and C+D, which represent that the system is full of cooperators, defectors and the coexistence of cooperators and defectors among the population, respectively. As a prominent feature, both  $C \leftrightarrow C+D$  and  $C+D \leftrightarrow D$  transition lines are increasing towards the large R limit. The spaces of C+D and D shrink while C phase is widely enlarged with the increase of  $\alpha$  from 0.1 to 0.2. It means that the growth of  $\alpha$  can control the spread of defection. The third party can choose the appropriate  $\beta$  and  $\alpha$  in the scenarios with different R according to this graph.

To further investigate the mechanism of the effect of  $\beta$  on the evolution of cooperation, we show the snapshots of strategy distribution at different time steps in figure 4.  $\beta = 3.2$ ,  $\beta = 2.8$ and  $\beta = 2.0$  are presented from the top to bottom, respectively. The first column represents a random initial state, where cooperators and defectors are randomly arranged across the network. As shown in the first row of figure 4, due to the large value of  $\beta$  ( $\beta = 3.2$ ), the third party has a low tolerance for the poor behaviour of the group. Plenty of defector clusters are formed in a rush during the first five MCS steps. The collaborators are surrounded by defector clusters and eventually disappeared over time. As the decrease of  $\beta$  in the second row of figure 4, the third party has a middle tolerance for the poor behaviour of the group cooperation. The evolution pattern of strategy distribution changes radically, which can be divided into two periods when  $\beta = 2.8$ . Similar to the situations mentioned in the first row, the defector clusters are grown quickly and the cooperators are largely eliminated. However, because of the increase of tolerance, defector clusters could not prevent the formation of cooperator clusters. It is clear that several cooperator clusters have been formed when MCS = 100. Therefore, the evolution direction of the entire system has been changed, and the collaborators dominate the entire network at the steady state. In the third row of figure 4, collaborators also experience a process of rapid decline at first, and then many cooperator clusters are formed. Compared with  $\beta = 2.8$ , the cooperator clusters are obviously more when  $\beta = 2.0$  at MCS = 100. In the end, the defectors fall victim to the invasion of cooperators, and thus cooperators rise to complete dominance. The comparison of different value of  $\beta$  illustrates that the synergy effect of punishment and reward on cooperation is significantly advanced with the decrease of  $\beta$ .

In order to explore the robustness of our conclusions obtained above, we introduce time interval of punishment and reward ( $\tau$  = 2, 3, 4, 5) during the evolution process as represented in figures 5 and 6. In general, we find that  $\tau$  has a noticeable effect on the synergy effect of punishment and reward in different scenarios. Longtime intervals will reduce the synergy effect of punishment and reward on cooperation. For example, from the diagram presented in figure 5a,

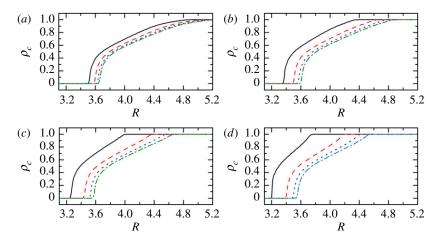


**Figure 4.** Snapshots of typical distributions of cooperators (red) and defectors (blue) with the synergy effect of punishment and reward at different Monte Carlo time. From the top to bottom,  $\beta = 3.2$ ,  $\beta = 2.8$  and  $\beta = 2.0$ , respectively. All the results are obtained with the combination of  $\alpha = 0.2$  and  $\beta = 2.5$ . (Online version in colour.)



**Figure 5.** Fractions of cooperation in the dependence of *R* and different time interval of punishment and reward  $(\tau)$  with the consideration of the different value of  $\beta$ .  $\tau=2$  (black solid line),  $\tau=2$  (red dashed line),  $\tau=3$  (blue dotted line),  $\tau=4$  (olive dash-dotted line). (a-d) are obtained by  $\beta=1$ ,  $\beta=2$ ,  $\beta=3$  and  $\beta=4$  with  $\alpha=0.1$ , respectively. (Online version in colour.)

we can see that the system changes from all-D to C + D when R = 3.23 with  $\tau = 2$ , instead of R = 3.42 with  $\tau = 3$ . On the other hand, the coexistence state ends up with all-C when R = 4.18 with  $\tau = 2$ , instead of R = 4.52 with  $\tau = 3$ . However, the curves become denser as  $\tau$  increases, which means that the differences of the synergy effect of punishment and reward become narrow. Accordingly, the effect of  $\tau$  on the evolution process becomes weak as  $\tau$  increases.



**Figure 6.** Fractions of cooperation in the dependence of *R* and different time interval of punishment and reward  $(\tau)$  with the consideration of different values of intensity factor  $(\alpha)$ .  $\tau=2$  (black solid line),  $\tau=2$  (red dashed line),  $\tau=3$  (blue dotted line),  $\tau=4$  (olive dash-dotted line). (a-d) are obtained by  $\alpha=0.05$ ,  $\alpha=0.10$ ,  $\alpha=0.15$  and  $\alpha=0.20$  with  $\beta=2.8$ , respectively. (Online version in colour.)

Figure 5 supports that our conclusion is robust for the varying  $\beta$  after considering the time interval. The patterns in figure 5a–d are similar, and the uprise of  $\beta$  harms the cooperation regardless of  $\tau$ . In detail,  $\tau$  cannot change the attributes of the effect of  $\beta$  on cooperation but merely influence the magnitude of this effect. Unlike the case of figure 5, it is interesting to note that the curves in figure 6a–d become more and more sparse as  $\alpha$  increases, which means the effect of the time interval on the cooperative process increases as  $\alpha$  increases. From the perspective of  $\alpha$ , it is found that a larger  $\alpha$  contributes to the emergence of cooperation as mentioned above in figure 3. Furthermore, simulations performed on the network of a various number of players reveal that the effect of time interval on cooperation is similar regardless of network size.

## 4. Conclusion

Inspired by the second-order free-riders of the traditional punishment or reward mechanism in the spatial public goods game, we considered the punishment and reward under the same framework by introducing a third party besides cooperator and defector, and constructed a mathematical model based on public goods game. The threshold parameter of punishment and reward  $(\beta)$  is used to control when to punish or reward. At the same time, it can reflect the tolerance of group defection. While a smaller  $\beta$  value has the greater positive influence on cooperation, large  $\beta$  value can also promote cooperation. For example, cooperators can survive only if R > 3.74 and all-C appears for R > 5.49 in the default case, while cooperators survive if R > 3.48 and all-C occurs for R > 4.62 when  $\beta = 4$  with the synergy effect of punishment and reward. On the other hand, the intensity factor ( $\alpha$ ) also has a visible effect on cooperation. The synergy effect becomes more significant as  $\alpha$  increases, which is more conducive to the emergence of cooperation. In addition, we find that the time interval of punishment and reward  $(\tau)$  also influences the cooperative evolution. Essentially, it is easier to promote cooperation when the intervention of punishment and reward is implemented more frequently. We hope these results can help public administrations design more effective policies and inspire more studies towards resolving social dilemmas.

Data accessibility. This article has no additional data.

Authors' contributions. Y.F., T.P.B., M.P., H.X. and Q.T. designed and performed the research as well as writing the paper.

Competing interests. The authors declare no competing interests.

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#### References

- 1. Trivers RL. 1971 The evolution of reciprocal altruism. *Q. Rev. Biol.* 46, 35–57. (doi:10.1086/406755)
- 2. Axelrod R, Hamilton WD. 1981 The evolution of cooperation. *Science* **211**, 1390–1396. (doi:10.1126/science.7466396)
- 3. Henrich J, Boyd R, Bowles S, Camerer C, Fehr E, Gintis H, McElreath R. 2001 In search of homo economicus: behavioral experiments in 15 small-scale societies. *Am. Econ. Rev.* **91**, 73–78. (doi:10.1257/aer.91.2.73)
- 4. Fehr E, Fischbacher U. 2004 Social norms and human cooperation. *Trends Cogn. Sci.* **8**, 784–790. (doi:10.1016/j.tics.2004.02.007)
- 5. Gächter S, Herrmann B, Thöni C. 2010 Culture and cooperation. *Phil. Trans. R. Soc. B* **365**, 2651–2661. (doi:10.1098/rstb.2010.0135)
- 6. Rand DG, Nowak MA. 2013 Human cooperation. *Trends Cogn. Sci.* 17, 413–425. (doi:10.1016/j.tics.2013.06.003)
- 7. Capraro V, Smyth C, Mylona K, Niblo GA. 2014 Benevolent characteristics promote cooperative behaviour among humans. *PLoS ONE* **9**, e102881. (doi:10.1371/journal.pone.0102881)
- 8. Hauser OP, Rand DG, Peysakhovich A, Nowak MA. 2014 Cooperating with the future. *Nature* 511, 220–233. (doi:10.1038/nature13530)
- 9. Perc M, Jordan JJ, Rand DG, Wang Z, Boccaletti S, Szolnoki A. 2017 Statistical physics of human cooperation. *Phys. Rep.* **687**, 1–51. (doi:10.1016/j.physrep.2017.05.004)
- 10. Capraro V, Perc M. 2018 Grand challenges in social physics: in pursuit of moral behavior. *Front. Phys.* **6**, 107. (doi:10.3389/fphy.2018.00107)
- 11. Hardin G. 1968 The tragedy of the commons. *Science* **162**, 1243–1248. (doi:10.1126/science. 162.3859.1243)
- 12. Naudts K, Chen Y, McGrath MJ, Ryder J, Valade A, Otto J, Luyssaert S. 2016 Europe's forest management did not mitigate climate warming. *Science* **351**, 597–600. (doi:10.1126/science.aad7270)
- 13. Burns DA, Aherne J, Gay DA, Lehmann C. 2016 Acid rain and its environmental effects: recent scientific advances. *Atmos. Environ.* **146**, 1–4. (doi:10.1016/j.atmosenv.2016.10.019)
- 14. Grundmann R. 2016 Climate change as a wicked social problem. *Nat. Geosci.* **9**, 562. (doi:10.1038/ngeo2780)
- 15. Van der Knijff J, Jones R, Montanarella L. 2000 Soil erosion risk: assessment in Europe. Joint Research Centre, European Soil Data Centre.
- 16. Chen X, Fu F. 2018 Social learning of prescribing behavior can promote population optimum of antibiotic use. *Front. Phys.* **6**, 193. (doi:10.3389/fphy.2018.00139)
- 17. Chen X, Fu F. 2019 Imperfect vaccine and hysteresis. *Proc. R. Soc. B* **286**, 20182406. (doi:10.1098/rspb.2018.2406)
- 18. Nowak MA. 2006 Five rules for the evolution of cooperation. *Science* 314, 1560–1563. (doi:10.1126/science.1133755)
- Perc M, Gómez-Gardeñes J, Szolnoki A, Moreno YLM. 2013 Evolutionary dynamics of group interactions on structured populations: a review. J. R. Soc. Interface 10, 20120997. (doi:10.1098/rsif.2012.0997)
- 20. Javarone MA. 2018 Statistical physics and computational methods for evolutionary game theory. Berlin, Germany: Springer.
- 21. Li X, Jusup M, Wang Z, Li H, Shi L, Podobnik B, Stanley HE, Havlin S, Boccaletti S. 2018 Punishment diminishes the benefits of network reciprocity in social dilemma experiments. *Proc. Natl Acad. Sci. USA* **115**, 30–35. (doi:10.1073/pnas.1707505115)
- 22. Tanimoto J, Sagara H. 2007 Relationship between dilemma occurrence and the existence of a weakly dominant strategy in a two-player symmetric game. *BioSystems* **90**, 105–114. (doi:10.1016/j.biosystems.2006.07.005)

- 23. Ito H, Tanimoto J. 2018 Scaling the phase-planes of social dilemma strengths shows game-class changes in the five rules governing the evolution of cooperation. *R. Soc. Open Sci.* **5**, 181085. (doi:10.1098/rsos.181085)
- 24. Wang Z, Jusup M, Wang RW, Shi L, Iwasa Y, Moreno Y, Kurths J. 2017 Onymity promotes cooperation in social dilemma experiments. *Sci. Adv.* 3, e1601444. (doi:10.1126/sciadv.1601444)
- 25. Wang Z, Jusup M, Shi L, Lee JH, Iwasa Y, Boccaletti S. 2018 Exploiting a cognitive bias promotes cooperation in social dilemma experiments. *Nat. Commun.* **9**, 2954. (doi:10.1038/s41467-018-05259-5)
- 26. Wang Z, Kokubo S, Jusup M, Tanimoto J. 2015 Universal scaling for the dilemma strength in evolutionary games. *Phys. Life Rev.* 14, 1–30. (doi:10.1016/j.plrev.2015.04.033)
- 27. Perc M, Szolnoki A. 2010 Coevolutionary games—a mini review. *BioSystems* 99, 109–125. (doi:10.1016/j.biosystems.2009.10.003)
- 28. Fotouhi B, Momeni N, Allen B, Nowak MA. 2019 Evolution of cooperation on large networks with community structure. *J. R. Soc. Interface* **16**, 20180677. (doi:10.1098/rsif.2018.0677)
- 29. Allen B, Lippner G, Chen YT, Fotouhi B, Nowak MA, Yau ST. 2017 Evolutionary dynamics on any population structure. *Nature* **544**, 227–230. (doi:10.1038/nature21723)
- 30. Wang Z, Wang L, Szolnoki A, Perc M. 2015 Evolutionary games on multilayer networks: a colloquium. *Eur. Phys. J. B.* 88, 124. (doi:10.1140/epjb/e2015-60270-7)
- 31. Fotouhi B, Momeni N, Allen B, Nowak MA. 2018 Conjoining uncooperative societies facilitates evolution of cooperation. *Nat. Human Behav.* 2, 492. (doi:10.1038/s41562-018-0368-6)
- 32. Tanimoto J. 2013 Difference of reciprocity effect in two coevolutionary models of presumed two-player and multiplayer games. *Phys. Rev. E* **87**, 062136. (doi:10.1103/PhysRevE.87.062136)
- 33. Wang Z, Kokubo S, Tanimoto J, Fukuda E, Shigaki K. 2013 Insight into the so-called spatial reciprocity. *Phys. Rev. E* 88, 042145. (doi:10.1103/PhysRevE.88.042145)
- 34. Kabir KA, Tanimoto J, Wang Z. 2018 Influence of bolstering network reciprocity in the evolutionary spatial Prisoner's Dilemma game: a perspective. *Eur. Phys. J. B* **91**, 312. (doi:10.1140/epjb/e2018-90214-6)
- 35. Nowak MA, Sigmund K. 1992 Tit for tat in heterogeneous populations. *Nature* 355, 250–253. (doi:10.1038/355250a0)
- 36. Amaral MA, Perc M, Wardil L, Szolnoki A, da Silva Júnior EJ, da Silva JK. 2017 Role-separating ordering in social dilemmas controlled by topological frustration. *Phys. Rev. E* **95**, 032307. (doi:10.1103/PhysRevE.95.032307)
- 37. Fu F, Hauert C, Nowak MA, Wang L. 2008 Reputation-based partner choice promotes cooperation in social networks. *Phys. Rev. E* **78**, 026117. (doi:10.1103/PhysRevE.78.026117)
- 38. Milînski M, Semmann D, Krambeck HJ. 2002 Reputation helps to solve the 'tragedy of the commons'. *Nature* 415, 424–426. (doi:10.1038/415424a)
- 39. Wedekind C, Milinski M. 2000 Cooperation through image scoring in humans. *Science* **288**, 850–852. (doi:10.1126/science.288.5467.850)
- 40. Szolnoki A, Perc M. 2014a Defection and extortion as unexpected catalysts of unconditional cooperation in structured populations. *Sci. Rep.* 4, 5496. (doi:10.1038/srep05496)
- 41. Szolnoki A, Perc M. 2014b Evolution of extortion in structured populations. *Phys. Rev. E* 89, 022804. (doi:10.1103/PhysRevE.89.022804)
- 42. Hao D, Rong Z, Zhou T. 2015 Extortion under uncertainty: zero-determinant strategies in noisy games. *Phys. Rev. E* **91**, 052803. (doi:10.1103/PhysRevE.91.052803)
- 43. Rong Z, Wu ZX, Hao D, Chen MZQ, Zhou T. 2015 Diversity of timescale promotes the maintenance of extortioners in a spatial prisoner's dilemma game. *New J. Phys.* 17, 033032. (doi:10.1088/1367-2630/17/3/033032)
- 44. Vainstein MH, Silva ATC, Arenzon JJ. 2007 Does mobility decrease cooperation? *J. Theor. Biol.* **244**, 722–728. (doi:10.1016/j.jtbi.2006.09.012)
- 45. Helbing D, Yu W. 2009 The outbreak of cooperation among success-driven individuals under noisy conditions. *Proc. Natl Acad. Sci. USA* **106**, 3680–3685. (doi:10.1073/pnas.0811503106)
- 46. Buesser P, Tomassini M, Antonioni A. 2013 Opportunistic migration in spatial evolutionary games. *Phys. Rev. E* **88**, 042806. (doi:10.1103/PhysRevE.88.042806)
- 47. Cheng H, Dai Q, Li H, Zhu Y, Zhang M, Yang J. 2011 Payoff-related migration enhances cooperation in the prisoner's dilemma game. *New J. Phys.* 13, 043032. (doi:10.1088/1367-2630/13/4/043032)
- 48. Li Y, Ye H. 2015 Effect of migration based on strategy and cost on the evolution of cooperation. *Chaos Solitons Fractals* **76**, 156–165. (doi:10.1016/j.chaos.2015.04.006)

- 49. Sicardi EA, Fort H, Vainstein MH, Arenzon JJ. 2009 Random mobility and spatial structure often enhance cooperation. *J. Theor. Biol.* 256, 240–246. (doi:10.1016/j.jtbi.2008.09.022)
- 50. Wu T, Wang L, Fu F. 2017 Coevolutionary dynamics of phenotypic diversity and contingent cooperation. *PLoS Comput. Biol.* **13**, e1005363. (doi:10.1371/journal.pcbi.1005363)
- 51. Xia CY, Meloni S, Perc M, Moreno Y. 2015 Dynamic instability of cooperation due to diverse activity patterns in evolutionary social dilemmas. *EPL* **109**, 58002. (doi:10.1209/0295-5075/109/58002)
- 52. Ohdaira T, Terano T. 2011 The diversity in the decision facilitates cooperation in the sequential prisoner's dilemma game. *Adv. Complex Syst.* **14**, 377–401. (doi:10.1142/S0219525911002962)
- 53. Perc M, Szolnoki A. 2008 Social diversity and promotion of cooperation in the spatial prisoner's dilemma game. *Phys. Rev. E* 77, 011904. (doi:10.1103/PhysRevE.77.011904)
- 54. Santos FC, Santos MD, Pacheco JM. 2008 Social diversity promotes the emergence of cooperation in public goods games. *Nature* **454**, 213–216. (doi:10.1038/nature06940)
- 55. Amaral MA, Javarone MA. 2018 Heterogeneous update mechanisms in evolutionary games: mixing innovative and imitative dynamics. *Phys. Rev. E* **97**, 042305. (doi:10.1103/PhysRevE. 97.042305)
- 56. Szolnoki A, Perc M. 2010 Reward and cooperation in the spatial public goods game. *EPL* **92**, 38003. (doi:10.1209/0295-5075/92/38003)
- 57. Sasaki T, Unemi T. 2011 Replicator dynamics in public goods games with reward funds. *J. Theor. Biol.* 287, 109–114. (doi:10.1016/j.jtbi.2011.07.026)
- 58. Szolnoki A, Perc M. 2012 Evolutionary advantages of adaptive rewarding. *New J. Phys.* **14**, 093016. (doi:10.1088/1367-2630/14/9/093016)
- 59. Chen X, Sasaki T, Brännström Å, Dieckmann U. 2014 First carrot, then stick: how the adaptive hybridization of incentives promotes cooperation. *J. R. Soc. Interface* **12**, 20140935. (doi:10.1098/rsif.2014.0935)
- 60. Sasaki T, Uchida S. 2014 Rewards and the evolution of cooperation in public good games. *Biol. Lett.* **10**, 20130903. (doi:10.1098/rsbl.2013.0903)
- 61. Okada I, Yamamoto H, Toriumi F, Sasaki T. 2015 The effect of incentives and meta-incentives on the evolution of cooperation. *PLoS Comput. Biol.* **11**, e1004232. (doi:10.1371/journal. pcbi.1004232)
- 62. Sasaki T, Uchida S, Chen X. 2015 Voluntary rewards mediate the evolution of pool punishment for maintaining public goods in large populations. *Sci. Rep.* **5**, 8917. (doi:10.1038/srep08917)
- 63. Yamagishi T. 1986 The provision of a sanctioning system as a public good. *J. Pers. Soc. Psychol.* **51**, 110–116. (doi:10.1037/0022-3514.51.1.110)
- 64. Fehr E, Gächter S. 2000 Cooperation and punishment in public goods experiments. *Am. Econ. Rev.* **90**, 980–994. (doi:10.1257/aer.90.4.980)
- 65. Gurerk O, Irlenbusch B, Rockenbach B. 2006 The competitive advantage of sanctioning institutions. *Science* **312**, 108–111. (doi:10.1126/science.1123633)
- 66. Rockenbach B, Milinski M. 2006 The efficient interaction of indirect reciprocity and costly punishment. *Nature* 444, 718–723. (doi:10.1038/nature05229)
- 67. Sigmund K. 2007 Punish or perish? Retailation and collaboration among humans. *Trends Ecol. Evol.* 22, 593–600. (doi:10.1016/j.tree.2007.06.012)
- 68. Gächter S, Renner E, Sefton M. 2008 The long-run benefits of punishment. *Science* **322**, 1510. (doi:10.1126/science.1164744)
- 69. Ule A, Schram A, Riedl A, Cason TN. 2009 Indirect punishment and generosity toward strangers. *Science* 326, 1701–1704. (doi:10.1126/science.1178883)
- 70. Wang Z, Xia CY, Meloni S, Zhou CS, Moreno Y. 2013 Impact of social punishment on cooperative behavior in complex networks. *Sci. Rep.* **3**, 3055. (doi:10.1038/srep03055)
- 71. Jordan JJ, Hoffman M, Bloom P, Rand DG. 2016 Third-party punishment as a costly signal of trustworthiness. *Nature* **530**, 473–476. (doi:10.1038/nature16981)
- 72. Panchanathan K, Boyd R. 2004 Indirect reciprocity can stabilize cooperation without the second-order free rider problem. *Nature* 432, 499–502. (doi:10.1038/nature02978)
- 73. Helbing D, Szolnoki Å, Perc M, Szabó G. 2010 Evolutionary establishment of moral and double moral standards through spatial interactions. *PLoS Comput. Biol.* **6**, e1000758. (doi:10.1371/journal.pcbi.1000758)
- 74. Hilbe C, Traulsen A. 2012 Emergence of responsible sanctions without second order free riders, antisocial punishment or spite. *Sci. Rep.* **2**, 458. (doi:10.1038/srep00458)
- 75. Chen X, Szolnoki A, Perc M. 2014 Probabilistic sharing solves the problem of costly punishment. *New J. Phys.* **16**, 083016. (doi:10.1088/1367-2630/16/8/083016)

- 76. dos Santos M. 2015 The evolution of anti-social rewarding and its countermeasures in public goods games. *Proc. R. Soc. B* **282**, 20141994. (doi:10.1098/rspb.2014.1994)
- 77. Szolnoki A, Perc M. 2015 Antisocial pool rewarding does not deter public cooperation. *Proc. R. Soc. B* **282**, 20151975. (doi:10.1098/rspb.2015.1975)
- 78. Szolnoki A, Perc M. 2017 Second-order free-riding on antisocial punishment restores the effectiveness of prosocial punishment. *Phys. Rev. X* 7, 041027. (doi:10.1103/physrevx.7.041027)
- 79. Perc M, Szolnoki A. 2012 Self-organization of punishment in structured populations. *New J. Phys.* **14**, 043013. (doi:10.1088/1367-2630/14/4/043013)
- 80. Szolnoki A, Perc M. 2010 Impact of critical mass on the evolution of cooperation in spatial public goods games. *Phys. Rev. E* 81, 057101. (doi:10.1103/PhysRevE.81.057101)
- 81. Szolnoki A, Chen X. 2016 Cooperation driven by success-driven group formation. *Phys. Rev. E* **94**, 042311. (doi:10.1103/PhysRevE.94.042311)